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Abstract: This article describes a methodology for the substantiation of the choice of power output for solar collectors in their operation in compound heat supply systems. The author proposes as a criterion for optimization annual net heating costs incurred over the project period amid outstripping growth rates for prices on energy carriers. The author presents methodologies for assessing the use of the energy of the Sun in heat supply systems based on empirical formulas for calculating the amount of heat for heating dwellings the consumer needs and the amount of heat coming in from solar collectors monthly. These analytical dependencies make it possible to substantially simplify coming up with the optimum composition of equipment in a compound system. The author lays out the optimization sequence and puts forth the necessary limitations based on the application of the Heaviside step function for zeroing out the negative values of the difference between consumption and heat absorption by the solar collector. The article examines an example of optimizing a compound heat supply system operating in conditions of the sharply continental climate of Zabaykalsky Krai (the Russian Federation).

Key words: Helium heating unit • System • Heating • Centralized heating • Energy of the Sun • Optimization • Collector • Methodology • Climate

INTRODUCTION

Today, heat supply is one of the most costly areas of the utilities sector. Its primary objective is providing heat energy consumers with quality and energy-effective heat supply [1, 2]. This is especially topical for northern countries with tough climatic conditions [3]. Currently, certain traditional and renewable power generation units are close in terms of the cost. The increase in the cost of traditional power generation units is the effect of stiffening the requirements on ecology. The development of technology has, in turn, led to a decrease in the cost of renewable power generation units. Many countries are engaged in stimulating the adoption of non-traditional and renewable sources of energy. In the Russian Federation, the “Ecologically Clean Power Generation” program calls for the development of solar heating and hot water supply systems for individual residential houses, agricultural, resort and certain production facilities and residential buildings (the focus area being solar heat supply).

What points in the favor of using renewable sources of energy is that up to 40% of expenditures on fuel used for supplying scattered consumers located far from fuel bases and railroad stations with heat goes for transportation. A substantial part of expenditures goes for building fuel warehouses and storage facilities, as well as putting up ash-disposal areas. Besides, burning large amounts of fuel in boiler plants and furnaces pollutes the environment with combustion products. Using the energy of the Sun even in modest volumes will help compensate for all known types of primary energy. The most effective way of using the energy of the Sun nowadays is helium-heating water using various types of solar collectors [4, 5].
It should be noted that the use of solar energy has its peculiarities. The technical-economic characteristics of helium-units mainly depend on the following factors: natural-climatic conditions in the area, which define the amount of energy output (the level of solar radiation, overcast conditions, the number of sunny days in a year, etc.); unit investment in helium-units; the availability and costs of fuel and energy. A low density of distribution of solar energy over the territory, irregularity of the flux of energy in time and dependence on natural climatic conditions are the major factors that large the size of helium-units and complexity of their design, which, in turn, causes increases in unit investment and material consumption in building helium-units [6].

This article is dedicated to a methodology for determining the optimum number of solar collectors. In selecting the number of solar collectors (SC), it is important to take account of two crucial factors - the consumer’s maximum heating load and the solar heating system’s reliability. If solar energy is the only source of heat, one will need to set up a large number of both SC’s proper and heat-accumulating containers to be able to guaranteedly supply the consumer under the maximum load and in case there is no solar irradiance. Note that for the most part of the heating period the equipment will be used at partial capacity. Consequently, this approach is not effective and SC’s should be combined with regular sources of thermal energy. The objective is to come up with a technical-economic determination of the optimum number of SC’s, when the base load is carried by solar collectors and additional heating is provided in case of need by the base source.

The determination of the optimum number of collectors is performed in the following sequence:

- Determine heat consumption by months.
- Determine the amount of incoming solar energy.
- Determine the necessary number of solar collectors at different values for the share of consumed heat being filled in for by heat coming in from solar collectors.
- Determine the optimum number of solar collectors based on the condition of fulfilling the chosen extremum of the adopted efficiency parameter.

