Solution of the Problem of Steady Motion of the Perfect Biphasic Media in the Open Uniform Cross-Section Channels

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Abstract: The steady motion of the perfect biphasic media is considered in this research. The formulas for definition of velocity and phase concentration were determined. Based on the analysis of the formulas that have been proved it may be deduced the following: by motion of a biphasic medium the velocities of both phases tend with distance from the motion onset to the same constant number. At the same time the phase concentration tends to the different constant numbers.

Key words: Steady motion · Perfect biphasic medium · Incompressible phases · True phase density · Reduced phase density · Phase concentration · Phase velocity · Hydrodynamic pressure · Phase interaction factor · Channel cross-section

INTRODUCTION

Over the last years certain progress was achieved in hydromechanics in the area of research of flow dynamics in the open channels [1, 2]. However, these models do not completely cover the physics of the process since in the Central Asia water used for irrigation is non-homogenous and contains certain amounts of solids [3]. It is commonly known that presence of a small number of solids in the flow substantially changes the process nature and structure [4, 5]. In this case the macroscopic parameters appear including reduced densities, interacting forces between the phases as well as other mechanical characteristics. These flow parameters breach the basic phase component conservation law, the components interact which results in the redistribution of velocity, concentration of particular components, changes the mixture flow. This is why study of the multiphase media motion is one of the topical issues of the modern continuum mechanics. We consider the elementary case of the steady motion of the perfect biphasic media. But even for this case the research is not completed. For example, the analytic formulas for velocities and phase concentrations are still not determined. The object of this article is to fill this gap.

Solution Procedure: The variable separation method was used for solution of the obtained ordinary differential equation and the calculations were made in the Mathcad environment [6-8].

The Main Part: The problem is solved on the basis of the “interpenetrating” model of the biphasic media according to which the equations of motion appear as follows [4, 5, 9, 10]:

\[
\rho_n \frac{\partial u_n}{\partial t} + \rho_n \mu_n \frac{\partial u_n}{\partial x} + \rho_n v_n \frac{\partial u_n}{\partial y} = -f_n \frac{\partial p_n}{\partial x} + \frac{\partial}{\partial y} \left( 2 f_n \mu_n \left( \frac{\partial u_n}{\partial x} + \frac{1}{3} \text{div} V_n \right) \right) + \\
+ \frac{\partial}{\partial y} \left( f_n \mu_n \left( \frac{\partial u_n}{\partial y} + \frac{\partial v_n}{\partial x} \right) \right) + \sum_{i=1}^{2} K(u_i - u_n) + \rho_n X_n,
\]

\[
\rho_n \frac{\partial v_n}{\partial t} + \rho_n \mu_n \frac{\partial v_n}{\partial x} + \rho_n u_n \frac{\partial v_n}{\partial y} = -f_n \frac{\partial p_n}{\partial y} + \frac{\partial}{\partial x} \left( 2 f_n \mu_n \left( \frac{\partial u_n}{\partial y} + \frac{1}{3} \text{div} V_n \right) \right) + \\
+ \frac{\partial}{\partial x} \left( f_n \mu_n \left( \frac{\partial u_n}{\partial x} + \frac{\partial v_n}{\partial y} \right) \right) + \sum_{i=1}^{2} K(v_i - v_n) + \rho_n Y_n,
\]

and the equation of continuity

\[
\frac{\partial \rho_n}{\partial t} + \frac{\partial}{\partial x} \left( \rho_n u_n \right) + \frac{\partial}{\partial y} \left( \rho_n v_n \right) = 0,
\]

\[f_1 + f_2 = 1, \rho_n = \rho_n \frac{f_n}{f_n},\]
Let us consider the case of the steady onedimensional flow of the perfect biphasic media in the open channels. By doing so let us assume that both components are incompressible and the body force can be neglected. In such case the equations of motion appear as follows:

\[
\begin{align*}
\rho_1 u_1 \frac{du_1}{dx} &= -f_1 \frac{dp}{dx} + K (u_2 - u_1), \\
\rho_2 u_2 \frac{du_2}{dx} &= -f_2 \frac{dp}{dx} + K (u_1 - u_2),
\end{align*}
\]

and the equation of continuity due to flow uniformity and according to the flow formula

\[
\begin{align*}
\frac{d}{dx} \left( \rho_1 u_1 \omega \right) &= 0, \\
\frac{d}{dx} \left( \rho_2 u_2 \omega \right) &= 0,
\end{align*}
\]

where \(\omega\) is the useful flow area.

