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Modeling and Stabilization of Nonlinear Systems Using PDC Approach

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Abstract: This paper studies and synthesis the parallel distribution compensation PDC controller for discrete-time nonlinear systems that which is represented by Takagi-Seguno (T-S) fuzzy models. This work is based on two steps: the first one is modeling and identification of the studied systems by using the clustering method. In particular, we use Fuzzy C-Means (FCM) algorithm. In second one, we interest in the first part to the stability analysis of multi-model by Lyapunov functions using convex optimization algorithms at the linear matrix inequality LMI and in the second part, we focus on synthesis of PDC control law. Finally, the study methodology is implemented on an inverted pendulum system.

Key words: Fuzzy clustering · Fuzzy C-Means · T-S fuzzy model · Fuzzy control · Linear Matrix Inequality (LMI) · Quadratic stability

INTRODUCTION

The diversity of problems in automatics, especially in control theory, has evolved considerably during the last decades. A substantial amount of research has focused on automatic control problems for nonlinear systems. This is due to the importance of the control theory applied to complex systems. However, before addressing the control problem, a large interest is devoted to modeling and identification, which reflects the dynamics of studied systems. For this reason, several researches have focused on the modeling and control of nonlinear systems. In [1-4], the authors are interested on some particular classes of nonlinear models. Other attempts were geared toward large systems [5-6]. Indeed, the difficulty of stability analysis and controller synthesis is related to the complexity of considered model. Hence, it's necessary to think of simpler models. In this context, several works modeling the non linear system used the set of linear models in recent years. Then the obtained models have merged using fuzzy rules which lead to the T-S fuzzy system [7-9]. This technique offers the possibility of designing partial control laws such that each one is associated with a local model. The global control law is, then, deduced by a suitable fusion method. Popular approaches exist in the literature to identify the parameters involved in the T-S model, mainly neuro-fuzzy technology [10] neural networks [11] and the clustering techniques [12-16].

Fuzzy clustering is an important problem which is the subject of active research in several real world applications.

In particular, the method of clustering is viewed as an interesting technique for modeling and identification of nonlinear systems. In this work, we used the clustering techniques. In this context, several clustering algorithms have been proposed in the research in order to estimate the parameters of the T-S fuzzy model. We can cite for example the Gustafson Kessel algorithm (GK) [17] the (GG) algorithm [18] and the Fuzzy C-Means algorithm (FCM) [19-20], In this paper we are interested only in the FCM algorithm. Fuzzy c-means (FCM) algorithm is one of the most popular fuzzy clustering techniques because it is efficient, straightforward and easy to implement.

The control theory applied to complex systems is the most important issue in the field of automation. In the last years, the multi-model approach attracted the attention of the automation engineers' community and various problems were treated. Mainly, those related to stability analysis and controller synthesis. Numerous works concerning the stability of the multi-modal have been carried out by using control techniques. The most

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important parts of these works are based on the Lyapunov approach. Indeed, in [21-22] the stability analysis by Lyapunov quadratic function has been studied. In [23-24] a non-quadratic function was used. In other attempts like [25], the property of M matrices is used to establish the stability of multi-model systems. Other works use the geometric approach and the Popov criterion [23] to develop stability criteria.

The problem of controller synthesis of multi-model systems is also addressed in the literature. The synthesis of the controller is handled by the uncertain systems techniques [26]. In [27] the sliding mode technique is used while in [28] the techniques of interconnected systems are employed and adaptive controller laws are designed in [29]. However, several methods are still unable to give the desired performances. This paper only concerns the stability analysis and synthesis of control laws for multi-models. Our approach is exclusively based on the second method of Lyapunov and its LMI formulation.

This present work is organized as follows:In the following section, we develop a brief review of TS model fuzzy clustering. The stability analysis of multi model approach is introduced in section 3. Section 4 is devoted to the synthesis of a control law which is the parallel distributed compensation PDC. The simulations results and validation model are presented in section 5. The last section is reserved to conclude our work.