**MATERIALS AND METHODS**

According to the existing methodology for assessing the efficiency of investment projects, it is customary to use an aggregate of a number of major indicators:

- The Net Present Value (NPV) or the integral effect;
- The Profitability Index (PI);
- The Internal Rate of Return (IRR);
- The Payback Period (PBP).

These economic criteria in company with conditions they have to satisfy form a system for objective qualitative assessment of the economic worthwhileness of investment projects. Note that none of the above criteria is sufficient for accepting the project. The sole objective criterion for economic worthwhileness is concurrent fulfillment of all the conditions. Compared with economic worthwhileness, attaining the maximum quantity of the net present value is a condition for a solution or a project to be considered optimum.

In examining the economic worthwhileness of activities aimed at boosting efficiency, it is important to keep to the principle of concurrent fulfillment of all of the above conditions. The exceptions are low-cost and no-cost activities. Applying this aggregate of criteria for determining the optimum composition of a compound heat supply system is not possible, since it is proposed that we consider as a criterion annual net heating costs incurred over the period of carrying out the project:

\[
C = \sum_{t=0}^{L} I_t \cdot \frac{1}{(1+R)^t} + \sum_{t=0}^{L} E_t^* \cdot \frac{1}{(1+R)^t}
\]  

where \( E_t^* \) is operating expenditures in the \( t \)-th year, which are needed for the system to function, rub.;

\( I_t \) is capital investment in the \( t \)-th year, which is needed for adopting collectors, rub.;

\( R \) is the acceptable rate of return on capital, year\(^{-1} \);

\( L \) is the service life of the investment facility under study, years;

\( t \) is the number of the step (year) of the calculation from \( t=0 \) to \( L \).

For comparing various ratios of heat sources, we can use changes in annual net costs in the system:

\[
\Delta C = \sum_{t=0}^{L} I_t \cdot \frac{1}{(1+R)^t} + \sum_{t=0}^{L} (E_t^* - E_t) \cdot \frac{1}{(1+R)^t}
\]  

where \( E_t^* \) is operating expenditures at the \( t \)-th step, which are needed for the system’s operation without renovation, rub.
If the system is operating amid an inflationary growth in prices and one-time capital investment, the formula (2) takes on the following form:

\[ \Delta C = I + \sum_{t=0}^{L} \left( E_i^* - E_i \right) \left( 1 + G \right) \left( 1 + R \right)^t \]

(3)

where \( G \) is the rate of inflationary growth in prices, year\(^{-1}\).

The present worth factor \( \sum_{t=0}^{L} \frac{(1 + G)}{(1 + R)^t} \) which is part of the formulas (1-3) at constant values for the rate of return on capital and rate of inflation, according to the theory of series can be found from the ratio

\[ a = \frac{(1 + G)}{(R - G)} \left[ 1 - \left( \frac{1 + G}{1 + R} \right)^L \right]. \]

Then the equation (3) takes on the following form:

\[ \Delta C = I + \left( E_i^* - E_i \right) \left( 1 + G \right) \left( R - G \right) \left[ 1 - \left( \frac{1 + G}{1 + R} \right)^L \right] \]

(4)

The criterion for the choice of the optimum number of solar collectors is \( \Delta C \)-min.

In the case when the financial outcomes of the investment project are defined by fuel and energy savings, in calculating the net present value one needs to take account of the fact that the rate of growth in prices on energy carriers across the world is ahead of the average rate of inflationary growth in prices by about 2% per year [7]. In this regard, the resulting formula takes on the following form:

\[ \Delta C = I_{\text{rec}}^{\text{cons}} + 1.02 \cdot \Delta E_{\text{ers}} \cdot \frac{(1 + G)}{R - (1.02 \cdot G + 0.02)} \cdot \left[ 1 - \left( \frac{1.02 + G \cdot 1.02}{1 + R} \right)^L \right] \]

(5)

where \( I_{\text{rec}}^{\text{cons}} \) is capital investment in renovating the consumer’s heat supply system, which matches the calculated number of collectors, rub.;

\( \Delta E_{\text{ers}} \) is a decrease in costs due to a boost in efficiency as a result of renovating the consumer’s heat supply system, which matches the calculated number of collectors, rub.

