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Fuzzy Approach for Optimization of Cold-Formed Cross Sections

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Abstract: This paper presents a fuzzy procedure for obtaining optimal design of channel section shapes under bending moment. The proposed approach refers to an ideal, equivalent single objective representation for the bicriteria objectives, where the cross-sectional area of thin-walled beam is the first objective, while the deflection of the beam is the second one. The design variables were selected as the geometric parameters of cross section. The set of constraints includes global and stability conditions and the strength condition. The membership functions for each objective are constructed and by the proposed formulation, the bicriteria problem will be transformed to an ideal equivalent single problem. By the proposed formulation for the equivalent objective, the best compromise solution is obtained. An illustrative example is presented and the results were discussed and compared with other approaches.

Key words: Cold-formed structures · Multicriteria optimization · Optimal design · Thin-walled beams.

INTRODUCTION

Cold-formed thin-walled beams are widely used in many applications such that mechanical industry and civil engineering, as they enable obtaining any shape of the beam cross-section profiles. Recently, a great interest for improving these profiles with regard to their shapes and manufacturing. The beams may be subject to longitudinal and transverse loads. Recently, the thin-walled beams of open cross sections are usually made of higher-strength steel, hence their walls may be thinner and the whole beam may be relatively light. The characteristics of such constructions are limited mainly by general and local stability conditions and also by strength conditions. The general approach to the stability of compressed beams or buckling arising during their bending is also known for a long time. Davies [1] presented a review of studies on the general and local stability of cold-formed beams. He presented an application of a generalized beam theory to buckling problems. Hancock [2] reviewed and summarized major research developments in cold-formed steel structures. Who presented the development of the North American Specification for the Design of Cold-Formed Steel Structural Members and provided a brief summary of the direct strength method. Bazant and Cedolin [3] have discussed the problems of strength, general and local stability. They have used a classical theory of warping torsion of thin-walled beams. Trahair [4] presented mathematical models of flexural-torsional buckling and their solutions in various load cases. Ma and Hughes [5] have used nonlinear elastic theory to lateral buckling behavior of beams under distributed vertical load, taking into account distortion of the web. Bradford and Ge [6] have considered elastic distortional buckling of two-span continuous I-beam with a concentrated load in each span. Hsu and Chi [7] focused on flexural behavior of cold-formed steel beams subject to monotonic and cyclic loading. Mohri *et al.* [8] have presented overall stability of unrestrained thin-walled elements of open cross-sections. They have compared the obtained results for lateral buckling stability to the European steel code and have proposed new Wagner's coefficients. Magnucki *et al.* [9] have considered strength and stability problems of a simply supported beam subject to uniformly distributed transverse load. The solutions were supported on Vlasov's theory. The local stability was described in accordance with the theory of thin plates and shells. Magnucki and Ostwald [10] have presented the problem of multicriteria optimization of thin-walled cold-formed beams. New results on the optimal design of

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cold-formed beams are presented by Magnucki and Ostwald [11]. Manevich and Raksha [12] have discussed the problem of bicriteria optimization of thin-walled open cross section columns subject to compression. The axial compressing force and bending moment were applied as the optimization criteria. The solution was found by means of the dimensionless parameters of mass, load and stress. The approach was continued by Raksha [13]. He has optimized thin-walled beams columns subject to compression, taking into account three criteria. Magnucki *et al.* [14] studied mono-and anti-symmetrical open I-sections of cold-formed thin-walled beams with double flanges. The beams were under uniformly distributed vertical load and simply supported at both ends. A dimensionless objective function was suggested and results of the study were presented graphically. Procedures of multicriteria optimization to the optimal design of thin-walled cold-formed beams were presented by Kasperska *et al.* [15-17]. Tain and Lu [18] optimized cold-formed open-channel sections with and without the lips subjected to compressive load. Liu *et al.* [19] optimized cross-section shapes of cold-formed columns. They proposed a two-stage optimization method. The new approach consists in using classification trees that generate near-optimal cross-sections subject to further local optimization. Lee *et al.* [20] used micro Genetic Algorithm in order to find an optimal cross-section of a cold-formed steel beam under uniformly distributed load.

