

## Impact of Oil Revenues Accumulation on Socio-Economic Indicators

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**Abstract:** The country that is rich with natural energy resources such as Kazakhstan has a significant source of revenues for well-being of its population. On the one hand, full use of all oil revenues for current consumption is associated with risks due to uncertainties in the development of global economy. Also unrestricted inflow of oil money into economy could lead to higher inflation in the country. On the other hand, accumulation of all oil revenues in a special fund to be used in emergency situations in the future is inefficient, as it deprives the country from using those revenues to improve living standards, to implement innovative projects and to improve the prospects for long-term economic development. A small dynamic stochastic general equilibrium model of open Kazakhstan's economy is built. The model corresponds to the new Keynesian tradition. It is assumed that in the country there are firms producing common goods and a sector that produces oil. Oil revenues are divided into two parts. One part is used for current consumption and the other is sent to an accumulation fund. The model is estimated on data of Kazakhstan. Some of parameters are found by calibration, the others are evaluated using the Bayesian approach. The research concerns how a change in the share of oil revenue accumulation influences on responses to various shocks. It was found how due to a change in the share of oil revenue accumulation shocks have strengthening or debilitating effects on indicators. The taken approach could be considered as one of the tools for determining the most acceptable level of oil revenues accumulation for an oil exporting country.

**Key words:** Economy • Modeling • General-equilibrium • Dynamic model • Stochastic model • Monetary policy • New Keynesian model

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### INTRODUCTION

In recent years, scientists and representatives of central banks and international organizations are interested in dynamic stochastic general equilibrium models (DSGE models). DSGE term was originally introduced in the work of Nobel laureates F. Kydland and E. Prescott [1], in which they made a fundamental contribution to construction of a real business cycles (RBC) model. This model is based on the analysis of microeconomic agents that optimize their behavior under flexible price condition. Price flexibility leaves room only for real values to cause fluctuations in economy. These can be technological shocks or abrupt changes in government spending.

Later in more recent studies elements of the Keynesian approach containing nominal rigidities were included in DSGE models. In particular, it is worth to point

the work of Calvo [2] in which a mechanism of pricing by firms as some stochastic process of decision-making about maintaining price at the same level or its changing was proposed. A new paradigm for modeling dynamic stochastic general equilibrium originated. These models were called New Keynesian DSGE models. This new approach takes into account microeconomic foundations of decision-making by households, optimization behavior of monopolistically competitive firms and regulatory functions of the state. Nominal price and wage rigidities provide more consistency of calculation results of the model on real data of short-term macroeconomic fluctuations in economy [3].

Compared with traditional macroeconomic models, in particular, econometric models, the main advantage of dynamic stochastic general equilibrium models is that they are not subject to the Lucas critique [4] which applies to all econometric models. For example, commonly

used method of vector autoregression and error correction model, although sometimes useful, have significant drawbacks [5].

To estimate traditional macroeconomic models rather long time series at constant monetary policy are required, which is not always possible, especially for developing economies. Unstructured approach to modeling based on statistical analysis of data does not allow receiving recommendations for changing conducted macroeconomic policy. Inflation expectations are not accounted, which play a crucial role in behavior of agents. It is hardly possible to build a reliable inflation equation, which contains current values of variables and their lags, when inflationary expectations are not considered.

Over the past 10-20 years a whole series of new Keynesian dynamic stochastic general equilibrium models were created. Among the most famous works, designed for policy analysis and forecasting, models of several European central banks [3, 6, 7] and developing countries [8, 9] could be noted. DSGE model has microeconomic rationale with nominal and real rigidities [10]. In such a model, households consume, determine the volume of investments and deliver labor on monopolistically competitive labor market. DSGE models have been developed and evaluated in several other studies [11-19, 34-35].

The purpose of the study is to build a dynamic stochastic general equilibrium model for the economy of Kazakhstan interconnected with the rest of the world. One of the features of the economy of Kazakhstan is that a significant portion of its gross domestic product is oil production, which serves as the basis for economic development and welfare of the population. Along with firms manufacturing common goods oil production sector is introduced into the model. In the Republic of Kazakhstan the National Fund was established, which accumulates most of the country's oil revenues. The model allows varying the proportion of oil revenues used for current consumption in order to assess the impact of such changes on effects of shocks.

Part of parameters of the model was determined by calibration of statistical data and the remaining part was estimated by Bayesian method. Also Metropolis-Hastings algorithm was applied. Calculations were performed by simulating the model and analyzing the impact of various macroeconomic policy options, forecasting responses of economic indicators to various shocks in the country and abroad. The impact of changes in share of oil accumulation on them was investigated.

Section 2 presents the model. Statistical data, calibration methods and parameter estimation are described in Section 3. The fourth section presents the results of the impulse responses of the main macroeconomic indicators for various internal and external shocks. The impact of oil revenues accumulation on these impulse responses is discussed. Section 5 concludes the paper.

**Model:** The country's economy is modeled as a small open economy that interacts with the rest of the world. The world economy is considered to consist of a large number of small open economies and is modeled as a continuum of economies, indexed by  $j \in [0, 1]$ . Domestic policy decisions of each small economy do not have a significant impact on the rest of the world. It is assumed that they have the same preferences, technology and market structure. Here, following mainly studies [11, 20-21], a description of model structure begins with a presentation of one country populated by a continuum of households, indexed by  $i \in [0, 1]$ . Because later attention will be focused on the behavior of one economy and its interaction with the global economy, for it in order to simplify the notation, all variables will be used without the  $j$ -index and for the rest countries such index will be applied. It will be called the country H.

