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Application of the exp (-Φ(η))-expansion Method to Find Exact Solutions for the Solitary Wave Equation in an Unmagnatized Dusty Plasma

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Abstract: In this paper, we implement the exp (-Φ(η))-expansion method to construct exact traveling wave solutions of the solitary wave equation in an unmagnatized dusty plasma and find out the approximate solution of the electrostatic wave potential equation. The procedure is simple, direct and constructive without the help of a computer algebra system. The obtained results show that the exp (-Φ(η))-expansion method is straightforward and effective mathematical tool for nonlinear evolution equations in mathematical physics and engineering.

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INTRODUCTION

A solitary structure is a hump or dip shaped nonlinear wave of permanent profile. To distinguish it from a soliton, we note that a soliton is a special type of solitary waves which preserve their shape and speed after interaction. It arises because of the balance between the effects of the nonlinearity and the dispersion (when the effect of dissipation is negligible in comparison with those of the nonlinearity and dispersion). However, when the dissipative effects are comparable to or more dominant than the dispersive effects, one encounters shock waves. The small but finite amplitude solitary waves are governed by a KdV type equation, while the shock waves are described by a KdV-Burgers type equation. In the recent years, the exact traveling wave solutions of nonlinear partial differential equations have been investigated by many authors who are interested in nonlinear phenomena which exist in all fields including either the scientific works or engineering fields, such as fluid mechanics, solid-state physics, plasma physics, plasma waves, chemical physics, elastic media, optical fibers, atmospheric, oceanic phenomena, biology and so on. The research of traveling wave solutions of some nonlinear evolution equations derived from such fields played an important role in the analysis of some phenomena. To obtain traveling wave solutions, many effective methods have been presented in the literature, such as the Backlund transformation method [1], the Adomian decomposition method [2, 3], the inverse scattering transform [4], the sine-cosine method [5], the Jacobi elliptic function expansion method [6, 7], the Darboux transformation method [8], the complex hyperbolic function method [9, 10], the rank analysis method [11], the ansatz method [12, 13], the exp-functions method [14], the modified simple equation method [15, 16], the (G′/G)-expansion method [17-26], the F-expansion method [27, 28], the homogeneous balance method [29-31], the auxiliary equation method [32, 33], the He's homotopy perturbation method [34, 35], the exp(-φ(η))-expansion method [36-38] and so on.

The rest of the paper is organized as follows: In Section 2, we give the description of the exp(-φ(η))-expansion method. In Section 3, we apply this method to the KdV-Burgers type equation in an unmagnatized dusty plasmas pointed out above; in section 4, physical explanations and in section 5 conclusions are given.

DESCRIPTION OF THE exp (-Φ(η))-EXPANSION METHOD

Let us consider a general nonlinear PDE in the form

$$F(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \ldots) \qquad (1)$$

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where $u = u(x,t)$ is an unknown function, F is a polynomial in $u(x,t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. In the following, we give the main steps of this method:

Step 1: We combine the real variables x and t by a complex variable η

$$u(x,t) = u(\eta)\,,\ \eta = x \pm Vt \tag{2}$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Eq. (1) into an ordinary differential equation (ODE) for $u = u(\eta)$:

$$\Re(u, u', u'', u''', \cdots) \tag{3}$$

where $\Re$ is a polynomial of u and its derivatives and the superscripts indicate the ordinary derivatives with respect to η.

Step 2: Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$u(\eta) = \sum_{i=0}^{N} A_i (\exp(-\Phi(\eta)))^i \tag{4}$$

where $A_i (0 \le i \le N)$ are constants to be determined, such that $A_N \neq 0$ and $\Phi = \Phi(\eta)$ satisfies the following ordinary differential equation:

$$\Phi'(\eta) = \exp(-\Phi(\eta)) + \mu \exp(\Phi(\eta)) + \lambda \tag{5}$$

Eq. (5) gives the following solutions:

Family 1: When $\mu \neq 0$, $\lambda^2 - 4\mu > 0$

$$\Phi(\eta) = \ln\left(\frac{-\sqrt{(\lambda^2 - 4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}(\eta + E)\right) - \lambda}{2\mu}\right) \tag{6}$$