This paper presents a fuzzy approach for obtaining optimum design of cold-formed thin-walled beams for open cross sections under pure bending moment. The fuzzy approach [21, 22] solves the bicriteria optimization problem, where the cross-sectional area of the thin-walled beam is the first objective function, while the deflection of the beam is the second objective function. The proposed fuzzy approach is referred to a new equivalent objective representation for the two objectives. The membership functions are formulated for the objectives. The bicriteria problem will then be transformed into a single problem by adding addition variables equal to the number of objective functions. By the proposed formulation for the equivalent objective for both cross-sectional area and deflection, the best compromise solution is obtained. An illustrative example is presented and the results were discussed and compared with other approaches.

Methodology of Thin-walled Beams with Open Cross Sections: The model of the thin-walled beam is formulated for a beam with two pure bending moments M applied to the beam ends. Two suggested models of beam cross sections are presented (Fig. 1 and 2). The shapes of these models are based on a typical channel section (Model-1) and a typical channel section with single bent flanges (Model-2).

The Strength Condition: The strength condition of thin-walled beams is based on the classical Vlasov's theory. A strength condition for the two beam models subject to a pure bending moment is in the form:

$$\frac{MH}{2I} \le \tau_{allowable} \tag{1}$$

Where:

- is the pure bending moment.
- H is the depth of a beam.
- I_z is the moment of inertia of a cross section around z-axis.
- $\tau_{allowable}$ is the allowable stress.

Thus, the strength condition can be written in the following form:

$$M_1 = \frac{2I_z\tau_{allowable}}{H} \tag{2}$$

$$M \le M_1 \tag{3}$$

$$I_z = \frac{3}{4}x_1\,(x_3\text{-}x_1\,)+\frac{x_1^3\left(x_1+x_2\right)^3}{12}+x_1x_3^2(x_1+x_2) + \frac{x_1\left(x_4+\frac{x1}{2}\right)^3}{12}+\left(x_4+\frac{x1}{2}\right)\left(x_3-\frac{3x_1}{4}-\frac{x_4}{2}\right)^2 \tag{4}$$

The Lateral Stability Condition: The lateral stability condition of thin-walled beams is based on the classical Vlasov's theory. A lateral stability condition for the two beam models subject to a pure bending moment is in the form:

$$M_2 = \frac{\pi\ E}{f_{sb}\ L}\sqrt{\frac{\pi^2 I_y I_\omega}{L^2}+\frac{I_y I_t}{2\left(1+\upsilon\right)}} \tag{5}$$

$$M \le M_2 \tag{6}$$

Where:

- M_2 is the critical moment of lateral buckling.
- L is the length of the beam.
- I_y is the moment of inertia around y-axis.
- I_t is the torsion moment of inertia.

- I_ω is the warping moment of inertia.
- E is the Young's modulus.
- ν is the Poisson's ratio.
- f_{sb} is the factor of safety with respect to lateral buckling.

The Local Stability Conditions: The local stability conditions of thin-walled beams are based on the classical Vlasov's theory. A first local stability condition for the two beam models that allows in determining the critical bending moment of a flange with single bend and can be written is in the form:

$$M_3 = \frac{6 I_z}{H}\left[\frac{1}{f_{sbl}(1-2\frac{y}{H})(x_1 x_2^3 + 3 x_1 x_2^2 x_4)}\right]\left[G\ I_t + \frac{\pi^2 x_2^2 E\ I_{flange}}{L^2}\right] \tag{7}$$

$$H = x_1 + 2\,x_3 \tag{8}$$

$$I_t = \left[\frac{x_1^3 (x_2 + x_4)}{3}\right] \tag{9}$$

$$y = \frac{x_4^2}{2\,(x_2 + x_4)} \tag{10}$$

$$I_{flange} = \frac{x_1\ x_4^3}{3} - \frac{x_1\ x_4^3}{4\,(x_2 + x_4)} \tag{11}$$

$$M \le M_3 \qquad 12)$$

Where:

- M_3 is the critical bending moment of local stability of a flange with a single bend.
- y is the location of the central axis of the flange cross section.
- I_{flange} is the moment of inertia of the flange with respect to z-axis.
- I_t is the torsion moment of inertia.
- G is the shear modulus of elasticity.
- f_{sbl} is the factor of safety with respect to local buckling.