The country's economy consists of a representative household, firms producing final goods, a firm producing oil and an authority that sets monetary policy in the country. The household uses a variety of final goods, offers its labor on market. Labor resources are fully mobile between sector producing final goods and oil sector. There are many firms in the country. They produce diversified products under monopolistic competition and nominal price rigidity. The representative household owns firms and receives profit from them. All firms use oil in their production and purchase it at a price prevailing on the world oil market. Oil sector determines the volume of oil production so as to maximize its profits. Part of the volume of produced oil is directed to meet domestic demand of firms producing final goods and the rest is exported.

**Households:** The representative household seeks to maximize the discounted utility obtained from consumption and required expenditures for this labor costs:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - \gamma C_{t-1})^{1-\sigma} - 1}{1-\sigma} - \theta_t \frac{L_t^{1+\varphi}}{1+\varphi} \right] \quad (1)$$

under constraints

$$P_t C_t + E_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t L_t + \Pi_t + \Pi_{ot}, \quad t = 0, 1, \dots \quad (2)$$

Here  $C_t$  – consumption,  $L_t$  – labor supply,  $\beta$  – intertemporal discount factor,  $\sigma$  – inverse elasticity of intertemporal substitution of consumption,  $\varphi$  – inverse elasticity of labor supply on wages,  $\gamma C_{t-1}$  – habit formation in consumption,  $\gamma$  – non-negative coefficient,  $\theta_t$  – preference shock variable that affects labor supply,  $P_t$  – consumer price index (CPI),  $Q_{t,t+1}$  – discounted value of dividends on securities in the end of period  $t$ ,  $W_t$  – rate of nominal wage,  $\Pi_t$ ,  $\Pi_{ot}$  – profits from the production of final goods and oil,  $E_t$  – operator of rational expectations at affordable information in period  $t$ .

It is assumed that the behavior of a variable labor supply shock is described by first-order autoregression process:

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t},$$

and a random variable  $\varepsilon_{\theta t}$  is a white noise. Hereinafter  $E_t$  is an operator of rational expectations on all available information at time  $t$ .

In problem (1) - (2) of the optimal choice of the household, using the necessary optimality conditions of the first order and considering that  $Q_{t,t+1} = 1/(1 + i_t)$ , we can obtain equations that define optimal allocation of consumption and labor:

$$\beta(1 + i_t) E_t \left\{ \frac{(C_{t+1} - \gamma C_t)^{-\sigma} P_t}{(C_t - \gamma C_{t-1})^{-\sigma} P_{t+1}} \right\} = 1 \quad (3)$$

$$\frac{\theta_t L_t^\varphi}{C_t^{-\sigma}} = \frac{W_t}{P_t} \quad (4)$$

In the model equations are written in the log-linearized form. We define  $\rho = -\ln \beta$ ,  $i_t = -\ln Q_{t,t+1}$ . Equations (3) and (4) after the log-linearization are recorded as

$$c_t = \frac{1-\gamma}{\sigma(1+\gamma)} (\rho - i_t + E_t \pi_{t+1}) + \frac{1}{1+\gamma} E_t c_{t+1} + \frac{\gamma}{1+\gamma} c_{t-1} \quad (5)$$

$$w_t - p_t = \sigma [\ln(1 - \gamma) + \frac{1}{1+\gamma} (c_t - \gamma c_{t-1})] + \varphi l_t + \xi_t \quad (6)$$

where hereinafter small letters denote logarithms of values that presented by capital letters, except the rate of inflation, for which

$$\pi_{t+1} = P_{t+1}/P_t - 1 \approx \ln(P_{t+1}/P_t) = p_{t+1} - p_t$$

Equation (5) is the Euler equation. This rate of inflation is based on the consumer price index, i.e. it reflects changes in prices of both domestic goods and imported goods. Equation (5) contains expectations of future values of inflation and consumption variables. Availability of expectations in equations generates a major challenge to find solutions.

**Firms' Behavior:** This subsection describes behavior of firms. There is a continuum of monopolistically competitive firms producing final goods and indexed by  $i \in [0,1]$  in the country H. Each firm uses a technology described by the production function

$$Y_{it} = A_t \min \left\{ N_{it}, \frac{1}{\zeta} O_{it} \right\},$$

where  $Y_{it}$  – output of final goods of firm  $i$ ,  $O_{it}$  – volume of oil used for production,  $A_t$  – coefficient reflecting the effect of technological progress, i.e. total factor productivity,  $N_{it}$  – number of employees in a firm,  $\zeta$  – factor that determines fixed proportions of production factors. Value  $A_t$  varies according to the autoregression process AR (1) of the following form:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{at},$$

and  $\varepsilon_{at}$  is a "white noise". Since a firm does not make an excessive expenditures, equalities  $Y_{it} = A_t N_{it}$ ,  $O_{it} = \zeta N_{it}$  will take place. According to Calvo [2] a firm in each period with probability  $1-\theta$  changes the price of its product and with probability  $\theta$  keeps it unchanged. Optimal level of price that a firm sets in period  $t$  is:

$$\bar{P}_t = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{k=0}^{\infty} \beta^k \theta^k C_{t+k}^{1-\sigma} P_{t+k}^\varepsilon MC_{t+k|t}^r}{E_t \sum_{k=0}^{\infty} \beta^k \theta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon-1}} \quad (7)$$

Here  $MC_{t+k|t}^r = MC_{t+k|t}^n / P_t$  – real marginal costs of a firm in the period  $t+k$ , which changed its price for the last time in period  $t$ . The index  $i$  is omitted because all firms are assumed to be identical. In the case of flexible prices  $\theta = 0$ , then

$$\bar{P}_t = \frac{\varepsilon}{\varepsilon-1} MC_{t|t}^r$$

By decomposing into a Taylor series of the first order and transition to the logarithms of variables the equation (7) takes the following form

$$\bar{p}_t = m + (1 - \theta\beta) \sum_{k=0}^{\infty} \beta^k \theta^k [mc_{t+k}^r + p_{t+k}] \quad (8)$$

where  $m = -mc^r$ .