Takagi Sugeno Fuzzy Model: Traditional methods of modeling and identification used to estimate the parameters of such process can not satisfy the desired performance. However, other techniques such as T-S fuzzy model showed a very good result for modeling and identification. The T-S fuzzy model consists of several fuzzy if-then rules that can be represented as follows:

$$\begin{aligned} R_i &= if\ x_{k1}\ \ is\ \ A_{i1}\ \ and \cdots and\ x_{kn}\ is\ A_{in} \\ &\quad then\ y_i = a_i^T\ x_k + b_i \end{aligned} \tag{1}$$

The " if " rule function defines the premise part, while the " then" rule function constitutes the consequent part of the T-S fuzzy model.

where i = 1,...,c, (c is the number of fuzzy rules), $a_i \in \Re^n, b_i \in \Re$: are the polynomial coefficients that form the consequent parameters of the ith rules, $x_k = [x_{k1},..,x_{kn}]^T \in \Re^n$ is the input vector of the fuzzy model and $A_{i1}, A_{i2},\cdots, A_{in}$ are multidimensional antecedents of the fuzzy rules. Finally, the global estimated output is calculated by a weighting of the others output of local models according to the expression:

$$\hat{y}(k) = \sum_{i=1}^{c} \mu_{ik}.y_i(k) \tag{2}$$

where

$$\mu_i(k) = \frac{\prod_{j=1}^{n} w_{A_{ij}}(x_j)}{\sum_{i=1}^{c}\sum_{j=1}^{n} w_{ij}(x_j)} \tag{3}$$

where

$w_{Aij}(x_j)$ is the membership function of the fuzzy set A_{ij} in the antecedent of $R_i.u_i(k)$ are weighting function that ensures the transition between sub-models and have the following properties:

$$\begin{aligned} &\sum_{i=1}^{c} \mu_i(k) = 1\ \ \forall k \\ &0 \le \mu_i(k) \le 1\ \forall i = 1\ldots c\ \ \forall k \end{aligned} \tag{4}$$

To identify the parameters involved in the model, we construct the regression matrix and the output vector from the available data of the system.

where

$$X = [x_1^T, x_2^T, \ldots, x_N^T]\ ,\ y = [y_1^T, y_2^T, \ldots, y_N^T] \tag{5}$$

The goal of the identification is to construct the unknown nonlinear function y = F(X) from the data, where F is the TS model (1) [31]. The consequent parameters for each rule can be obtained using the weighted recursive least squares method (WRLS). The WRLS algorithm is described as follows:

$$\theta(k) = \theta(k-1) + G(k)[y(k) - x_e^T(k)\theta(k-1)]y(k) \tag{6}$$

$$G(k) = \frac{P(k-1)x_e^T(k)}{\frac{1}{\mu(k)} + x_e^T(k)P(k-1)x_e(k)} \tag{7}$$

$$P(k) = \frac{1}{\lambda}\left[P(k-1) - \frac{P(k-1)x_e^T(k)x_e(k)P(k-1)}{\frac{1}{\mu(k)} + x_e^T(k)P(k-1)x_e(k)}\right] \tag{8}$$

where x_e = [X,1] is the matrix augmented regression, P(k - 1) is a covariance matrix and G(k) referred to the estimator gain vector. A common choice of initial value is to take $\theta_i(0) = 0$ and $P_i(0) = \alpha I$, where α is a large number.

Fuzzy Clustering Algorithms: The Fuzzy clustering algorithms are used to obtain a T-S model. To approximate nonlinear function by c linear functions, these algorithms are intended to minimize the objective function expressed by [30].

$$J = \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m \cdot d_{ik}{}^2 \tag{9}$$

where

$$d_{ik}{}^2 = \|x_k - v_i\|^2 = (x_k - v_i)^T (x_k - v_i) \tag{10}$$

d_{ik} is the distance between the observation x_k and the cluster center v_i. m is a weighting exponent which affects the cluster shape. In fact, when m is close to 1, the membership function becomes almost boolean and when m is greater, the membership function will be fuzzy (μ_{ik} proportional to 1/c). Based on [30], the weighting exponent is chosen between 1.5 and 2.5. In addition, other authors chose it between 2 and 4. In this study, we will be focused on the Fuzzy C-Means algorithm (FCM).

Fuzzy C-Means Algorithm: The Fuzzy C-Means algorithm FCM proposed by Bezdek is the most widely used of fuzzy clustering algorithms. The aim of this algorithm is to minimize the criterion given by (9) which has been improved by adding the normalization constraint and the new criterion expressed by:

$$J = \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m \cdot d_{ik}{}^2 - \lambda(\sum_{i=1}^{c}\mu_{ik} - 1) \tag{11}$$

where
λ is the Lagrange multiplier.