By integrating this equation under the initial condition \(x = x_0, u_1 = u_{10}\) we deduce formula for the velocity of the first component

\[
\left( \frac{u_1}{u_{10}} \right) \frac{A}{\rho_1 u_{10}} e^{\frac{u_1 - u_{10}}{u_{10} - f_{10} u_{10}}} \frac{\rho_1 u_{10}}{\rho_1 u_{10} + K (u_1 - f_{10} u_{10} - f_{20} u_{20}) \frac{D_1}{\rho_1 u_{10}}} \frac{K (x - x_0)}{\rho_1 u_{10}} = \frac{K (x - x_0)}{\rho_1 u_{10}}.
\]

Likewise, for the velocity of the second component:

\[
\left( \frac{u_2}{u_{20}} \right) \frac{A}{\rho_2 u_{20}} e^{\frac{u_2 - u_{20}}{u_{20} - f_{10} u_{20}}} \frac{\rho_2 u_{20}}{\rho_2 u_{20} + K (u_2 - f_{10} u_{10} - f_{20} u_{20}) \frac{D_2}{\rho_2 u_{20}}} \frac{K (x - x_0)}{\rho_2 u_{20}} = \frac{K (x - x_0)}{\rho_2 u_{20}}.
\]
where

\[
A_i = \frac{\rho_{10} f_{10} \mu_{10}^2 (f_{10} \mu_{10} + 2 f_{20} \mu_{20}) + \rho_{20} f_{20} \mu_{20}^2 (2 f_{10} \mu_{10} + f_{20} \mu_{20})}{(f_{10} \mu_{10} + f_{20} \mu_{20})^2}
\]

\[
B_{11} = \frac{\rho_{20} f_{20} \mu_{20}^2 - f_{10} \mu_{10}^2}{f_{10} \mu_{10} + f_{20} \mu_{20}}
\]

\[
D_i = \frac{f_{10} \mu_{10} \rho_{20} \mu_{20} + f_{20} \mu_{20} \rho_{10} \mu_{10}}{(f_{10} \mu_{10} + f_{20} \mu_{20})^2}
\]

\[
B_{22} = \frac{\rho_{10} f_{10} \mu_{10} - f_{20} \mu_{20}^2}{f_{10} \mu_{10} + f_{20} \mu_{20}}
\]

For concentrations of the first and second components:

\[
\ln \left[ \frac{f_1}{f_0} \left( \frac{f_{10}}{f_{10} - 1} \right) \right] + \frac{f_{10}^2 \mu_{10}^2 \rho_{20} \mu_{20} + f_{20}^2 \mu_{20}^2 \rho_{10} \mu_{10}}{(f_{10} \mu_{10} + f_{20} \mu_{20})^2} \ln \left[ \frac{(f_{10} \mu_{10} + f_{20} \mu_{20}) f_{10} - f_{10} \mu_{10}}{(f_{10} \mu_{10} + f_{20} \mu_{20}) f_{10} - f_{10} \mu_{10}} \right] +
\]

\[+ \frac{\rho_{20} f_{20} \mu_{20}^2 - \rho_{10} f_{10} \mu_{10}^2}{(f_{10} \mu_{10} + f_{20} \mu_{20}) \rho_{10} \mu_{10}} \left( f_{10} - f_{10} \right) = \frac{K(x - x_0)}{\rho_{10} \mu_{10}} \]

\[
\ln \left[ \frac{f_2}{f_0} \left( \frac{f_{20}}{f_{20} - 1} \right) \right] + \frac{f_{20}^2 \mu_{20}^2 \rho_{10} \mu_{10} + f_{10}^2 \mu_{10}^2 \rho_{20} \mu_{20}}{(f_{10} \mu_{10} + f_{20} \mu_{20})^2} \ln \left[ \frac{(f_{10} \mu_{10} + f_{20} \mu_{20}) f_{20} - f_{20} \mu_{20}}{(f_{10} \mu_{10} + f_{20} \mu_{20}) f_{20} - f_{20} \mu_{20}} \right] +
\]

\[+ \frac{\rho_{10} f_{10} \mu_{10}^2 - \rho_{20} f_{20} \mu_{20}^2}{(f_{10} \mu_{10} + f_{20} \mu_{20}) \rho_{20} \mu_{20}} \left( f_{20} - f_{20} \right) = \frac{K(x - x_0)}{\rho_{20} \mu_{20}} \]

**CONCLUSIONS**

By moving of the biphasic media the velocities of the both components tend with distance from the motion onset to the same constant number \(f_{10} \mu_{10} + f_{20} \mu_{20}\). In this case the velocity of the component with a larger initial velocity always remains more than this number and the velocity of the component with a smaller velocity is always less. The velocity of the component with a larger density tends to \(f_{10} \mu_{10} + f_{20} \mu_{20}\) slower than that of the component with a smaller density. At the same time the concentrations of the components tend to different constant numbers.

**REFERENCES**