Let's separately consider the *oil sector* in the country H. The firm producing oil maximizes profit at the given wage rate  $W_t$  and the world price of oil  $P_{ot}$ :

$$\max_{N_{ot}} [P_{ot} O_{st} - W_t N_{ot}] \quad (9)$$

provided

$$O_{st} = A_{ot} N_{ot}^\mu, \quad 0 < \mu < 1 \quad (10)$$

Here  $A_{ot}$  – coefficient reflecting the impact of technological development in the oil sector, dynamics of which is described by an autoregression process AR (1) of the following form:

$$\ln A_{ot} = \rho_{ao} \ln A_{ot-1} + \varepsilon_{ao},$$

where  $\varepsilon_{ao}$  is a "white noise". Value  $N_{ot}$  is the number of employed in oil production and  $O_{st}$  – the volume of oil supply both for domestic consumption and export. Let's substitute  $O_{st}$  from the production function (9) into the target function (10):

$$\max_{N_{ot}} [P_{ot} A_{ot} N_{ot}^\mu - W_t N_{ot}]$$

and by writing the first order optimal conditions we obtain the dependence of number of employed in the oil sector and overall supply of oil from oil price and wage rate, respectively:

Then the real effective exchange rate in logarithmic form is defined as:

$$q_t = \int_0^1 q_{jt} dj = \int_0^1 (e_{jt} + p'_{jt} - p_t) dj = e_t + p_{jt}^* - p_t = p_{Ft} - p_t = p_{Ft} - (p_{Ht} + as_t)$$

Hence, the real exchange rate is linearly expressed through terms of trade:

$$q_t = (1 - \alpha)s_t. \quad (14)$$

Obviously, the parameter  $\alpha$ , which ranges from 0 to one, can not be equal to one, since this would mean that only imported products are consumed in the country H.

Under the assumption of perfect markets of securities under first-order condition, similar to (3), must be maintained for each country  $j$ . After rearranging it taking into account the relation of real exchange rate and terms of trade (14) following is obtained

$$N_{ot} = \left( \frac{W_t}{\mu P_{ot} A_{ot}} \right)^{\frac{1}{\mu-1}} = \left( \frac{\mu P_{ot} A_{ot}}{W_t} \right)^{\frac{1}{1-\mu}}$$

$$O_{st} = A_{ot}^{\frac{1}{1-\mu}} \left( \frac{\mu P_{ot}}{W_t} \right)^{\frac{\mu}{1-\mu}}$$

It can be seen that volume of oil production as number of employees in its production depend positively on world oil price and negatively on wage in the economy.

### Inflation, Terms of Trade and International Risk

**Sharing:** Log-linearization of consumer price index relative to the steady state for which  $P_H = P_F = P$  allows interconnecting terms of trade with inflation. Using decomposition into a Taylor series of the first order with respect to the steady state can be obtained the following formula for logarithm of consumer price index

$$p_t = (1 - \alpha)p_{Ht} + \alpha p_{Ft}.$$

Terms of trade, by definition, are the ratio of price index of a partner country to index of domestic prices of the country H. Effective trading conditions take into account terms of trade with all countries. Since logarithmic form of terms of trade  $S_t = p_{Ft} - p_{Ht}$ , the logarithm of consumer price index is

$$p_t = p_{Ht} + as_t. \quad (11)$$

This formula links consumer price index with price index of domestic goods [11]. The relation between rates of CPI inflation with inflation on domestic good prices in the country H is easily derived from it

$$\pi_t = \pi_{Ht} + a\Delta s_t. \quad (12)$$

$$c_t - \gamma c_{t-1} = c_t^* - \gamma c_{t-1}^* + \frac{1-\gamma}{\sigma} s_t. \quad (15)$$

Households can invest by acquiring both domestic securities  $B_t$  and foreign securities  $B_t^*$ . Herein the following budget constraint should be held:

$$P_t C_t + Q_{t,t+1} B_{t+1} + Q_{t,t+1}^* E_{t+1} B_{t+1}^* \leq B_t + E_t B_t^* + W_t L_t + T_t, \quad t = 0, 1, \dots$$

Through  $T_t$  transfers are defined. Log-linearization of optimality conditions for  $B_t, B_t^*$  gives the relation that links interest rate in the country with the world interest rate:

$$i_t = i_t^* + E_t \Delta e_{t+1} \quad (16)$$

**Equilibrium:** Real income  $Y_{ct}$  of the country H producing oil consists of income  $Y_b$  received from the production of goods by firms and income  $Y_{ot}$  received from oil sales abroad. The country cannot use all oil revenues for current consumption. Through  $\delta$  let's denote the share of oil revenues, which are sent to current consumption of the country H. The rest of the oil revenues are accumulated in a special fund for future use. In Kazakhstan such a fund is the National Fund of Kazakhstan. Then income for current consumption is

$$Y_{ct} = Y_t + \delta Y_{ot} = \left( \int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} + \delta \frac{P_{ot} O_t}{P_t}$$

For current consumption income  $Y_{ct}$  with account of oil revenues is used, not just revenues from firms that produce final goods. From market clearing condition of each good [11] the following formula is derived

$$y_t = c_t + \frac{\alpha\omega}{\sigma} s_t. \quad (17)$$

Then equation (15) can be rewritten as:

$$c_t - \gamma c_{t-1} = y_t^* - \gamma y_{t-1}^* + \frac{1-\gamma}{\sigma} s_t,$$

and using it we obtain

$$y_{ct} = \gamma y_{ct-1} + y_t^* - \gamma y_{t-1}^* + \frac{1}{\sigma_\alpha} s_t - \frac{\gamma\alpha\omega}{\sigma} s_{t-1} \quad (18)$$

where  $\sigma_\alpha = \frac{\sigma}{1-\alpha+\alpha\omega}$ ,  $y_{ct} = g_Y + \psi_Y y_t + (1-\psi_Y) y_{ot}$ ,  $0 < \psi_Y < 1$ .