**Main Part:** Let us consider, as an example, a more simple heating system, which is based on combining two sources of heat supply: a solar collector in this scheme is a basic source of heat; the missing part is compensated for by an electric boiler, mainly in the night-time and rainy weather. Instead of electric boilers, water-heating boilers can be used, which work on gas, mazut and solid fuels (wood, coal). In this scheme, the accumulator tank concurrently serves as an intermediate heat exchanger between the circuits of the solar collector and the heating circuit. Firstly, such division makes it possible to use as a heat transfer agent non-freezing fluids in the circuit of the solar collector and water in the heating circuit. Secondly, there is simplification in the way of process automation. Thirdly, there appears a possibility of adding a third circuit for hot water supply. With no substantial transformations, this optimization methodology can also be applied to helium-air heating systems [8-10].

Determining a facility’s heat consumption by months.

Heat consumption is determined by calculating heat losses through enclosure walls, by a set heat load depending on the temperature of ambient air, or by the readings of the heat meter:

\[ Q_m = Q_a \left( r_{\text{in}} - r_{\text{air}} \right) \cdot \tau_m \]

(6)

where \( Q_a \) is the heat load of a building, kW; \( r_{\text{in}} \) is the assumed temperature inside the building, ºC; \( r_{\text{air}} \) is the assumed temperature of outside air, ºC; \( r_{\text{m}} \) is the average monthly temperature of outside air, ºC; \( \tau_m \) is the number of hours of the heating system’s operation, h.

**Determining the Amount of Incoming Solar Energy:**

To determine the amount of incoming solar energy, we need data characteristic of the region in question:

\( S \) is total solar radiation (direct and diffused) on a horizontal surface under a cloudless sky, MJ/m\(^2\); \( S_v \) is total solar radiation (beam and diffused) on a vertical surface under a cloudless sky, MJ/m\(^2\); \( S_m \) is the monthly average daily income of solar radiation on a horizontal surface beyond the Earth’s atmosphere, MJ/m\(^2\). /3/

Then, based on data obtained, we determine the ratio between the monthly average daily incomes of total radiation on sloping and horizontal surfaces. Since the collector can be set up not perpendicular to the direction of solar rays, we need to correct the capacity of direct solar irradiance on a sloping surface.
For a more accurate calculation, we need to take account of the characteristics of the heat exchanger and the accumulator tank.

**Determining the Number of Heat Collectors:** The optimum amount of solar collectors will match the ratio of traditional heating and helium-heating under the condition $\Delta C_{\text{min}}$.

The amount of heat coming in from solar collectors monthly, kWh:

$$Q_{\text{s.co.}}^{i} = n_{\text{co}} \cdot U_{\text{co}} \cdot \eta_{\text{co}} \cdot M_{\text{co}}^{i}$$  \hspace{1cm} (7)

where $n_{\text{co}}$ is the number of collectors;

$\eta_{\text{co}}$ is the collector’s efficiency;

$M_{\text{co}}^{i}$ is the monthly income of solar radiation, W/m$^2$;

$U_{\text{co}}$ is the unit area of the collector’s surface that receives solar energy, m$^2$.

Determining the cost of energy resources spent by the traditional source:

$$P_{s}^{i} = p_{\text{ce}} \cdot (Q_{m}^{i} - Q_{s.co.}^{i})$$  \hspace{1cm} (8)

where $p_{\text{ce}}$ is the cost of energy, rub. /kW.