After the minimization of (11) by canceling the derivative of J with respect to λ, μ_{ik} and v_i.

$$\begin{cases} \dfrac{\partial J}{\partial \lambda} = 0 \\ \dfrac{\partial J}{\partial \mu_{ik}} = 0 \\ \dfrac{\partial J}{\partial v_i} = 0 \end{cases} \tag{12}$$

we obtain the following expressions

$$v_i = \frac{\sum_{k=1}^{N}(\mu_{ik})^m . x_k}{\sum_{k=1}^{N}(\mu_{ik})^m} \tag{13}$$

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{c}(\frac{d_{ik}}{d_{jk}})^{\frac{2}{m-1}}} \tag{14}$$

Fuzzy C-Means Steps: Initialize the fuzzy partition matrix randomly

Given a data set ($x(k)$, $y(k)$), the algorithm is described by the following steps.

Step1: Initialize 1 = 0 (l is the number of iterations).

- ✓ Choose the number of clusters c.
- ✓ Choose the weighting exponent m.
- ✓ Choose stop criterion ε.
- ✓ Initialize the fuzzy partition matrix U randomly

Step 2: Calculate the cluster centers v_i

Step 3: Calculate the distances d_{ik} for each cluster.

Step 4: Update the fuzzy partition matrix.

Step 5: If $\left\|U^{(l)} - U^{(l-1)}\right\| > \varepsilon$ return to step 2 else stop.

Stability Analysis: A continuous T-S [32], model is based on the interpolation between several LTI local models as follows:

$$\dot{x}(t) = \sum_{i=1}^{c}\mu_i\left[A_i x(t) + B_i u(t)\right] \tag{15}$$

where c is the number of submodels, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input vector, $A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times n}$ Basic stability conditions based on the quadratic Lyapunov functions are given by the following result. The continuous T-S model described by (15) is globally asymptotically stable if there exists a common matrix.

$$\begin{aligned} &P = P^T \succ 0 \\ &A_i^T P + PA_i < 0; i = 1,...,c \end{aligned} \tag{16}$$

The existence of a defined symmetric positive matrix depends on two conditions P. The first is related to the stability of all local models (each matrix A_i is of Hurwitz). And the second requires that the sum $\sum_{i=1}^{c} A_i$ is of Hurwitz also.

T-S Controller Design: In order to stabilize the T-S model (15), a T-S controller can be designed using the PDC technique [33]. In this case, the global control law is obtained by interpolation of local linear feedback laws related with each submodel. For the T-S controller design, it is supposed that the system (15) is locally stabilisable, i.e. the pairs $(A_i, B_i), \forall i \in \{1,...,c\}$ are stabilisable. The resulting global controller when all decision variables are measurable is:

$$u(t) = -\sum_{i=1}^{c} \mu_i K_i x(t) \tag{17}$$

where μ_i has to respect constraint (4). Substituting (17) in (15), we obtain the closed loop continuous T-S model:

$$\dot{x}(t) = \sum_{i=1}^{c}\sum_{j=1}^{c} \mu_i \mu_j G_{ij} x(t) \tag{18}$$

where

$$G_{ij} = A_i - B_i K_j \tag{19}$$

The multi-model described by (15) is globally asymptotically stabilized via the PDC control law (17), if there exists a symmetric and positive definite matrix P and Q_i matrices [34].

Theorem 1: Suppose that there exists symmetric positive definite matrices $P > 0$, Q_{ij} and matrices K_i, i = 1,...,c such that.

$$£_c(G_{ii}, P) + Q_{ii} < 0 \forall i = 1,...,c \tag{20}$$

$$£_c(G_{ij}, P) + Q_{ij} \le 0 \forall (i,j) = 1,...,c, i < j \tag{21}$$

$$\begin{pmatrix} Q_{11} & \cdots & Q_{1L} \\ \vdots & \ddots & \vdots \\ Q_{1L} & \cdots & Q_{LL} \end{pmatrix} > 0 \tag{22}$$

where $\mu_i \mu_j \ne 0, G_{ij} = A_i - B_i K_j$ and

$$£_c(G_{ii}, P) = (\frac{G_{ij} + G_{ji}}{2})^T P + P(\frac{G_{ij} + G_{ji}}{2}) \cdot$$