Further, for simplification we consider the case of absence of habit formation in consumption, assuming  $\gamma = 0$ . let's substitute  $c_t$  into the Euler equation

$$y_{ct} = E_t y_{ct+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) - \frac{\alpha\omega}{\sigma} E_t \Delta s_{t+1} \quad (19)$$

This equation is a dynamic curve IS. It can be rewritten in another form replacing rate of inflation measured by consumer price index by price index of goods produced in the country:

$$y_{ct} = E_t y_{ct+1} - \frac{1}{\sigma} (i_t - E_t \pi_{Ht+1} - \rho) - \frac{\alpha(\omega-1)}{\sigma} E_t \Delta s_{t+1} \quad (20)$$

The real marginal costs of firm

$$MC_t^r = \frac{W_t + \zeta P_{ot}}{A_t P_{Ht}}$$

From here we can get their representation in logarithmic form:

$$mc_t^r = g_W + \psi_W w_t + (1 - \psi_W) p_{ot} - p_{Ht} - a_t, 0 < \psi_W < 1.$$

As for the closed economy, the equation is valid

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \lambda \widehat{mc}_t^r \quad (21)$$

where  $\widehat{mc}_t^r = \frac{(1-\beta\theta)(1-\theta)}{\theta} mc_t^r - mc^r$  and  $mc^r$  – marginal costs in the steady state [11]. Let  $\tau$  be subsidies per unit of production. Then marginal costs are adjusted as following:

$$MC_t^r = (1 - \tau) \frac{W_t + \zeta p_{ot}}{A_t P_{Ht}}$$

Given the optimality condition in the oil sector and necessary conditions for optimality in the problem of the household, this formula is transformed to

$$mc_t^r = g_W + \ln(1 - \tau) + \psi_W (\sigma c_t + \varphi l_t + \xi_t) + \psi_W \alpha s_t + (1 - \psi_W) (\sigma c_t + \varphi l_t + \xi_t + \alpha s_t - \ln \mu - a_{ot} - (\mu - 1) n_{ot}) - a_t. \quad (22)$$

In oil production sector entire volume of its revenues is divided by domestic consumption of domestic firms and export abroad

$$O_{st} = O_t + O_t^*$$

Log - linearization relative to steady state leads to the formula

$$o_{st} = g_O + \psi_O o_t + (1 - \psi_O) o_t^*, 0 < \psi_O < 1.$$

From the production function for the oil sector it follows that  $o_{st} = a_{ot} + \mu n_{ot}$  and  $o_t = \ln \zeta + n_t$ . Then

$$n_{ot} = \frac{1}{\mu} [g_O + \psi_O \ln \zeta + \psi_O y_t - \psi_O a_t + (1 - \psi_O) o_t^* - a_{ot}].$$

Labor resources are also divided between firms producing final goods and the oil sector:

$$L_t = N_t + N_{ot}, l_t = g_N + \psi_N n_t + (1 - \psi_N) n_{ot}, 0 < \psi_N < 1.$$

Given these formulas and relations (17) and (18), the real marginal costs can be expressed as follows:

$$\begin{aligned} mc_t^r = & g + (\sigma - \sigma_\alpha) y_t^* + \left[ \sigma_\alpha \psi_Y + \varphi \psi_N + \varphi (1 - \psi_N) \frac{1}{\mu} \psi_O + (1 - \psi_W) \frac{1-\mu}{\mu} \psi_O \right] y_t - \\ & - \left[ \varphi \psi_N + \varphi (1 - \psi_N) \frac{1}{\mu} \psi_O + (1 - \psi_W) \frac{1-\mu}{\mu} \psi_O + 1 \right] a_t + \\ & + \left[ \sigma_\alpha (1 - \psi_Y) + \varphi (1 - \psi_N) \frac{1}{\mu} (1 - \psi_O) + (1 - \psi_W) \frac{1-\mu}{\mu} (1 - \psi_O) \right] o_t^* - \\ & - \left[ \varphi (1 - \psi_N) \frac{1}{\mu} + (1 - \psi_W) \frac{1}{\mu} \right] a_{ot} + \sigma_\alpha (1 - \psi_Y) (p_{ot} - p_{Ht}) + \xi_t \end{aligned} \quad (23)$$



where  $g$  – a constant term. As could be noticed the production of goods and oil, as well as overseas production increase marginal costs of firms. Increasing productivity in the goods sector and in the oil sector reduce marginal costs. Let's assume that prices in the country are correlated with world oil prices, i.e. the difference between them is described by autoregression process of the first order:

$$p_{ot} - p_{Ht} = \rho_{po}(p_{ot} - p_{Ht}) + \varepsilon_{pot}$$

where  $\rho_{po}$  – positive coefficient less than one,  $\varepsilon_{pot}$  – a random variable "white noise".