Since the quantity $P_{s}^{i}$ during summer months can be negative on account of $Q_{s.co.}^{i}$ being greater than $Q_{m}^{i}$, we need to apply the Heaviside step function for zeroing out negative values.

$$P_{s}^{i} = \begin{cases} 0, & P_{s}^{i} \leq 0 \\ P_{s}^{i}, & P_{s}^{i} > 0 \end{cases}$$  \hspace{1cm} (9)

For the conditions of Zabaykalsky Krai (the town of Chita) we worked out analytical dependencies that make it possible to substantially simplify coming up with the optimum composition of equipment in a compound system.

In Zabaykalsky Krai, the complex $\theta$ during the year is presented in Figure 1.

The dependence of the complex $\theta$ on time for any facility can be with sufficient accuracy described using the following empirical formula:

$$\theta = a + b \cdot \sin \frac{2 \cdot \pi \cdot \tau}{d} + c$$  \hspace{1cm} (10)

where $\tau$ is time, h; $a$, $b$, $c$, $d$ are empirical coefficients.

For Zabaykalsky Krai, $a=0.4$; $b=0.4$; $c=1.5$; $d=9793$.

Thus, the amount of heat the consumer needs for heating dwellings will be expressed as follows:

$$Q_{m}^{i} = Q_{a} \cdot (0.4 + 0.4 \sin \frac{2 \cdot \pi \cdot \tau_{i}}{9793} + 1.5) \cdot \tau_{i}$$  \hspace{1cm} (11)

And the amount of heat coming in from solar collectors monthly will be derived as follows:

$$Q_{s.co.}^{i} = n_{\text{co}} \cdot U_{\text{co}} \cdot \eta_{\text{co}} \cdot (185.5 + 42.6 \sin \frac{2 \cdot \pi \cdot \tau_{i}}{4022} + 3.9)$$  \hspace{1cm} (12)

If there is no hot water supply at the facility, we have to raise the issue of utilizing heat in summer months. The dependence of the quantity of heat consumption on time can be with substantial accuracy determined through the methodology [11-12] using the formula:

$$Q_{\text{month}}^{i} = Q_{\text{max}} \cdot Q_{\text{year}} \left( \cos \left( \frac{\tau - \tau_{\text{max}}}{T} \right) + 1 + q_{\text{hws}} \right) \cdot \frac{Q_{\text{max}}}{2 \cdot b}$$  \hspace{1cm} (13)

where $\tau_{\text{max}}$ is the moment of time by which the highest boiler load $Q_{\text{max}}$ is attained, $T$ is the length of the year, $q_{\text{hws}}$ is the share of produced heat that goes for hot water supply, $a$ and $b$ are empirical coefficients.

Sufficient accuracy is attained at $a=3/2$ and $b=1.35$.

We can use any timespan - months, days, hours.

As an example of applying the developed methodology, Tables 1-2 present the results of calculating the optimum number of solar collectors for the compound heat supply system of the two-way automobile border checkpoint “Olochi” (the RF-Mongolia).
Table 1: The heat consumption of a facility

<table>
<thead>
<tr>
<th>Month</th>
<th>Temperature of outside air, C</th>
<th>Assumed temperature of inside air, C</th>
<th>Assumed heat load, kW</th>
<th>Heat consumption, Q_{rmax}, kWh</th>
<th>Number of hours of heat load operation, h</th>
<th>Heat consumption, Heat consumption, Gcal</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>-24</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>15.750</td>
<td>744</td>
</tr>
<tr>
<td>February</td>
<td>-19</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>13.875</td>
<td>672</td>
</tr>
<tr>
<td>March</td>
<td>-7.9</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>9.713</td>
<td>744</td>
</tr>
<tr>
<td>April</td>
<td>1.8</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>6.075</td>
<td>720</td>
</tr>
<tr>
<td>May</td>
<td>5.75</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>4.594</td>
<td>384</td>
</tr>
<tr>
<td>September</td>
<td>4.75</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>4.969</td>
<td>360</td>
</tr>
<tr>
<td>October</td>
<td>0.2</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>6.675</td>
<td>744</td>
</tr>
<tr>
<td>November</td>
<td>-11.7</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>11.138</td>
<td>720</td>
</tr>
<tr>
<td>December</td>
<td>-23.9</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>15.713</td>
<td>5832</td>
</tr>
<tr>
<td>Year</td>
<td>18</td>
<td>18</td>
<td>-38</td>
<td>21</td>
<td>5832</td>
<td>60870</td>
</tr>
</tbody>
</table>