The factor of safety of local buckling f_{sbl} exceeds the factor of safety of lateral buckling f_{sb} by 50 %. Thus, the factor of safety of local buckling f_{sbl} equals 1.5 f_{sb}. These values of both factors are different in order to preclude an interaction between both buckling forms. If an interaction arises, the critical bending moment will be lower. A second local stability condition for the two beam models that allows in determining the critical bending moment of a flange with single bend and can be written is in the form:

$$M_4 = \frac{2 I_z}{3 H}\left[\frac{\pi^2 x_1^2 E}{f_{sbl}\,(1-\upsilon^2)\,x_2^2}\right] \tag{13}$$

$$M \le M_4 \tag{14}$$

A third local stability condition for the two beam models that allows in determining the critical bending moment of a rectangular plate with three supported edges and one free edge and can be written in the form:

$$M_5 = \frac{2 I_z}{H}\left[\frac{x_1^2 G}{f_{sbl}\,(1-\frac{3}{4}\frac{x_4}{x_3})\,x_4^2}\right] \tag{15}$$

$$M \le M_5 \tag{16}$$

A fourth local stability condition for the two beam models that allows in determining the critical bending moment of a web considered as a rectangular plate supported at four edges and compressed with a linearly varying distributed load and can be written is in the form:

$$M_6 = \frac{I_z}{H}\left[\frac{\pi^2 x_1^2 E}{f_{sbl}\,(1-\upsilon^2)\,x_3^2}\right] \tag{17}$$

$$M \le M_6 \tag{18}$$

Mathematical Model of Bicriteria Problem: In this paper, the weight of the beam is assumed as the first objective function. It is expressed by the area of the beam cross section as follows:

$$A\,(X) = 2\,x_1\,(x_2 + x_3 + x_4) \tag{19}$$

While, the deflection of the beam is assumed as the second objective function and is expressed as follows:

$$D\,(X) = \frac{M\ L^2}{8\ E\ I_z} \tag{20}$$

Where, I_z is the moment of inertia of a cross section around z-axis and is given by formula (4).

Membership Functions: In this case, it is necessary to construct the membership function for both objectives. In order to construct the membership function for the first objective A (X) given by Liu *et al.* [19], the individual minimum A_{min} (X) and maximum A_{max} (X) are calculated under the given stress and stability constraints. Also, in order to construct the membership function for the second objective D (X) given by Lee *et al.*[20], the individual minimum D_{min} (X) and maximum D_{max} (X) are calculated under the given stress and stability constraints. Thus, the membership function for the first objective function λ_1 (X) may be represented as follows:

$$\lambda_1(X) = \begin{cases} 0 & A(X) > A(X)_{max} \\ \dfrac{A(X)_{max} - A(X)}{R_1} & A(X)_{min} < A(X) < A(X)_{max} \\ 1 & A(X) \le A(X)_{min} \end{cases} \tag{21}$$

$$R_1 = A_{max}(X) - A_{min}(X) \tag{22}$$

While, the membership function for the second objective function λ_2 (X) may be represented as follows:

$$\lambda_2(X) = \begin{cases} 0 & D(X) > D(X)_{max} \\ \dfrac{D(X)_{max} - D(X)}{R_2} & A(X)_{min} < D(X) < D(X)_{max} \\ 1 & D(X) \le D(X)_{min} \end{cases} \tag{23}$$

$$R_2 = D_{max}(X) - D_{min}(X) \tag{24}$$

The additional two variables λ_1 (X) and λ_2 (X) represent the degree of satisfaction for both cross sectional area A (X) and the deflection D (X), respectively. Each additional variable shows the value of one when the corresponding objective is completely satisfied and shows the value of zero when this objective is not satisfied.