Family 2: When $\mu \neq 0$, $\lambda^2 - 4\mu < 0$

$$\Phi(\eta) = \ln\left(\frac{\sqrt{(4\mu - \lambda^2)}\tan\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\eta + E)\right) - \lambda}{2\mu}\right) \tag{7}$$

Family 3: When $\mu = 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu > 0$

$$\Phi(\eta) = -\ln\left(\frac{\lambda}{\exp(\lambda(\eta + E)) - 1}\right) \tag{8}$$

Family 4: When $\mu \neq 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu = 0$

$$\Phi(\eta) = \ln\left(-\frac{2(\lambda(\eta + E) + 2)}{\lambda^2(\eta + E)}\right) \tag{9}$$

Family 5: When $\mu = 0$, $\lambda = 0$ and $\lambda^2 - 4\mu = 0$

$$\Phi(\eta) = \ln(\eta + E) \tag{10}$$

Here, E, λ, μ are constants to be determined latter and the positive integer N can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3).

Step 3: We substitute Eq. (4) into Eq. (3) and then we account the function $\exp(-\Phi(\eta))$. As a result of this substitution, we get a polynomial of $\exp(-\Phi(\eta))$. We equate all the coefficients of same power of $\exp(-\Phi(\eta))$ to zero. This procedure yields a system of algebraic equations whichever can be solved to find A_N,……, V, λ, μ. Substituting the values of A_N,……, V, λ, μ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

SOLITARY WAVES SOLUTION OF THE KDV-BURGERS TYPE EQUATION IN AN UNMAGNETIZED DUSTY PLASMA

In this section, we will apply the $\exp(-\Phi(\eta))$-expansion method to construct many new and more general traveling wave solutions for the DIA shock waves in unmagnatized plasma. The governing nonlinear equations for the DIA shocks in terms of normalized variable are [39]

$$\frac{\partial n_i}{\partial \tau} + \frac{\partial (n_i U_i)}{\partial z} = 0 \tag{11}$$

$$\frac{\partial U_i}{\partial \tau} + U_i \frac{\partial U_i}{\partial z} = -\frac{\partial \varphi}{\partial z} - 3\sigma_i n_i \frac{\partial n_i}{\partial z} + \eta_i \frac{\partial^2 U_i}{\partial z^2} \tag{12}$$

and

$$\delta \frac{\partial^2 \varphi}{\partial z^2} = \exp(\varphi) - \delta n_i + (\delta - 1) \tag{13}$$

where U_i is the ion fluid speed, φ is the electrostatic wave potential and $\eta_i = \delta \mu_d / \omega_{pi} \lambda_{De}^2$ (in which μ_d is the kinematic viscosity). Here the time and space variables are units of the ion plasma period ω_{pi}^{-1} and electron Debye length $\lambda_{De}/\sqrt{\delta}$, respectively. If we expand

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots\cdots\cdots \tag{14}$$

$$U_i = \epsilon n U_i^{(1)} + \epsilon^2 U_i^{(2)} + \cdots\cdots\cdots, \tag{15}$$

$$\varphi = \epsilon\varphi^{(1)} + \epsilon^2\varphi^{(1)} + \cdots\cdots\cdots \qquad (16)$$

and introduce the stretched variables $x = \epsilon^{\frac{1}{2}}(z - v_0\tau)$ and $t = \epsilon^{\frac{3}{2}}\tau$. Then we readily obtain the KdV-Burgers equation of the form

$$a_1u_t + u\,u_x + b_1u_{xxx} - c_1u_{xx} = 0 \qquad (17)$$

where

$$u = \varphi^1, a_1 = \frac{2v_0}{a}, b_1 = \frac{\delta^2}{a}, c_1 = \frac{v_0 n_c}{a}$$

and

$$a = (3\delta - \delta^2 + 12\sigma)/\delta.$$

As $v_0>0$ and $n_0>0$, the sign of the coefficients a_1, b_1, c_1 are determined by the sign of a.
Upon using the transformation

$$u(x,t) = v(\xi) \;\; ; \xi = x + Vt \qquad (18)$$

where V are speed of travel, equation (17) is transformed to

$$a_1Vv' + v\,v' + b_1v''' - c_1v'' = 0 \qquad (19)$$

where the prime denotes differentiation with respect to ξ
Eq. (19) is integrable, therefore, integrating, we obtain

$$a_1Vv + \frac{v^2}{2} + b_1v'' - c_1v' + P = 0 \qquad (20)$$

where P is an integral constant which is to be determined.