In the case of flexible prices, as obtained above,  $mc^f = m$ . Through  $\tilde{y}_t = y_t - y_t^n$  we denote deviation of output under nominal price rigidity from output under flexible prices. From equation (23) for the case of flexible prices the equation for the deviation of real marginal costs from their values in the steady state is obtained

$$\widehat{mc}_t^r = \left[ \sigma_\alpha \psi_Y + \varphi \psi_N + \varphi(1 - \psi_N) \frac{1}{\mu} \psi_O + (1 - \psi_W) \frac{1-\mu}{\mu} \psi_O \right] \tilde{y}_t \quad (24)$$

Let's substitute it into the right side of equation (21) and obtain the following equation:

$$\pi_{Ht} = \beta \mathbb{E}_t \pi_{Ht+1} + \lambda_\alpha \tilde{y}_t \quad (25)$$

where the coefficient

$$\lambda_\alpha = \lambda \left[ \sigma_\alpha \psi_Y + \varphi \psi_N + \varphi(1 - \psi_N) \frac{1}{\mu} \psi_O + (1 - \psi_W) \frac{1-\mu}{\mu} \psi_O \right]$$

This equation is the New Keynesian Phillips curve for the considered here a small open economy. And for dynamic IS curve from the Euler equation (20) follows that

$$\tilde{y}_{ct} = \mathbb{E}_t \tilde{y}_{ct+1} - \frac{1}{\sigma_\alpha} (r_t - r_t^n)$$

where

$$\begin{aligned} r_t^n = & \rho + \sigma_\alpha [\alpha(\omega - 1) + g_1](\rho_{y*} - 1)y_t^* + \sigma_\alpha g_2(\rho_\alpha - 1)a_t + \sigma_\alpha g_3(\rho_o - 1)o_t^* + \\ & + \sigma_\alpha g_4(\rho_{ao} - 1)a_{ot} + \sigma_\alpha g_5(\rho_\xi - 1)\xi_t + \sigma_\alpha g_6(\rho_{po} - 1)(p_{ot} - p_{Ht}) \end{aligned}$$

Taking into account the equality  $\tilde{y}_{ct} = \psi_Y \tilde{y}_t$  the dynamic equation of IS curve can be written for the variable  $\tilde{y}_t$ :

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\psi_Y \sigma_\alpha} (i_t - \mathbb{E}_t \pi_{Ht+1} - r_t^n). \quad (26)$$

Another important component of the dynamic stochastic general equilibrium model is usually monetary policy rule. In this model we adhere to the well-known Taylor rule

$$i_t = \rho + \varphi_\pi \pi_{Ht} + \varphi_y \tilde{y}_t + v_{mt}$$

where  $\varphi_\pi, \varphi_y$  – non-negative coefficients and  $v_{mt}$  is a random variable reflecting the monetary policy shocks. Its dynamics is given by a first-order autoregression process  $v_t = \rho_{mv} v_{t-1} + \varepsilon_{mt}$  and  $\varepsilon_{mt}$  is a "white noise".

**Data and Estimation of the Model Parameters:** Following data sources were used: the official websites of government agencies (Statistics Agency, Ministry of Economic Development and Trade, National Bank, Ministry of Finance of the Republic of Kazakhstan), as well as data from the World Bank and the IMF.

Table 1: Estimates of parameters

Parameters	Prior mean	Prior p.d.f.	Post. mean
rhooz	0.700	<i>beta</i>	0.9468
phi	0.000	<i>gamma</i>	3.0143
alpha	0.660	<i>beta</i>	0.6900
eta	5.000	<i>gamma</i>	5.8618
sigma	1.000	<i>norm</i>	1.0005
nu	5.000	<i>gamma</i>	5.0775
eps	6.000	<i>norm</i>	6.0045
theta	0.700	<i>beta</i>	0.6312
rhoao	0.900	<i>beta</i>	0.9008
rhoksi	0.700	<i>beta</i>	0.6993
rhov	0.500	<i>beta</i>	0.4723
rhoyz	0.700	<i>beta</i>	0.6897
rhopo	0.800	<i>beta</i>	0.8011

There are various ways of assessing or calibrating parameters of the linearized DSGE model. The model presented in the last section includes 24 main parameters. The remaining parameters are calculated in the program based on them. Estimates obtained during calibration and econometric estimation were then refined by Bayesian method using the Metropolis-Hastings' algorithm. Table 1 shows the estimates of the main parameters.

### Impulse-response Analysis

**Internal and External Shocks:** Kazakhstan's oil revenues are sent to the National Welfare Fund, of which about 10 percent of annual income are spent on current expenditures in the economy. Hence, the parameter  $\delta$  in the first calculation is assumed to be 0.1. Let's consider responses of variables of macroeconomic models on various shocks.

*The oil price shock.* Upsurge of world oil price increases costs of firms producing goods. Figure 1 shows that the production of goods in the economy is reduced, with a stronger effect under flexible prices. There is reduction in the use of oil in the economy and in its production, the number of employed falls both in the oil sector and in other sectors of the economy.

In accordance with Figure 2 it could be noticed that the inflation rate increases sharply as on goods produced in the country, as on the consumer price index. The consequence of this is a reduction in real wages. At the same time due to rising oil revenues consumption and interest rates increase. The terms of trade improve, the real exchange rate and net exports increase. Graphs of impulse responses for other shocks are not presented.

*Shock of external demand for oil.* Oil production sharply increases and its internal consumption reduces in the country. Accordingly, the number of employees in the sector of oil rises, while it decreases in sectors producing common goods. Overall employment in the economy reduces. Under rigid prices there is a sharp decline in production of goods other than oil, although under flexible prices the volume of production of these goods would have increased due to a rapid price correction by firms.