Then the multi model (18) is globally asymptotically stable. The control design problem is to find the feedback gains (K_i) such that the closed loop system (18) is stable. The conditions (20) - (22) are not convex in P and K_i. In order to convert them into an LMI problem, these inequalities are multiplied in the left and the right by P^{-1}. Then, taking into account the definition (19), the constraints (20) - (22) become:

$$X > 0 \tag{23}$$

$$XA_i^T + A_i X - N_i^T B_i^T - B_i N_i + Y_{ii} < 0, \forall i = 1,...,c \tag{24}$$

$$XA_i^T + A_i X + XA_j^T + A_j X - N_i^T B_i^T - B_i N_i - N_i^T B_j^T - B_j N_i + 2Y_{ij} \le 0, \forall (i,j) = 1,...,c, i < j \tag{25}$$

$$\begin{pmatrix} Y_{11} & \cdots & Y_{1c} \\ \vdots & \ddots & \vdots \\ Y1c & \cdots & Y_{cc} \end{pmatrix} > 0 \tag{26}$$

where
$Y_{ij} = X Q_{ij} X$ and $K_i = N_i X^{-1}, \forall i = 1,...,c$

Example
System Description:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{g \sin(x_1) - a\, m_1\, l\, x_2^2 \sin(2x_1)/2 - a \cos(x_1)\, u}{4l/3 - a\, m_1\, l \cos^2(x_1)} \end{cases} \tag{27}$$

where x_i is the pendulum angle (in radians) from the vertical, x_2 is the angular velocity, g = 9.8 m/s^2 is the constant gravity, m_l is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum and u is the force applied to the cart (Newton). a = 1/(m1 + M) where m1=0.1 Kg, M =1 Kg and 21 = 1.0 m.

Results of System Identification: In this section, we analyze the identification results of the inverted pendulum system. The variables $y(k)$ and $u(k)$ are output and input data, respectively. We choose y(k-1), y(k-2), u(k-1), as the variables of the fuzzy model (regression vector). Also, the value of weighting exponent m is set at 2.5. The sequences of input and output signal used for the identification process are shown in Figure 1. Figure 2

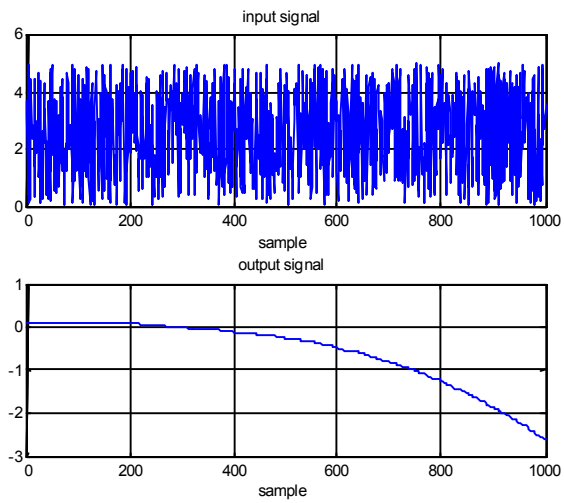

Fig. 1: Input and output signal

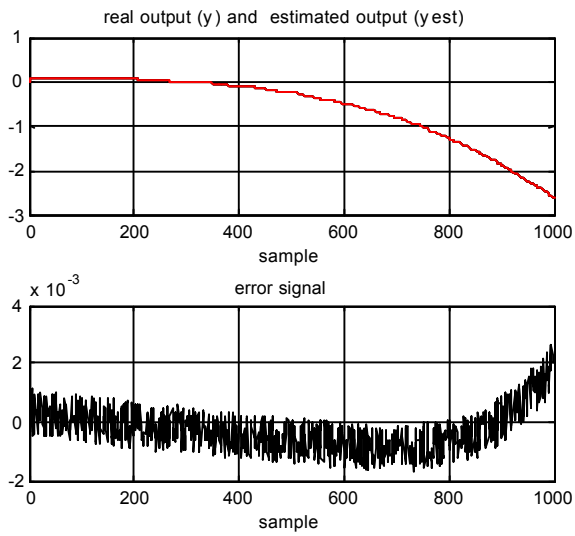

Fig. 2: Simulation results of FCM algorithm

Table I: validation results of cluster number

	C=2	C=3	C=4	C=5
$C_{EC}(10^{-4})$	-3.3657	-3.1620	-3.4657	-3.2189

Table II: results of model validation

	RMSE	VAF
FCM	0.0012	99.99

shows the real output superposed with the estimated output and the signal error generated by the difference between the real and estimated outputs. The obtained results show that the proposed FCM algorithm provides a good approximation modeling accuracy.

It can be seen from both Figure 2, the good identification of the system and it shows that the fuzzy model is very adequate.