Proposed Fuzzy Approach: The aim of the proposed fuzzy approach is to determine the best compromise solution. This proposed fuzzy approach is referred to an ideal equivalent single objective representation for the both objectives. By the proposed equivalent formulation for the two objectives, the bicriteria problem will be transformed to an ideal equivalent single problem by adding two additional variables. These additional variables are presented in such a way that they represent the degree of satisfaction for each objective function. The equivalent membership function for both objectives can be represented as follows:

$$\text{Maximize } \lambda(X) = \frac{\lambda_1(X) + \lambda_2(X)}{2} \tag{25}$$

Also, this equivalent representation shows the value one when the both objectives are completely satisfied and shows the value zero when the both objectives are not satisfied. Thus, the best compromise solution for both objectives can be obtained by solving the following preference problem P (PBP):

P (PBP): Maximize $\lambda(X) = \dfrac{\lambda_1(X) + \lambda_2(X)}{2}$

Subject to:

(21), (23)

$$\begin{aligned} &M \le M_j,\ j = 1, 2, \ldots 6 \\ &x_i \ge 0,\ i = 1, 2, 3, 4 \text{ and integer.} \end{aligned} \tag{26}$$

Note that the sets of geometric constraints must be considered such as the values of all decision variables must be greater than zero and other standard requirements. These standard requirements may be summarized as follows:

$$\begin{aligned} &x_3 \ge x_4 \\ &x_1 + x_2 \le H,\ H = x_1 + 2x_3,\ H_{max} = 200 \text{ mm.} \\ &H \le 200 \text{ mm},\ x_1 \le 16 \text{ mm} \\ &\frac{H}{x_1} \ge 10,\ \frac{2x_3}{x_1} \ge 10 \\ &\frac{x_2}{x_1} \ge 10,\ \frac{x_4}{x_1} \ge 10 \end{aligned} \tag{27}$$

The set of constraints (28) are referred to standard requirement constraints for optimal design of cold-formed cross section. The main advantage of the proposed preference problem P (PBP) is to obtain only one solution called the best compromise solution. While, other approaches used parametric study and present Pareto-solution according to the value of the proposed parameters. Also, other approaches present infinite number of Pareto-solution referred to the value of the proposed parameters. The proposed fuzzy approach presented in this paper obtains only one solution and thus no need to do parametric study as in other approaches.

Numerical Analysis Calculations: Numerical investigation was carried out for a family of thin-walled beams subject to pure bending moment with the following parameters:

- Pure bending moment M = 10, 20, 30 x 10^6 N. mm, Young's modulus E = 205 000 N /mm^2.
- Poisson's ratio ν = 0.3, Allowable stress $\tau_{allowable}$ = 0.0015 E = 307.5 N / mm^2.
- Shear modulus G = 78846.1538 N /mm^2, Beam length L = 1, 2, … …10, 12 m.
- Factor of safety for lateral buckling f_{sb} = 1.8.
- Factor of safety for local buckling f_{sbl} = 2.7.

Results of single objective optimization for both cross section area A(X) and deflection D(X) for different cases of L = 1, 2, …10, 12 m at M = 10, 20, 30 x 10^6 N. mm are given in Tables 1-3.

Results of the proposed fuzzy approach for the problem given by (25) - (27) for the equivalent preference function with the corresponding values of membership functions for different cases of L = 1, 2, …,10,12m at M = 10, 20, 30 x 10^6 N. mm are given in Tables 4-6. Comparison between other approaches and the proposed fuzzy approach is summarized in Table 7.

Figs 1-3 show the lower and upper deflection for different cases of L = 1, 2,..…, 10, 12 m at M = 10, 20, 30 x 10^6 N. mm. While, Figs 4-6 show the lower and upper area for different cases of L = 1, 2, ……0, 12 m at M = 10, 20, 30 x 10^6 N. mm. Figs 7-9 show the results of membership function of the fuzzy approach for different cases of L = 1, 2, …10, 12 m at M = 10, 20, 30 x 10^6 N. mm. While, Figs 10-12 show the results of dimensionless objectives (Deflection/Length) and (ML/E Iz) for different cases of L = 1, 2 …10, 12 m at M = 10, 20, 30 x 10^6 N. mm.

