Application of Stochastic Dynamic Programming in Water Allocation, Case Study: Latian Dam

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Abstract: Optimal allocation and dynamic and stochastic factors in water allocation are important. So in this paper, stochastic dynamic programming (SDP) is used to determine the optimum of water allocation to farmers of Varamin plain and municipal of Tehran city from Latian dam during 1991-2012. Objective function is consumer welfare surplus that obtained from the area under the agricultural and municipal water demand functions. To estimate the demand function a panel data approach is used. Given that the operation of the reservoir dam has always been a conflict between the decision makers, the purpose of this paper is to determine the optimal allocation of water. Hence, it is assumed the amount of water allocation to farmers is not optimal and it must be increased. According to results, optimal of water allocation for agriculture and municipal is 220.704 and 383.561 MMC, respectively. Hence, the amount of water allocated to agriculture can be increased. Also, according to demand curve of water, agricultural water supply in drought conditions is necessary.

JEL Classification: Q25, D6, C6, C23

Key words: Optimal Water Allocation, Stochastic Dynamic Programing, Latian Dam

INTRODUCTION

Reservoir system managements are designed for determining water release by considering the interests of the reservoir stakeholders, inflows, impounded water volume, release capacity, water demands and downstream constraints [1]. This multiple activities and objectives complicated supply-demand conflicts [2]. While the demand of water reaches a limit of what the natural system can provide, water shortage will become a major obstacle to social and economic development for one region [3]. More urban regions where demands outstrip water resources availabilities have suffered from chronic severe shortages, particularly when faced with the rapid population increase and speedy economic development. This competitiveness can strengthen the agricultural water shortage and serious problems (e.g. agricultural sustainability concerned) can thus arise from poorly planned water-management systems when merely limited water resources are available for multiple competing users [4]. For years, controversial water-allocation issues among municipal, industrial and agricultural users have been intensified [5].

In past years, water scarcity has become threat in most arid and semiarid regions around the world, including reservoir [6] and much research has been conducted using reservoir optimization models to identify optimal operating strategies [7]. But, in water resources management, uncertainties could intensify the conflict-laden issue of water allocation among competing municipal, industrial and agricultural interests. Consequently, to address the uncertain parameters Stochastic Dynamic Programming (SDP) was applied to reservoirs management [8-12]. In stochastic programming uncertain parameters are treated as random variables with a known probability density function (PDF). Hence, the consequences of the decisions taken at present re not known until the unknown data is realized. Application of value iteration approach to obtain the value function that solves the Bellman equation, which is adopted mainly as an acceleration method [13-15].

The objective of this paper is to analyze the optimal water allocation of Latian dam reservoir between municipal and agricultural users. For this purpose, Stochastic Dynamic Programming (SDP) has been applied. The basic part of this study are presented as follows.
In Section 2, data collection and case study have been described. In section 3, the methodology of the paper is presented along with the value iteration approach and SDP model. The empirical analysis and the model application results in the study area are analyzed in Section 4. Finally, a conclusion is given in Section 5.

Case Study and Data: The Latian Dam is located on Jajrood River in the northeast of Tehran, the capital of Iran (Figure 1). It is one of the most important municipal and agricultural water supply reservoirs in Tehran city and Varamin Plain. Also, the Latian Reservoir is a moderately eutrophic, with an area of 3.3 km² and a drainage area of 670 km². This reservoir water to provide 6000 hectares of arable Varamin plain and 30% of Tehran drinkable water. Data is collected from Ministry of Agriculture, Water Resources Management Company of Iran and 240 questionnaires in 2011-2012.

MATERIALS AND METHODS

The Value Iteration Approach: In this approach, a numerical approximation to the infinite horizon value function has used that maximizes the value of the problem resulting from decisions carried out in the future. For a generic objective function $f(x, w)$ and an equation of motion for the state variable $x_{t+1} = g(x, w)$ the Bellman equation is:

$$
\begin{align*}
\text{Max} V(x_t) &= f(x_t, w_t) + \left( \frac{1}{1+r} \right) V(x_{t+1}) \\
\text{Subject to} &
\end{align*}
$$

$$
\begin{align*}
x_{t+1} &= g(x_t, w_t), \quad w_t \geq 0 \quad (1)
\end{align*}
$$

The functional form that chosen for the polynomial approximation to the infinite-horizon is a Chebychev Polynomial, which belongs to a family of orthogonal polynomials described by Judd [13] and implemented by Provencher and Bishop [15]. The approximation takes the form:

$$
V(x_t) = \sum \alpha_i \varphi_i(M(x)) \quad (2)
$$

Where $\varphi$ is the coefficient of the $i$th polynomial term $\varphi_i(M(x))$ which is defined over the interval given by the mapping $\tilde{z} = M(x)$, that is $[-1, 1]$ in the case of the Chebychev polynomial. The expressions of the Chebychev polynomial are sinusoidal in nature and are given (for the $n$th term) by the relationship $\tilde{z} = \cos \left( n \cos^{-1}(\tilde{z}) \right)$.