Model Validation: The validation of fuzzy Takagi-Sugeno model can be obtained using the following criteria:

- Entropy classification:

In order to validate the cluster number a several criterion was used in the literature quoting as example Entropy classification defined by the following equation [34].

$$C_{EC}(c) = \frac{1}{N}\sum_{k=1}^{N}\sum_{i=1}^{C}\mu_{ik}\log(\mu_{ik}) \quad \forall\, 1 \le i \le c,\ 1 \le K \le N \tag{28}$$

The rule of using this criterion is as follows:

$$C^* = \min_{C=2,\ldots.N-1}\left[C_{EC}(C)\right]$$

- Root Mean Square Error (RMSE):

This test calculates the mean squared error between the measured and model output.

$$RMSE = \frac{1}{N}\sqrt{\sum_{k=1}^{N}(y_k - \hat{y}_k)^2} \tag{29}$$

where

N : Number of observations.
Y_k : Real output.
$\hat{y}_k$: Estimated by the model output.

When the value of RMSE tends to zero the two outputs are combined.

- Variance Accounting For (VAF)

$$VAF = 100\%.\left[1 - \frac{\mathrm{var}(y_k - \hat{y}_k)}{\mathrm{var}(y_k)}\right] \tag{30}$$

If the value of VAF is around 100% then the model will be validated [25]. The simulation results of the validation tests used (Table II) have shown good performance of this algorithm.

In this section, we use another type of tests, the statistical tests, to validate a fuzzy model. Firstly, we present the residues autocorrelation function. Secondly, we are interested in the cross-correlation test between residues and in-put system. These tests are mentioned in the relation (31) and (32).

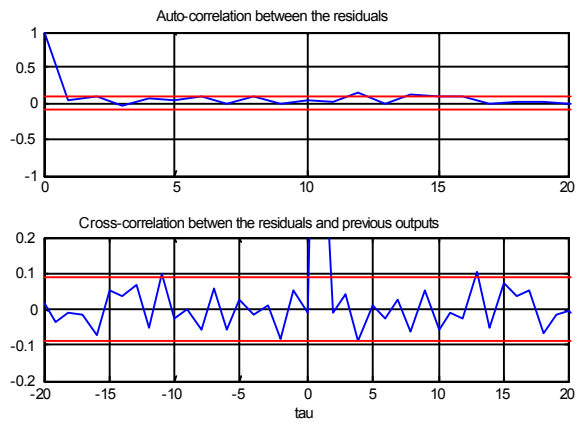

Fig. 3: Correlation test results

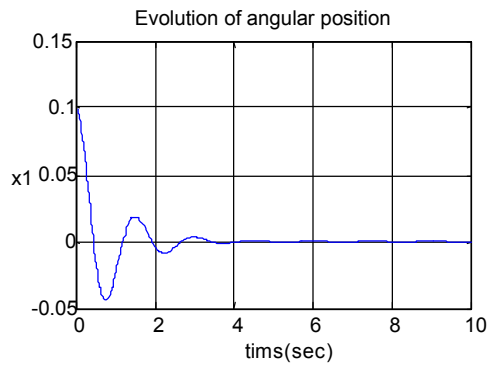

Fig. 4: Evolution of angular position

- Residue auto-correlation function:

$$\hat{r}_{\varepsilon\varepsilon}(\tau)=\frac{\sum_{k=1}^{N-\tau}\left(\varepsilon(k,\hat{\theta})-\bar{\varepsilon}\right)\left(\varepsilon(k-\tau,\hat{\theta})-\bar{\varepsilon}\right)}{\sum_{k=1}^{N}\left(\varepsilon(k,\hat{\theta})-\bar{\varepsilon}\right)^2} \tag{31}$$

- Cross-correlation function between residues and previous input:

$$\hat{r}_{u\varepsilon}(\tau)=\frac{\sum_{k=1}^{N-\tau}\left(u(k)-\bar{u}\right)\left(\varepsilon(k-\tau,\hat{\theta})-\bar{\varepsilon}\right)}{\sqrt{\sum_{k=1}^{N}\left(u(k)-\bar{u}\right)^2}\sqrt{\sum_{k=1}^{N}\left(\varepsilon(k,\hat{\theta})-\bar{\varepsilon}\right)^2}} \tag{32}$$

ε : Is the prediction error and u is the system input. The validity of model requires the following results:

$$\hat{r}_{\varepsilon\varepsilon}=\begin{cases}1,\tau=0\\0,\tau\neq 0\end{cases}\text{and } \hat{r}_{u\varepsilon}(\tau)=0,\forall\tau \tag{33}$$

In general, the correlation functions are zero when is the interval [-20, 20] with a confidence interval of 95%, i.e

$$\frac{-1.96}{\sqrt{N}}<\hat{r}<\frac{1.96}{\sqrt{N}}$$

Figure 3 shows the correlation-based model validation test results for the FCM algorithm.