Table 2: The characteristics of incoming solar radiation

<table>
<thead>
<tr>
<th>S*, mJ/m²</th>
<th>S*, mJ/m²</th>
<th>N</th>
<th>NE/NW</th>
<th>E/W</th>
<th>SE/SW</th>
<th>S</th>
<th>n, 24 hr</th>
<th>S**av, mJ/(m² 24 hr)</th>
<th>Sav, mJ/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>164</td>
<td>143</td>
<td>371</td>
<td>495</td>
<td>31</td>
<td>9</td>
<td>279</td>
<td></td>
<td></td>
</tr>
<tr>
<td>February</td>
<td>270</td>
<td>210</td>
<td>424</td>
<td>566</td>
<td>28</td>
<td>14.5</td>
<td>406</td>
<td></td>
<td></td>
</tr>
<tr>
<td>March</td>
<td>528</td>
<td>365</td>
<td>572</td>
<td>692</td>
<td>31</td>
<td>22.3</td>
<td>691.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>April</td>
<td>678</td>
<td>459</td>
<td>557</td>
<td>558</td>
<td>30</td>
<td>31.2</td>
<td>936</td>
<td></td>
<td></td>
</tr>
<tr>
<td>May</td>
<td>850</td>
<td>512</td>
<td>573</td>
<td>497</td>
<td>31</td>
<td>38.1</td>
<td>1181.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>June</td>
<td>880</td>
<td>512</td>
<td>514</td>
<td>427</td>
<td>30</td>
<td>41.2</td>
<td>1236</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July</td>
<td>882</td>
<td>518</td>
<td>511</td>
<td>452</td>
<td>31</td>
<td>39.6</td>
<td>1227.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>August</td>
<td>719</td>
<td>457</td>
<td>542</td>
<td>520</td>
<td>31</td>
<td>33.8</td>
<td>1047.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>September</td>
<td>540</td>
<td>371</td>
<td>530</td>
<td>584</td>
<td>30</td>
<td>25.4</td>
<td>762</td>
<td></td>
<td></td>
</tr>
<tr>
<td>October</td>
<td>344</td>
<td>263</td>
<td>490</td>
<td>611</td>
<td>31</td>
<td>16.7</td>
<td>517.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>November</td>
<td>194</td>
<td>166</td>
<td>392</td>
<td>543</td>
<td>30</td>
<td>10.3</td>
<td>309</td>
<td></td>
<td></td>
</tr>
<tr>
<td>December</td>
<td>126</td>
<td>121</td>
<td>305</td>
<td>475</td>
<td>31</td>
<td>7.6</td>
<td>235.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on the calculation results, the study found the optimum number of solar collectors to be 10 (under the collector’s unit area of 1.38 m²).

Inferences. The proposed methodology for optimizing the composition of equipment in compound heat supply systems makes it possible to expeditiously determine the optimum number of solar collectors without the need for performing multiple complex astronomical calculations. Applying it in conducting energetic examinations of heat supply facilities makes it possible to provide technically and economically substantiated recommendations on adopting renewable sources of energy. Besides, the proposed approach can be employed in calculations for assessing the ecological efficacy of adopting renewable sources of energy (e.g. [13]).

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REFERENCES

13. Pinigin, V.V. and A.G. Batukhtin. Reaffirming the methods of reducing the harmful discharge of the TES with the use of the exergetic analysis. - LAP LAMBERT Academic Publishing. Saarbrücken, Germany.