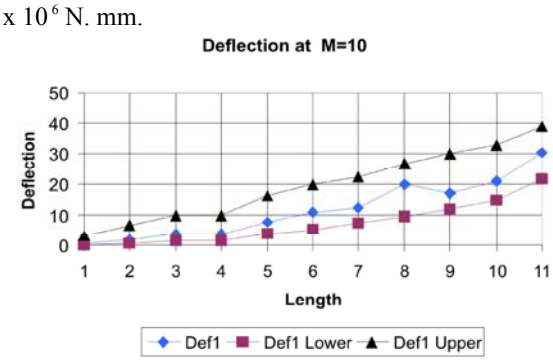

Fig. 1: lower and upper deflection for L = 1, 2……10, 12 m at M = 10 x10^6 N. mm.

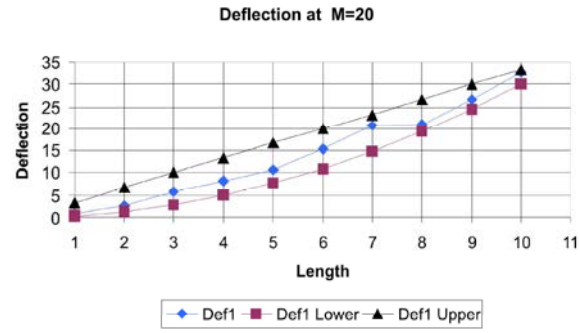

Fig. 2: lower and upper deflection for L = 1, 2….10 m at M = 20 x10^6 N. mm.

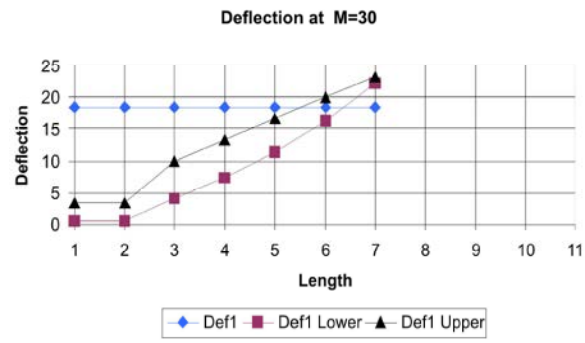

Fig. 3: lower and upper deflection for L = 1, 2…..7 m at M = 30 x10^6 N. mm.

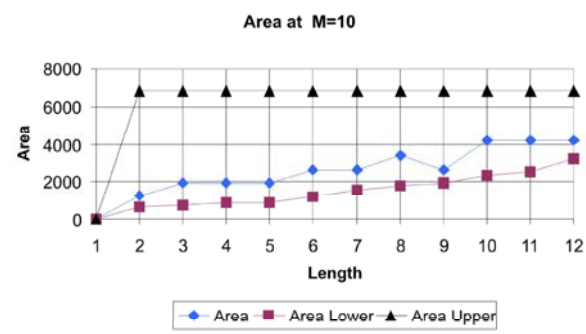

Fig. 4: lower and upper area for L = 1, 2.….10, 12 m at M = 10 x10^6 N. mm.

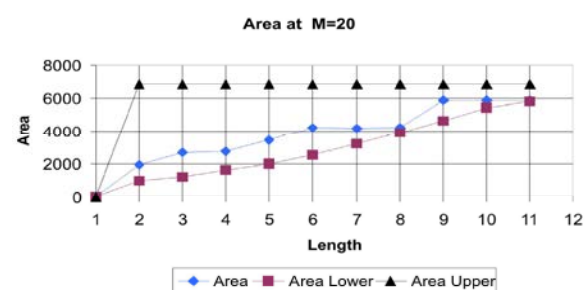

Fig. 5: lower and upper area for L = 1, 2…..10 m at M = 20 x10^6 N. mm.

