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Elzaki Transform of Derivative Expressed by Heaviside Function

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Abstract: Lzaki transform, whose fundamental properties are presented in this paper, is still not widely known, nor used, ELzaki transform may be used to solve problems without resorting to a new frequency domain. In this article, we will study ELzaki transform of derivative expressed by Heaviside function. Related to this topic, the proposed idea can be also applied to other transforms.

Key words: ELzaki transform of derivative · Integral transform · Heaviside function

INTRODUCTION

Most of the available transform theory books, if not all, do not refer to ELzaki transform. On the other hand, for historical a accountability, we must note that a related formulation, called s-multiplied Laplace transforms, was announced as early as 1948 if not before, Tarig M. ELzaki in some papers [1-12], showed ELzaki transform applications to partial differential equations, ordinary differential equations, system of ordinary and partial differential equations and integral equations.

In [5] Elzaki transform solve differential equations with variable coefficients which were not solved by Sumudu transform and Laplace transform; this means that Sumudu and Laplace transforms failed to solve these types of differential equations.

Also Tarig M. ELzaki showed that ELzaki transform can be effectively used to solve ordinary differential equations [1] and engineering control problems. M. Elzaki extended this transform method to variables with emphasis on Solutions to partial differential equations.

The method of integral transform is usually considered a valuable tool to deal with problems concerning integral equations. In this paper, we handle the problem by ELzaki transform derivative expressed by Heaviside function. And this plays a role to solve directly initial value problems without first determining a general solution and non-homogeneous ordinary differential equations without first solving the corresponding homogeneous.

Elzaki transform of derivatives have been researched in many ways to solve differential equations. The principal contents are;

$$E\left[f'\right]=\frac{T(s)}{s}-sf(0);$$

$$E\left[f''\right]=\frac{T(s)}{s^2}-f(0)-sf'(0);$$

For ELzaki transform of the first and second derivatives of $f(t)$.

In this work, we would like to propose the new approach of $E(f')$ by changing the choice of function of differential form in integration by parts. The obtained result is $E(f')$ can be represented by an infinite series or Heaviside function.

Preliminaries

Definition 2.1: If $f(t)$ is function defined for all $t \geq 0$, its Elzaki transform is the integral of $f(t)$ times $e^{-\frac{t}{s}}$ form $t = 0$ to ∞. It is a function of s and is defined by $E[f]$;

Thus;

$$E\left[f(\mathrm{t})\right]=T(s)=s\int_0^\infty f(t)e^{-\frac{t}{s}}dt$$

Provided the integral of $f(t)$ exists [1].

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Definition 2.2: A function $f(t)$ has ELzaki transform if it satisfies the growth restriction

$$|f(t)| < Me^{\frac{|t|}{k}}$$

For all $t \geq 0$ and some constant M and k..

Main Results: We would like to propose $E[y']$ can be represented as an infinite series of s^k by changing the choice of function of differential form in integration by parts and deal with the expression of Heaviside function of it.

Theorem 3.1: ELzaki transform of the first derivative of y(t) satisfies;

$$E[y'] = \sum_{k=1}^{m} s^{(k+1)} y^k(0) + s^m E\left(y^{(m+1)}\right)$$

For $y^{(k)}$ is the k–th derivative of a given function $y(t)$. As, $m \to \infty$,we have;

$$E(y') = \sum_{k=1}^{m} s^{k+1} y^{(k)}(0)$$

The above formula holds if $y(t)$ and $y'(t)$ are continuous for all $t \geq 0$ and satisfies the growth restriction.

Proof: We would like to establish the statement by the mathematical induction. For, $k = 1$, by integration by parts,

$$\begin{aligned} E(y') &= \int_0^\infty e^{\frac{-t}{s}} y'(t)dt \\ &= s\left[\left(-se^{\frac{-t}{s}} y'(t)\right)_0^\infty + s\int_0^\infty e^{\frac{-t}{s}} y''(t)dt\right] \\ &= s\left[sy'(0) + E(y'')\right] = s^2 y'(0) + sE(y'') \end{aligned} \tag{1}$$

In the general case. Next, we suppose that,

$$E[y'] = \sum_{k=1}^{m} s^{(k+1)} y^k(0) + s^m E\left(y^{(m+1)}\right) \tag{2}$$