Taking the homogeneous balance between v^2 and v'' in eq. (20), we obtain N = 2. Therefore, the solution of Eq. (20) is of the form:

$$v(\xi) = A_0 + A_1(\exp(-\Phi(\xi)) + A_2(\exp(-\Phi(\xi))^2 \qquad (21)$$

where A_0, A_1, A_2 are constant to be determinate such that $A_N \neq 0$.

Substituting eq. (21) into eq. (20) and then equating the coefficients of exp (-Φ(ξ)) to zero, we obtain

$$-6c_1A_2 + \frac{1}{2}A_2^2 + 6b_1A_2 = 0 \qquad (22)$$

$$A_1A_2 + 10b_1A_2\lambda - 2c_1A_1 + 2b_1A_1 - 10c_1A_2\lambda = 0 \qquad (23)$$

$$\begin{gathered}-6c_1A_2\mu\lambda + a_1VA_1 - c_1A_1\lambda^2 + 6\,b_1A_2\mu\lambda \\ + 2b_1A_1\mu + A_0A_1 - 2c_1A_1\mu + b_1A_1\lambda^2 = 0\end{gathered} \qquad (24)$$

$$\begin{gathered}P + b_1A_1\mu\lambda + 2b_1A_2\mu^2 + \frac{1}{2}A_0^2 \\ - c_1A_1\lambda\mu - 2c_1A_2\mu^2 + a_1VA_0 = 0\end{gathered} \qquad (25)$$

Solving these equations (22) to (25), we obtain

$$\begin{aligned}P = &-8\,\mu^2c_1^2 - \frac{1}{2}\lambda^4\,b_1^2 - 8\,\mu^2b_1^2 - \frac{1}{2}\lambda^4\,c_1^2 \\ &+ \frac{1}{2}\,a_1^2V^2 + 4\mu c_1^2\lambda^2 + 16\,\mu^2b_1c_1 \\ &+ \lambda^4b_1c_1 + 4\lambda^2\mu b_1^2 - 8\mu c_1\lambda^2b_1\end{aligned}$$

$$A_0 = X_0, A_1 = X_1\lambda, A_2 = X_1$$

where

$$X_0 = -a_1V - 8\mu b_1 + 8\mu c_1 + \lambda^2c_1 - \lambda^2b_1,$$
$$X_1 = 12(c_1 - b_1)$$

Substituting these values A_0, A_1, A_2, in eq. (21), we obtain

$$v(\xi) = X_0 + X_1\lambda\,(\exp(-\Phi(\xi)) + X_1(\exp(-\Phi(\xi))^2 \qquad (26)$$

Now substituting equations (6)-(10) into (26) respectively, we get the following five traveling wave solutions of the third order KdV-Burgers equation.
When $\mu \neq 0$, $\lambda^2-4\mu>0$

$$\begin{aligned}v_1(\xi) = X_0 - X_1&\left(\frac{2\lambda\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)+\lambda}\right) \\ &+ X_1\left(\frac{2\mu}{-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)-\lambda}\right)^2\end{aligned} \qquad (27)$$

where $\xi = x+Vt$ and E is an arbitrary constant.
When $\mu \neq 0$, $\lambda^2-4\mu<0$

$$\begin{aligned}v_2(\xi) = X_0 + X_1\lambda&\left(\frac{2\lambda\mu}{\sqrt{4\mu-\lambda^2}\tanh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)\right)-\lambda}\right) \\ &+ X_1\left(\frac{2\lambda\mu}{\sqrt{4\mu-\lambda^2}\tanh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+E)\right)-\lambda}\right)^2\end{aligned} \qquad (28)$$