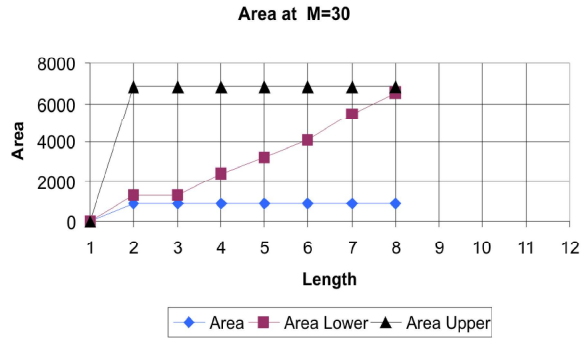

Fig. 6: lower and upper deflection for L = 1, 2….7 m at M = 30 x10 ^{6} N. mm.

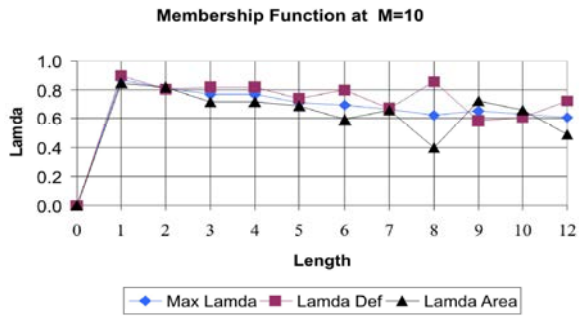

Fig. 7: Results of fuzzy approach for L = 1, 2….10, 12 m at M = 10 x10 ^{6} N. mm.

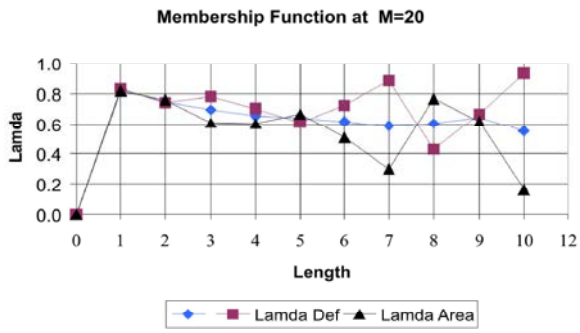

Fig. 8: Results of fuzzy approach for L = 1, 2….10 m at M = 20 x10 ^{6} N. mm.

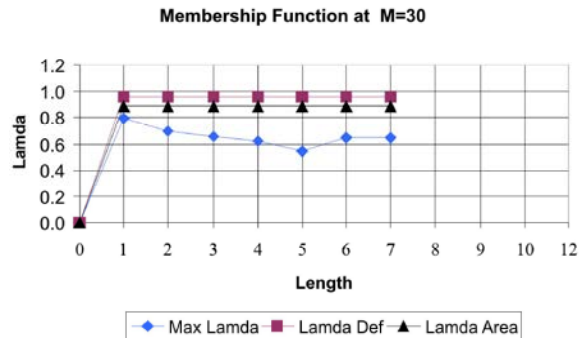

Fig. 9: Results of fuzzy approach for L = 1, 2…..7 m at M = 30 x10 ^{6} N. mm.

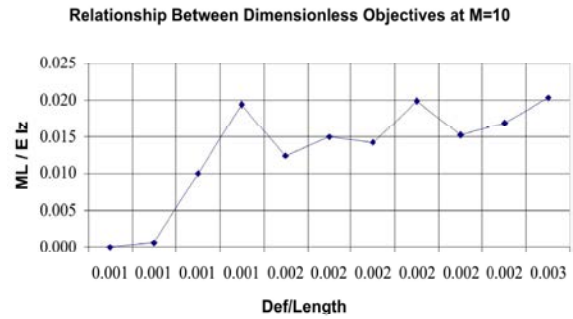

Fig. 10: Dimensionless objective functions Def / L and ML / EI $_z$ at M = 10 x10 ^{6} N. mm.