Stochastic Dynamic Programming Model: Uncertainty in water resources management could be due to the dynamics of the ecosystem or exogenously caused by weather, prices, or institutions. SDP approach is the dominant method to solving the discrete time stochastic dynamic equations of motion. This method has been concentrated in optimal normative intertemporal water allocation.

Equation 3, represented a resource network based on a single state representation in a network system. The dynamics of the system are given by:
\[ \Delta X_t = \tilde{e}_t - W_t \]  

(3)

The change reservoir stock must balance the stochastic change \( \tilde{e}_t \) and resource use \( w_t \). The index \( t \) in equation (3) denotes time period. The final demand for resources/services is satisfied by resource flows from \( X_t \), namely \( w_t \).

We define the following timing of information and controls. First, the decision-maker observes the realization of the exogenous stochastic stock change variable \( \tilde{e}_t \) and hence \( X_t \). Second, the decision-maker chooses the control \( w_t \), the level of resource extraction or harvest. \( P(W) \) is the intermediate value of flow resources that defined by the inverse demand function. This function is \( P = g + kW \), where \( P \) is water price, \( W \) is water consumption for agriculture and municipal sector and \( g \) and \( k \) are the intercept and the price coefficient. The net surplus, \( CS(w) \) derived from water consumption is denoted by:

\[ CS(w) = \int P(w), dw \]  

(4)

The Surplus of Water Consumption Is a Concave Increasing Function: The optimal decision for all years is same, if the time horizon is infinite. The stochastic dynamic recursive equation that defines the optimal water management is:

\[
\text{Max} V_t(X_t, \hat{e}_t) = CS_t(w_t) + \left( \frac{1}{1+r} \right) \left[ \int V_{t+1}(X_{t+1}, \hat{e}_{t+1}) dF \right] 
\]  

(5)

Subject to,

\[ \Delta X_t = \hat{e}_t - w_t \]  

(6)

\[ X_t \leq X_{t+1} \leq \bar{X} \]  

(7)

\[ w_t \geq 0 \]  

(8)

In this equation, \( X \) denote a vector of state variables, \( w \) a vector of control variables and \( \hat{e} \) a vector of random events that influence the state variables, the objective function or both. \( r \) is interest rate. The distribution of the stochastic vector is known, a priori. The objective function (5) is maximized to obtain the optimal set of controls \( \left[ w_1^*, \ldots, w_T^* \right] \) subject to (6), (7) and (8). The objective function defined as the discounted sum of net consumer surplus for agriculture and municipal users. The objective of the decision maker is to obtain an optimal water release in order to maximize the accumulated value over time. At each year, the level of a state variable is a function of the state variable level at the previous year, the control and the realized stochastic variables, (6). The problem is limited by feasibility constraints. At every year, the current net surplus depends on the water releases. Consequently the objective function is the expectation of the current net surplus.

Instead of the traditional methods of optimizing the value function for discrete points in the probability, control and state spaces, two approximations have made, the value function and information accumulation by the decision-maker. We approximate the expected value function by supposing that it is a continuous function in state space. In addition, it is assumed that the decision-maker at any year regards the stochastic control problem as a closed loop problem based on the recent information. This information is obtained with respect to the amount of water release, to be updated each time by stochastic condition. Implicitly, this approximation suppose that the ability of the decision-maker to observe the updated level of the state variable in the future does not alter the current optimal control given the current state of the system.

Municipal Demand: Estimates of municipal water demand function are based on water price, per capita income and climate parameters and water demand in this sector is sensitive to price, but is inelastic\[16\],\[17\]. Municipal water demand is derived using Stone-Geary function and it is estimated using the random effects model. The price elasticity of municipal water demand function has obtained from this literature;\[18\],\[19\],\[20\],\[21\]. A simple linear form is fitted to the resulting data yielding.

Agricultural Demand: Agricultural water demand is calculated according to irrigation requirements based on crop areas and climate. Irrigation requirements of crops were estimated using the Penman-FAO-Monteith approach based on climate and crop culture in Varamin plain \[22\].

To estimate the agriculture water demand function in Varamin plain, a statistical survey has been conducted on a sample of 240 farmers with 8 crops. The water demand function is: \( W = \beta_0 + \beta X_{an} \), that \( W \) is water consumption, \( \beta \) is parameter and \( X \) denote the vector of independent variable (price of water per cubic (10 Rials), Net Revenue (Million Rilas per Hectare) and Climate parameters (temperature and Precipitation)). This equation was estimated with Evewis and Panel Data Approach.
RESULT

Agriculture and Municipal Water Demand Function:
The results of agriculture and municipal water demand function are presented in the Table 1.