Rates of inflation fall both on goods produced in the country and measured by consumer price index. Growth in oil production does not compensate for the downturn in other sectors of the economy. According to the Taylor rule, monetary authorities react to the decline in output and inflation by lowering the nominal interest

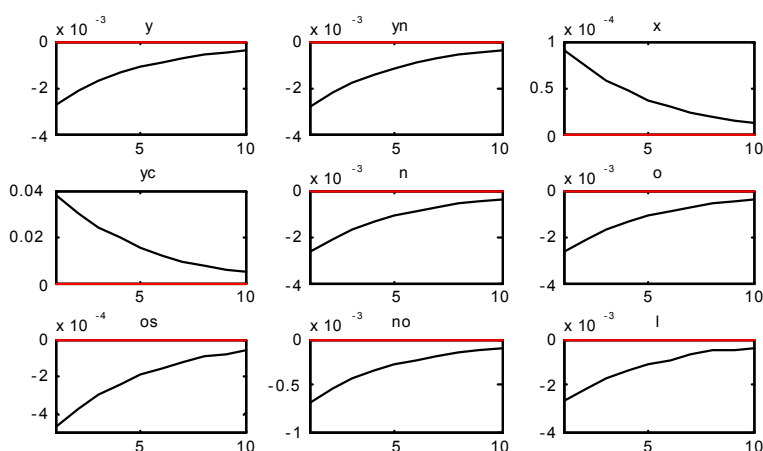


Fig. 1: Impact of oil price shock

Notes: y - output under rigid prices, yn - output under flexible prices, x - difference in the production under rigid and flexible prices, yc - total income used in the economy, n - number of employees in the production of goods, o - domestic oil consumption, os - volume of oil production, no - number of employees in the oil sector, l - total employment in logarithms.



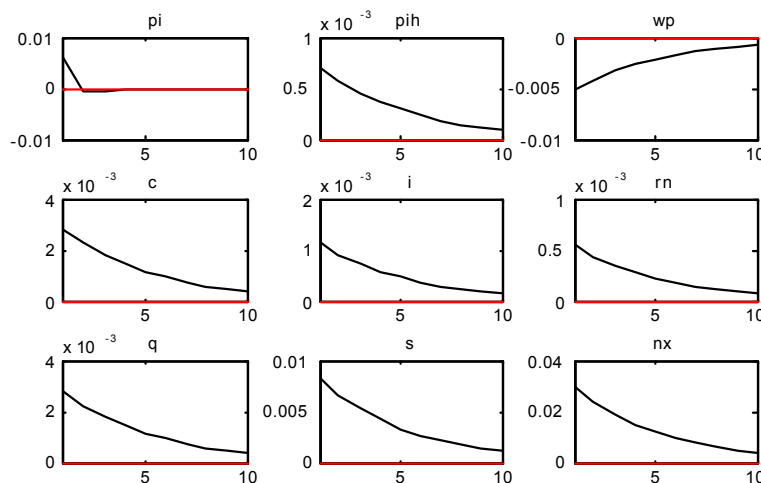


Fig. 2: Impact of oil price shock

Notes: pi - rate of inflation CPI, pih - rate of inflation for domestically produced goods, wp - real wages, c - consumption, i - nominal interest rate, rn - real interest rate under flexible prices, q - real exchange rate of the national currency, s - terms of trade, nx - net exports in logarithms.

rate. A reduction of the real interest rates takes place. A decrease in employment and output of goods leads to a reduction in average real wages. Accordingly, consumption decreases. Real depreciation of the national currency occurs. Negative impact of the shock of oil demand abroad on the economy is also revealed in the deterioration of terms of trade, which in turn, leads to a decrease in net exports.

*Productivity shock in the commodity sector.* This shock of increased productivity leads to a sharp increase in the output of goods under rigid and flexible prices. Besides, the growth under flexible prices is higher than under rigid prices. Increased production of goods causes a rise in domestic oil consumption and in number of employees both in production of goods and in the oil sector. A total number of employed in the economy increases. An output of oil production rises.

The rate of inflation decreases both on domestic products and consumer price index. As a result the total income in the economy, real wages and consumption increase. In response to the decline of inflation and a negative deviation of the output under rigid prices from the output under flexible prices monetary authorities lower interest rates. Strengthening of the national currency occurs, the terms of trade improve and the net export increases.

Let's consider implications of *productivity shock in the oil sector*. The marginal costs of oil production decrease. The volume of oil production increases while the number of employees in its production reduces.

This has a positive impact on output in the goods sector and the number of employed in the production of domestic goods increases. In general, employment in the economy rises. The total income used in the economy rises. It consists of income earned in the production of goods and the production of oil.

*Influence of oil sector productivity shock* on other variables. Inflation rates fall under both rigid and flexible prices. This has a positive impact on real wages. Consumption increases. Nominal and real interest rates reduce. The real appreciation of the national currency occurs. The terms of trade improve and net exports grow.

*The shock of monetary policy* leads to a decline in the production of goods and the number of employees occupied in their production. Correspondingly domestic oil consumption reduces and its total production falls. As a result there is a reduction in employment in the oil sector and ultimately a decline in the total number of employed in the economy.

The total income used in the economy decreases, since it consists of income derived from the production of goods and production of oil. Real wages decline and consumption reduces. In response to the decline of production and rate of inflation on domestic goods monetary authorities lower the interest rate. The national currency is experiencing real depreciation. Terms of trade deteriorate and net exports decline.

*Shock of labor preferences.* If people begin to appreciate more rest and leisure compared to consumption, this will have an impact on the economy.

Labor costs in the production of goods reduce and, as a result, domestic oil consumption goes down. The consequence is a decline in oil production in the country and the number of employees in the oil sector. Also the total number of employed in the economy contracts.

There is a rise of inflation rates on prices of domestically produced goods and consumer price index which takes into account prices of imported goods. A decline in production and growth rate of inflation will lead to a sharp decline in real wages. Consumption in the country shortens. In response to inflation growth monetary authorities reacts by lifting interest rates. Also real depreciation of the national currency happens. The terms of trade worsen and net exports of the country reduce.