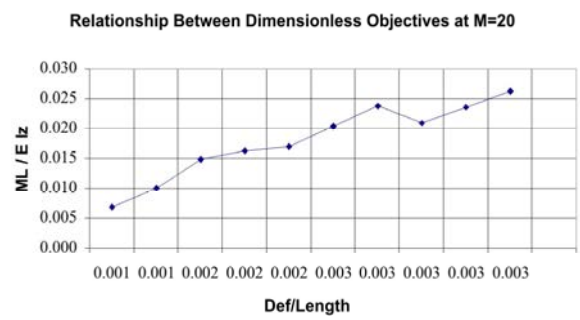

Fig. 11: Dimensionless objective functions Def / L and ML / EI $_z$ at M = 20 x10 ^{6} N. mm.

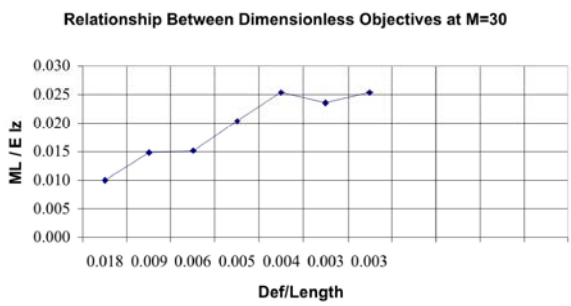

Fig. 12: Dimensionless objective functions Def / L and ML / EI $_z$ at M = 30 x10 ^{6} N. mm.

Table 1: Results of single objective optimization at moment M = 10 kN m.

Length m	1	2	3	4	5	6	7	8	9	10	12
A_{min} mm^2	612	756	864	864	1204	1578	1770	1974	2352	2536	3200
A_{max} mm^2	6840	6840	6840	6840	6840	6840	6840	6840	6840	6840	6840
R_1	6228	6084	5976	5976	5636	5262	5070	4866	4488	4304	3640
D_{min} mm	0.151	0.603	1.356	1.356	3.767	5.425	7.384	9.645	12.21	15.07	21.7
D_{max} mm	3.309	6.665	9.937	9.937	16.601	19.86	22.42	26.64	29.99	32.71	39.01
R_2	3.158	6.062	8.581	8.581	12.834	14.43	15.03	16.99	17.78	17.64	17.31

Table 2: Results of single objective optimization at moment M = 20 kN m.

Length m	1	2	3	4	5	6	7	8	9	10
A_{min} mm^2	984	1194	1578	1974	2536	3200	3876	4648	5424	5824
A_{max} mm^2	6840	6840	6840	6840	6840	6840	6840	6840	6840	6840
R_1	5856	5646	5262	4866	4304	3640	2964	2192	1416	1016
D_{min} mm	0.301	1.206	2.713	4.822	7.535	10.85	14.77	19.29	24.41	30.14
D_{max} mm	3.295	6.664	9.999	13.27	16.66	19.98	23.27	26.67	29.99	33.29
R_2	2.994	5.458	7.286	8.447	9.125	9.13	8.51	7.38	5.572	3.151

Table 3: Results of single objective optimization at moment M = 30 kN m.

Length m	1	2	3	4	5	6	7
A_{min} mm^2	1254	1578	2352	3200	4068	5424	6534
A_{max} mm^2	6840	6840	6840	6840	6840	6840	6840
R_1	5586	5262	4488	3640	2772	1416	306
D_{min} mm	0.452	1.808	4.069	7.234	11.302	16.275	22.153
D_{max} mm	3.312	6.666	9.989	13.32	16.641	19.99	23.275
R_2	2.86	4.858	5.92	6.086	5.339	3.715	1.122

Table 4: Results of fuzzy optimum cross section design at moment M = 10 kN m.

Length m	1	2	3	4	5	6	7	8	9	10	12
λ_{max}	0.873	0.814	0.769	0.671	0.713	0.695	0.669	0.626	0.654	0.633	0.608
Area	1268	1944	1944	1312	2672	2664	3410	2672	4212	4212	4212
λ_1	0.895	0.805	0.819	0.886	0.739	0.794	0.677	0.856	0.586	0.611	0.722
Deflection	0.623	1.680	3.780	9.731	7.775	11.24	12.48	19.90	17.13	21.15	30.45
λ_2	0.851	0.822	0.718	0.455	0.688	0.597	0.661	0.396	0.723	0.656	0.494
x_1	2	3	3	2	4	4	5	4	6	6	6
x_2	198	196	196	198	196	195	194	196	194	194	194
H	200	199	199	200	200	200	199	200	200	200	200
x_3	98	98	98	99	98	98	97	98	97	97	97
x_4	20	30	30	31	40	40	50	39	60	60	60
Def / L	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.003
ML / E Iz	.0007	.0101	.0195	.0124	.0150	.0143	.0199	.0152	.0169	.0203	.0007

Table 5: Results of fuzzy optimum cross section design at moment M = 20 kN m.