where $\xi = x+Vt$ and E is an arbitrary constant.
When $\mu = 0$, $\lambda \neq 0$ and $\lambda^2-4\mu>0$

$$v_3(\xi) = X_0 + X_1\lambda\left(\frac{\lambda^2}{\exp(\lambda(\xi+E))-1}\right) + X_1\left(\frac{\lambda^2}{\exp(\lambda(\xi+E))-1}\right)^2 \qquad (29)$$

where $\xi = x+Vt$ and E is an arbitrary constant.
When $\mu \neq 0$, $\lambda \neq 0$ and $\lambda^2-4\mu = 0$

$$v_4(\xi) = X_0 - X_1\left(\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E)+2)}\right) + X_1\left(\frac{\lambda^2(\xi+E)}{2(\lambda(\xi+E)+2)}\right)^2 \qquad (30)$$

where $\xi = x+Vt$ and E is an arbitrary constant.
When $\mu = 0$, $\lambda = 0$ and $\lambda^2-4\mu = 0$

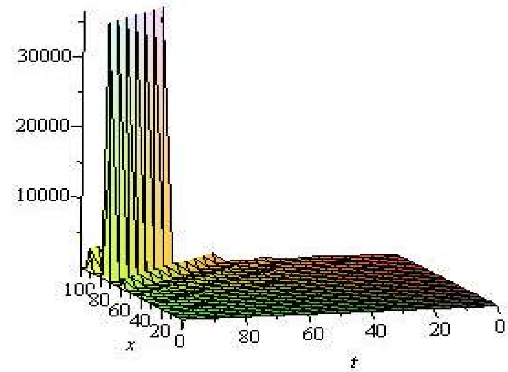

Fig. 1: Solitary wave solution $v_1(\xi)$ when $a_1 = 0.3$, $b_1 = 0.5$, $c_1 = 0.8$, $V = 1$, $\mu = 2$, $\lambda = 1$, $E = 1$ and $0\leq x$, $0\leq t$

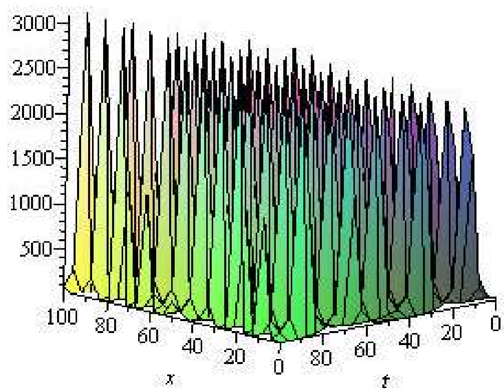

Fig. 2: Solitary wave solution $v_2(\xi)$ when $a_1 = 0.3$, $b_1 = 0.5$, $c_1 = 0.8$, $V = 3$, $\mu = 10$, $\lambda = 3$, $E = 1$ and $0\leq x$, $0\leq t$

$$v_5(\xi) = X_0 + \frac{X_1\lambda}{(\xi+E)} + X_1\left(\frac{1}{(\xi+E)}\right)^2 \tag{31}$$

where $\xi = x+Vt$ and E is an arbitrary constant.

Therefore the approximate solution of the electrostatic wave potential equation can be written as

$$\varphi = \epsilon\{X_0 + X_1\lambda\left(\exp\left(-\Phi(\xi)\right)\right) + X_1\left(\exp\left(-\Phi(\xi)\right)\right)^2\} + O(\epsilon^2) \tag{32}$$

PHYSICAL EXPLANATIONS

In this section we will discuss the physical explanations and graphical representation of the above determined five families of solutions.