*Shock of output abroad.* Since the economy is open one percent increase of output abroad will have an impact on the dynamics of macroeconomic variables in the country. In accordance with the condition of international risk sharing a jump in output in the rest world will increase consumption in the country. Balance of aggregate demand and aggregate supply in this situation provides deteriorating terms of trade and, consequently, a decrease in net exports, which will then grow. Here the effect of oil exports' growth due to rising production abroad, which was discussed above, is not taken into account. Production of goods in the country reduces, the total income used in the economy declines. The initial cut down of inflation rates will change abruptly up.

Consumption and real wages increase. Depreciation of national currency happens. As a result of reduction of output employment in the production of goods declines and domestic oil consumption reduces as well. In general, there will be a decline in oil production and a reduction in the number of employed in its production. Total employment also reduces.

**Changing of Rate of Oil Revenues Accumulation:** Now let's change the share of oil revenues, which are sent for current use and find out how it will affect consequences of various shocks. Let  $\delta = 0.9$ , i.e. 90 percent of oil revenues will be sent on the current use. This means that the share of accumulation of oil revenues in the National Welfare Fund is 10 percent. Comparison of impulse responses obtained for the case  $\delta = 0.9$  leads to the following conclusions.

*Shock of oil price.* Responses to this shock will strengthen for the following variables: production under rigid prices, production under flexible prices, total income used in the economy, number of employed in the

production of goods, domestic oil consumption, oil output, number of employed in the oil sector, total number of employed, rate of CPI inflation, inflation rate for domestically produced goods, real wages, consumption, nominal interest rate, real interest rate under flexible prices, real effective exchange rate of the national currency, terms of trade, net exports and weaken for deviations of output under rigid and flexible prices.

*Shock of external demand for oil.* Responses to the shock strengthen for the following variables: total income used in the economy, rate of CPI inflation, real wages, consumption, real exchange rate of the national currency, terms of trade, net exports. On the responses to the other variables the change in the share of accumulation of oil revenues will have no impact.

*Productivity shock in the final goods sector.* Responses to the shock strengthen for the following variables: number of employed in the production of goods, number of employed in the oil sector and weaken for the following variables: consumption, real effective exchange rate of the national currency, terms of trade, net exports. On the responses to the other variables the change in the share of accumulation of oil revenues will have no impact.

*Productivity shock in the oil sector and labor preference shock.* There is a debilitating impact on the consequences of these shocks only to responses of the following variables: consumption, real effective exchange rate of the national currency, terms of trade, net exports.

To the *shock of monetary policy* decrease of the share of oil revenues accumulation will have a debilitating impact only to responses of the following variables: consumption, real effective exchange rate of the national currency.

## CONCLUSION

In the article a small dynamic stochastic general equilibrium model for the economy of Kazakhstan, which produces oil and retain part of oil revenues in a special fund is presented. Along with the final goods sector the model includes the oil production sector, which plays an important role in the economy, provides a significant proportion of revenues to the state budget. Firms' factors of production are labor and oil as energy inputs. The oil sector also uses labor for production. The model corresponds to the new Keynesian tradition and the approach of Gali et al. with the formation of Calvo sticky prices.

The principal feature of the dynamic stochastic general equilibrium models is inclusion of equations with expectations of future values of variables. These are equations of New Keynesian Phillips curve and dynamic curve IS. Ignoring, for example, inflation expectations can lead to misleading results about the behavior of the economy in response to various internal and external shocks.

To estimate the parameters of the model mainly data of the Statistics Agency and the National Bank of Kazakhstan were used, while data for Russia and the U.S. were obtained from the database of the International Monetary Fund. Estimates of model parameters were obtained either by calibration and regression analysis or by applying Bayesian approach and Metropolis-Hastings algorithm. Values of individual parameters were taken from previous studies in other countries.

The provided calculations on the model allowed receiving forecasted responses of model variables to internal and external shocks of the economy of Kazakhstan. Internal shocks are productivity shocks in the final goods sector, productivity shock in the oil sector, labor preference shock, monetary policy shock, while external shocks are oil price shock, oil demand abroad shock and production abroad shock. Each endogenous variable responds to shocks differently. Particularly, inflation, consumption, output, employment by sectors, real wages, terms of trade and other indicators were considered.

Separately the influence of changes in the share of oil revenues accumulation in the National Fund Kazakhstan on forecasted responses of macroeconomic variables was studied. With the decrease of the share, i.e. with an increase in the share of oil revenues sent to the state budget, responses to shocks in oil prices and oil demand abroad are amplified, while responses to productivity shocks in the goods sector and the oil sector and the labor preference shock are attenuated.

Monetary policy in the model is represented by the Taylor rule. Relative increase in the coefficient of the inflation variable, i.e. strengthening central bank's reaction to deviations of the inflation rate from the target level affects responses of macroeconomic indicators. They are amplified for productivity shocks in the goods sector and in the oil sector and are weakened or change the direction for the external oil demand shock.

Consideration of responses of indicators to shocks will allow forecasting the consequences of decisions in the economy, as well as external influences on the economy.