Price is inelasticity in both function, but elasticity is agriculture is less than municipal. Using these parameters, intercept and slope can be achieved. So, the consumer surplus of water demand achieved for farmers and households (equation 4).

Application of Sdp in Latian Reservoir: The Latian Dam Reservoir can be modeled as a single aggregated reservoir and release to the Varamin Plain irrigators and Tehran population water. Accordingly, \( X \) is the stock of water in Latian reservoirs, \( \tilde{\epsilon} \) are the stochastic levels of inflow to the reservoir, \( w \) are the water releases from the reservoir that produce water supply for irrigation and drinking. It’s supposed that yearly inflows are i.i.d distributed with a log-normal distribution \( \tilde{\epsilon}_{i,j} \sim \text{lognormal}(\mu, \sigma^2) \).

The maximum and minimum storage capacity in the Latian dam each year is 685 and 209 MCM, respectively. Data on actual inflows, releases and storage are available for 1991 to 2012. A log-normal distribution was fitted to the observations and used to generate a set of 5 and 4 discrete probabilities for the associated inflow quantities for municipal and agriculture, respectively. The decision-maker is assumed to maximize the sum of the expected net present value of the water releases over this time period. The objective function maximizes subject to the equation of motion for the reservoir stock and the feasibility constraints. The problem is solved using NLP procedure with GAMS package.

A seven degree Chebychev polynomial approximation of the value function is obtained. This polynomial coefficients are iteratively computed using the Chebychev regression algorithm. Table 2 shows the Chebychev polynomial coefficients.

In Table 3 are shown the results of maximizing the objective function. The NPV is higher for urban consumers to Agricultural consumers. The optimal amount of water allocated to urban is more than farmers, because they won more prosperity and are willing to pay a higher price for water. The optimal amount of water allocated to farmers and urbanites are more than long-term averages (72.5 and 121.6 MCM, respectively). Hence, the two groups are demanding more water for consumption.

To evaluate the quality of fit of SDP model, it’s simulated the optimal predicted releases and storage for Latian reservoirs over the historic time period, using the actual realized inflows to set the initial conditions for each year’s optimization. Figure 1-A to 1-D presents the SDP policy simulations versus the observed ones for water release (the control) and storage (the state). The information used by the decision-maker, namely the current reservoir storage level, the current runoff and the probability distribution of stochastic inflows. The optimization at each time, the marginal value of current releases must be equal to the expected value of water stored and carried over for use in next years.

The comparison between the simulated and actual amounts allocated water to agriculture and drinking consumers, showed in figure 1-A and 1-B. As this chart shows, the dynamic stochastic programming model is well able to simulate the amount of allocated water and there is a little difference between the actual and simulated. Therefore, the output from the models is reliable.

In Figure 1-C and 1-D, the total amount of reservoir water in the two models are compared. Values simulated is more suitable in agricultural model. The amount of water allocated to farmers has more random change, but the amount of water allocated to farmers have not changed much. Because in the stochastic conditions, the amount of water allocated to agriculture is associated with greater changes.

The value of every unit of water in every year for agriculture and municipal presented in Figure 2-A and 2-B. This value obtained by dividing the current value to the amount of water allocated. This function is more elasticity for Municipal users to Agricultural users. Average value of municipal and agricultural water are 237.77and 413.74 (10 Rials), respectively. Given that the price elasticity of water demand is less for farmers, thus, every unit of water is more valuable to them and are willing to pay more price for water consumption. Therefore, in drought years, the agricultural water must be provided.
Table 3: Optimal water allocation and value of objective function

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<td>Annual Water Allocation (MCM)</td>
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Fig. 1-A: Compare between actual and simulation of agricultural water allocation

Fig. 1-B: Compare between actual and simulation of Municipal water allocation

CONCLUSION

In water resources management, uncertainties could intensify the conflict-laden issue of water allocation among competing municipal, industrial and agricultural interests. Hence, in this study, the optimal allocation between farmers and municipal has been investigated by using stochastic dynamic programming in latian Dam at 1992-2012. SDP approach is the dominant method to solving the discrete time stochastic dynamic equations of motion. This method has been concentrated in optimal normative inter temporal water allocation. The result show that optimal of water allocation for agriculture and municipal is 220.704 and 383.561 MMC, respectively. Hence, the amount of water allocated to agriculture and municipal can be increased. Also, according to demand curve of water, agricultural water supply in drought conditions is necessary.

REFERENCES