And show that $E(y')$ can be expressed by;

$$E(y') = \sum_{k=1}^{m+1} s^k y^k(0) + s^{m+1} E(y^{m+2})$$

Substituting this equation in (2), we have;

$$\begin{aligned} E(y^{m+1}) &= \int_0^\infty e^{\frac{-t}{s}} y^{(m+1)}(t)dt \\ &= s\left[\left(-se^{\frac{-t}{s}} y^{(m+1)}(t)\right)_0^\infty + s\int_0^\infty e^{\frac{-t}{s}} y^{(m+2)}(t)dt\right] \\ &= s\left[sy^{(m+1)}(0) + E(y^{(m+2)})\right] = s^2 y^{(m+1)}(0) + sE(y^{(m+2)}) \end{aligned}$$

Hence from (2), we get;

$$\begin{aligned} E(y') &= \sum_{k=1}^{m} s^{k+1} y^k(0) + s^m\left(s^2 y^{m+1}(0) + sE(y^{m+2})\right) \\ &= \sum_{k=1}^{m+1} s^{k+1} y^k(0) + s^{m+1} E(y^{m+2}) \end{aligned}$$

Thus, if the equality holds for k, it holds for $k + 1$. Therefore, by mathematical induction, the equality is true for all natural number n.

Theorem 3.2:

$$E(y') = se^{\frac{-n}{s}} y(n) - sy(0) + \int_0^n e^{\frac{-t}{s}} y(t)dt + sE\left[y'(t)u(t-n)\right].$$

For all n and for u is the unit step function.

Proof: We would like to verify by mathematical induction. IF $n = 1$

$$\begin{aligned} E(y') &= s\int_0^\infty e^{\frac{-t}{s}} y'(t)dt \\ &= s\left[\left(-se^{\frac{-t}{s}} y(t)\right)_0^1 + \frac{1}{s}\int_0^1 e^{\frac{-t}{s}} y(t)dt + \int_0^\infty e^{\frac{-t}{s}} y'(t)dt\right] \\ &= se^{-\frac{1}{s}} y(1) - sy(0) + \int_0^1 e^{-\frac{t}{s}} y(t)dt + sE\left[y'(t)u(t-1)\right] \end{aligned}$$

Next, we assume that the equality holds for $n = k$ i.e.,

$$E(y') = se^{-\frac{k}{s}} y(k) - sy(0) + \int_0^k e^{-\frac{t}{s}} y(t)dt + sE\left[y'(t)u(t-k)\right]. \tag{3}$$

Let us we show that;

$$= se^{-\frac{(k+1)}{s}} y(k+1) - sy(0) + \int_0^{k+1} e^{-\frac{t}{s}} y(t)dt + sE\left[y'(t)u(t-k-1)\right].$$

From (3);

$$E(y') = se^{-\frac{k}{s}} y(k) - sy(0) + \int_0^k e^{-\frac{t}{s}} y(t)dt + \int_k^\infty e^{-\frac{t}{s}} y'(t)dt. \tag{4}$$

Here;

$$\int_k^\infty e^{-\frac{t}{s}} y'(t)dt = \int_k^{k+1} e^{-\frac{t}{s}} y'(t)dt + \int_{k+1}^\infty e^{-\frac{t}{s}} y'(t)dt$$

$$= \left(e^{\frac{-t}{s}} y(t) \right)_k^{k+1} + \frac{1}{s} \int_k^{k+1} e^{-\frac{t}{s}} y(t)dt + E\left[y'(t)u(t-k-1)\right];$$

$$= e^{-\frac{(k+1)}{s}} y(k+1) - e^{-\frac{k}{s}} y(k) + \frac{1}{s} \int_k^{k+1} e^{-\frac{t}{s}} y(t)dt + E\left[y'(t)u(t-k-1)\right] \tag{5}$$

Substituting (5) into (4), to find;

$$E(y') = se^{-\frac{k}{s}} y(k) - sy(0) + \int_0^k e^{-\frac{t}{s}} y(t)dt + se^{-\frac{(k+1)}{s}} y(k+1) - se^{-\frac{k}{s}} y(k)$$

$$+ \int_k^{k+1} e^{-\frac{t}{s}} y(t)dt + sE\left[y'(t)u(t-k-1)\right].$$

$$= se^{-\frac{(k+1)}{s}} y(k+1) - sy(0) + \int_0^{k+1} e^{-\frac{t}{s}} y(t)dt + sE\left[y'(t)u(t-k-1)\right].$$

The validity of the equality for all natural number n follows by mathematical induction.
It is clear that theorem 3.2 be rewritten by

$$E(y') = se^{-\frac{n}{s}} y(n) - sy(0) + \int_0^n e^{-\frac{t}{s}} y(t)dt; \text{ For } t < n.$$

CONCLUSION

In this paper, we introduce Elzaki transform for derivative expressed by Heaviside function. The proposed method is successfully implemented by using this interesting transform, then we conclude that Elzaki transform considered as a nice refinement in existing numerical techniques.

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