Explanations: The introduction of dispersion without introducing nonlinearity destroys the solitary wave as different Fourier harmonics start propagating at different group velocities. On the other hand, introducing nonlinearity without dispersion also prevents the formation of solitary waves, because the pulse energy is frequently pumped into higher frequency modes. However, if both dispersion and nonlinearity are present, solitary waves can be sustained. Similarly to dispersion, dissipation can also give rise to solitary waves when combined with nonlinearity. Hence it is interesting to point out that the delicate balance between the nonlinearity effect of uu_x

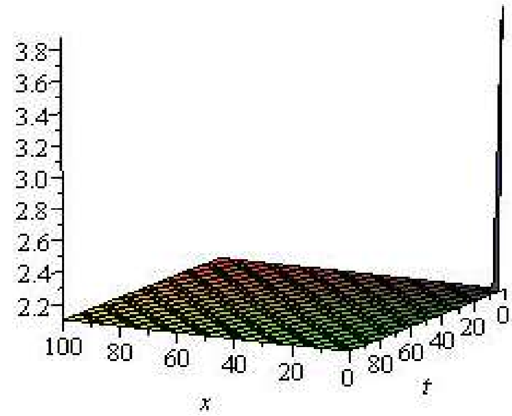

Fig. 3: Solitary wave solution $v_3(\xi)$ when $a_1 = 0.3$, $b_1 = 0.5$, $c_1 = 0.8$, $V = 2$, $\mu = 0$, $\lambda = 3$, $E = 1$ and $0\leq x$, $0\leq t$

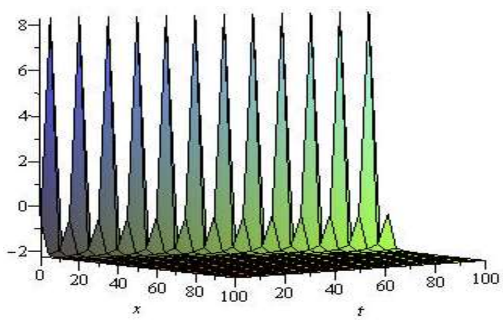

Fig. 4: Solitary wave solution $v_4(\xi)$ when $a_1 = 0.3$, $b_1 = 0.5$, $c_1 = 0.8$, $V = -0.5$, $\mu = 3$, $\lambda = 4$, $E = 1$ and $0\leq x$, $0\leq t$

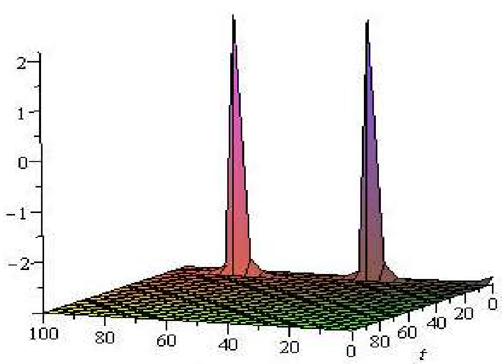

Fig. 5: Solitary wave solution $v_5(\xi)$ when $a_1 = 0.3$, $b_1 = 0.5$, $c_1 = 0.8$, $V = -10$, $\mu = 0$, $\lambda = 0$, $E = 5$ and $0\leq x$, $0\leq t$

and the dissipative effect of u_{xx} and u_{xxx} give rise to solitons, that after a fully interaction with others the solitons come back retaining their identities with the same speed and shape. The KdV-Burgers equation has solitary wave solutions that have exponentially decaying wings. If two solitons of the KdV-Burgers equation collide, the solitons just pass through each other and emerge unchanged. For special values of the parameters solitary wave solutions are originated from the obtained exact solutions.

Solitons are special kinds of solitary waves. The soliton solution is a specially localized solution, hence $u'(\xi)$, $u''(\xi)$, $u'''(\xi) \to 0$ as $\xi \to \pm\infty$, where ξ = x+Vt. Solitons have a remarkable property that it keeps its identity upon interacting with other solitons.

Graphical representation of the solutions: The graphical illustrations of the solutions of the KdV-Burgers type equation are given below in the figures (Fig.1 to 5) with the aid of Maple.

CONCLUSION

In this paper, the exp $(-\Phi(\eta))$-expansion method has been successfully applied to find the exact solutions for nonlinear partial differential equations such as the KdV-Burgers type equation in unmagnatized dusty plasmas. Also find the electrostatic wave potential in equation (32). The results show that the exp $(-\Phi(\eta))$-expansion method is a powerful mathematical tool to solve the solitary wave equation in an unmagnatized dusty plasmas; it is also a promising method to solve other nonlinear equations.

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