Length m	1	2	3	4	5	6	7	8	9	10
λ_{max}	0.828	0.752	0.696	0.656	0.639	0.620	0.593	0.603	0.645	0.552
Area	1944	2671	2720	3410	4212	4200	4212	5888	5888	5888
λ_1	0.836	0.738	0.783	0.705	0.611	0.725	0.887	0.434	0.672	0.937
Deflection	0.840	2.488	5.560	8.147	10.57	15.28	20.72	20.97	26.54	32.76
λ_2	0.820	0.765	0.609	0.606	0.667	0.514	0.300	0.772	0.619	0.168
X_1	3	4	4	5	6	6	6	8	8	8
X_2	196	196	196	194	194	193	194	192	192	192
H	199	200	200	199	200	200	200	200	200	200
X_3	98	97	98	97	97	97	97	96	96	96
X_4	30	40	48	50	60	60	60	80	80	80
Def / L	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.003
ML / E Iz	0.0067	0.0100	0.0148	0.0163	0.0169	0.0204	0.0237	0.0210	0.0236	0.0262

Table 6: Results of fuzzy optimum cross section design at moment M = 30 kN m.

Length m	1	2	3	4	5	6	7
λ_{max}	0.797	0.696	0.654	0.621	0.547	0.645	0.647
Area	876	876	876	876	876	876	876
λ_1	0.956	0.956	0.956	0.956	0.956	0.956	0.956
Deflection	18.440	18.440	18.440	18.440	18.440	18.440	18.440
λ_2	0.892	0.892	0.892	0.892	0.892	0.892	0.892
x_1	3	3	6	6	6	8	9
x_2	196	196	194	194	194	192	190
H	199	199	200	200	200	200	199
x_3	98	98	96	97	97	96	95
x_4	33	33	60	60	60	80	90
Def / L	0.018	0.009	0.006	0.005	0.004	0.003	0.003
ML / E Iz	0.0100	0.0148	0.0152	0.0203	0.0254	0.0236	0.0253

Table 7: Comparison between other approaches and the proposed fuzzy approach.

Weighting method	Proposed fuzzy approach
Needs parametric study	No need for any parametric study
Needs convexity assumption	No need for any convexity assumption
No information about the suitable weights for obtaining the efficient solution.	Leads directly to the best compromise solution by solving only one problem.
Solving a sequence of problems.	Solving only one problem.
Obtaining infinite number of efficient solutions.	Obtaining only one compromise solution.

CONCLUSION

This paper presents a fuzzy procedure for obtaining optimal design of channel section shapes under bending moment. The proposed approach refers to an ideal, equivalent single objective representation for the bicriteria objectives, where the cross-sectional area of thin-walled beam is the first objective, while the deflection of the beam is the second one. The membership functions are formulated for the objectives. The bicriteria problem will then be transformed into a single problem by adding addition variables equal to the number of objective functions. These additional two variables represent the degree of satisfaction for both cross sectional area and the deflection. Each additional variable shows the value of one when the corresponding objective is completely satisfied and shows the value of zero when this objective is not satisfied. By the proposed formulation for the equivalent objective for both cross-sectional area and deflection, the best compromise solution is obtained. By the proposed equivalent formulation for the two objectives, the bicriteria problem will be transformed to an ideal equivalent single problem. The main advantage of the proposed preference problem P (PBP) is to obtain only one solution called the best compromise solution. An illustrative example is presented and the results were discussed and compared with other approaches. It has been noted that the cross section areas for different beam lengths are the same, while the deflection functions are different.

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