## REFERENCES

1. Kydland, F.E. and E.C. Prescott, 1982. Time to build and aggregate fluctuations // *Econometrica*, 50: 1345-1370.
2. Calvo, G., 1983. Staggered prices in a utility maximizing framework // *Journal of Monetary Economics*, 12: 383-398.
3. Smets, F. and R. Wouters, 2003. An estimated stochastic dynamic general equilibrium model of the euro area // *Journal of the European Economic Association*/ 1(5): 123-1175.
4. Lucas, R.E., 1976. Econometric Policy Evaluation: A Critique // *Carnegie-Rochester Conference Series on Public Policy*, 1: 19-46.
5. Kumhof, M., D. Laxton, D. Muir and S. Mursula, 2010. The Global Integrated Monetary and Fiscal Model (GIMF) //IMF Working Paper. - 2010. - WP 1034.
6. Dib, A., 2001. An estimated Canadian DSGE model with nominal and real rigidities // *Bank of Canada Working Paper*, pp: 26.
7. Cuche-Curtia, N., H. Dellasb and J.M. cNatalc, 2009. DSGE-CH: a dynamic stochastic general equilibrium model for Switzerland, *Swiss National Bank* // *Economic Studies*, pp: 5.
8. Medina, J. and C. Soto, 2007. The Chilean business cycle through the lens of a stochastic general equilibrium model // *Central Bank of Chile Working Papers*, pp: 457.
9. Florian, D., and C. Montoro, XXXX. Development of MEGA-D: A DSGE model for policy analysis [http://www.frbatlanta.org/news/CONFERENCE/09qam\\_poe/montoro.pdf](http://www.frbatlanta.org/news/CONFERENCE/09qam_poe/montoro.pdf).
10. Tovar, C., 2008. DSGE models and central banks // *BIS Working Papers*, pp: 258.
11. Galí J. and T. Monacelli, 2005. Monetary policy and exchange rate volatility in a small open economy // *The Review of Economic Studies*, 72(3): 707-734.
12. Beidas-Strom, S. and T. Poghosyan, 2011. An estimated dynamic stochastic general equilibrium model of the Jordanian economy // *IMF working paper*. - 2011. - 52 p. <http://www.imf.org/external/ns/cs.aspx?id=28>.
13. Andrieu, M., 2010. A note on identification patterns in DSGE models // *ECB Working Paper*, pp: 123.
14. Guerron-Quintana, P., 2010. What you match does matter: The effects of data on DSGE estimation // *Journal of Applied Econometrics*. 25: 774-804.

15. Curdia, V. and M. Woodford, 2010. Credit spreads and monetary policy // *Journal of Money, Credit and Banking*, 42(1): 3-35.
16. Dees, S., M.H. Pesaran, L.V. Smith and R.P. Smith, 2009. Identification of new Keynesian Phillips curves from a global perspective // *Journal of Money Credit and Banking*, 41(7).
17. Kleibergen, F. and S. Mavroeidis, 2009. Weak instrument robust tests in GMM and the new Keynesian Phillips curve // *Journal of Business and Economic Statistics*, 27(3): 293-311.
18. Adjemian, S., M. Darracq Parins and S. Smets, 2009. A quantitative perspective on optimal monetary policy cooperation between the us and the euro area // *European Central Bank Working Paper*, pp: 884.
19. Del Negro, M. and F. Schorfheide, 2008. Forming priors for DSGE models (and how it affects the assessment of nominal rigidities) // *Journal of Monetary Economics*, 55: 1191-1208.
20. Kolasa, M., 2008. Structural heterogeneity or asymmetric shocks? Poland and the euro area through the lens of a two-country DSGE model// *National Bank of Poland. - 2008. - Working Paper*, pp: 49.
21. Liu, P.A., 2006. Small New Keynesian Model of the New Zealand economy//*Reserve Bank of New Zealand, Discussion Paper Series*, pp: 03/06.
22. Gali, J. and M. Gertler, 1999. Inflation dynamics: A structural econometric analysis// *Journal of Monetary Economics*, 44: 195-222.
23. National Bank of the Republic of Kazakhstan. Режим доступа: <http://www.nationalbank.kz>.
24. Taylor, J.B., 1993. Discretion versus Policy Rules in Practice, *Carnegie-Rochester Conference series on Public Policy*, 39: 195-214.
25. Vdovichenko, A.G. and V.G. Voronina, 2006. Monetary policy rules and their application in Russia // *Research in International Business and Finance*, 20: 145-162.
26. Clarida, R., J. Gali and M. Gertler, 1997. Monetary policy rules in practice: some international evidence // *NBER*, pp: 6254.
27. Drobyshevski, C. and A. Kozlovskaya, 2002. Internal aspects of monetary policy in Russia // *Proceedings of the IET*, pp: 45R.
28. Mukhamediyev, B., 2007. Rules of Monetary Policy of National Bank of Kazakhstan // *Quantile. - 2007. - 3. - C. 91-106. (Russia, Moscow)*.
29. Blanchard, O. and J. Galí, 2007. Real Wage Rigidities and the New Keynesian Model, *Journal of Money// Credit and Banking*, 39: 35-65.
30. Galí, J. and M. Gertler, 2007. Macroeconomic Modeling for Monetary Policy Evaluation // *Journal of Economic Perspectives*, 21: 25-45.
31. Smets, F. and R. Wouters, 2002. An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area. *ECB Working Paper Series*, pp: 171.
32. Smets, F. and R. Wouters, 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach//*The American Economic Review*, 97: 586-606.
33. Agency of the Republic of Kazakhstan on Statistics. <http://www.stat.kz>.
34. Alexey N. Gerasimov, Yevgeny I. Gromov, Svetlana A. Levchenko, Tamara V. Skrebtsova and Mikhail A. Kobozev, 2013. Modeling and Forecasting of Key Indicators of Socio-Economic Development of Traditionally Agrarian Regions, *World Applied Sciences Journal*, 27(10): 1282-1287.
35. Shahla Ramzan, Muhammad Inayat Khan, Faisal Maqbool Zahid and Shumila Ramzan, 2013. Regional Development Assessment Based on Socioeconomic Factors in Pakistan Using Cluster Analysis, *World Applied Sciences Journal*, 21(2): 284-292.