It can be seen from Figure 3 that practically all couple of points is inside the 95% confidence bounds and hence, the model can be regarded as being adequate for long-term predictions.

Results Synthesis of Control Laws: In this section, we are going to examine the performance of the proposed control developed above. Using the FCM algorithm and the weighted recursive least squares (WRLS) method, the local models are obtained as follows:

$$A_1=\begin{bmatrix}-1.9822 & -0.9822\\1 & 0\end{bmatrix}\quad A_2=\begin{bmatrix}-1.9817 & -0.9817\\1 & 0\end{bmatrix}$$

$$A_3=\begin{bmatrix}-1.9949 & -0.9949\\1 & 0\end{bmatrix}\quad A_4=\begin{bmatrix}-1.9965 & -0.9965\\1 & 0\end{bmatrix}$$

$$B_1=B_2=B_3=\begin{bmatrix}0.0313\\0\end{bmatrix}$$

$$B_4=0.0156$$

$$C_1=[-0.0260\quad 0.0129]\quad C_2=[-0.0155\quad 0.0077]$$

$$C_3=[-0.0119\quad 0.0060]\quad C_4=[-0.0203\quad 0.0101]$$

From conditions (23),(24),(25) and (26) defined above and with definition (20) we obtain the following feedback gains and the symmetric and positive definite matrices.

- The following gains are obtained:

$$K_1=[109.7853\quad 11.2950]$$
$$K_2=[114.8096\quad 12.5032]$$
$$K_3=[128.0720\quad 17.8235]$$
$$K_4=[282.8335\quad 54.9130]$$

- The symmetric and positive definite matrice.

$$P=\begin{bmatrix}12.6980 & -6.3277\\-6.3277 & 15.1351\end{bmatrix}$$

The simulation results relative to the evolution of state vector and control law are shown in Figures 4, 5 and 6.

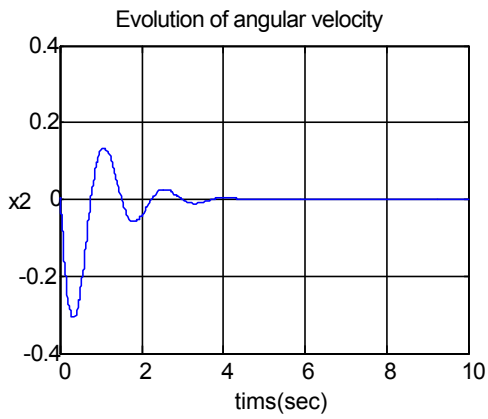

Fig. 5: Evolution of angular velocity

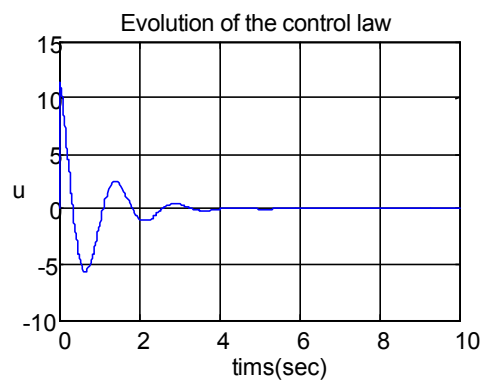

Fig. 6: Evolution of the control law

Figure 4 shown the angular position controlled with an initial conditions x1 (0)= 0.1 and x2 (0)=0 for the system, Figure 5 illustrated the velocity signal and the force control signal was given by Figure 6. After the simulation results, we can affirm that state feedback of multi model control allow a good stability of our system.

CONCLUSION

In this paper, we addressed the problem modeling of T-S fuzzy systems and the stability analysis of such models. To satisfy stability conditions, we used a quadratic Lyapunov function formulated as a set of LMI. Then, a state feedback controller based on PDC approach is designed to stabilize T-S models. Finally, simulation results proved that states of the studied processes have been stabilized with satisfactory performance which showed the effectiveness and the validity of the proposed synthesis approach.

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