Selection of Complex System in the Reduced Multiple Criteria Space

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Abstract: The paper describes the PAKS-M multi-stage technology for group ordering and sorting multi-attribute objects. This technology provides the multiple criteria choice by reducing the criteria space, constructing several hierarchical trees of various composite criteria and an integral index with numerical and/or verbal scales, which aggregate initial characteristics of the objects considered and using different methods of decision-making. The suggested approach was applied to selection of computing cluster. The PAKS-M technology allows a decision maker to compare different criteria systems and choose the most preferable criteria, reduce time for solving the decision problem, find and analyze the best computing cluster.

Key words: Multiple criteria choice • Reduction of criteria space • Aggregation of attributes • Computing cluster

INTRODUCTION

The typical problems of decision making are ordering and sorting objects by their properties. In many cases, the objects are described with tens and hundreds of diverse attributes. For example, these are complex technical systems and, in particular, high-performance computing systems or clusters. Now computing clusters are used to solve various research and applied problems in manifold areas. Modern standard and relatively cheap microprocessors, network communication and peripheral devices allow users to build different cluster configurations, which can be flexibly transformed and reconstructed according to user needs.

A comparison of different clusters and selection of the most preferable option is a weakly formalized and ill-structured complex problem. The best cluster is determined by many criteria both quantitative and qualitative. But direct ordering and sorting alternatives with a large number of attributes is a rather difficult procedure for a decision maker (DM) or expert. Additional difficulties appear, firstly, when objects are described with verbal attributes, convolution of which is impossible and mathematically incorrect. Secondly, DM considers often a few objects, which are usually incomparable by their properties. Therefore, the well-known methods of multiple criteria decision making [1-6] are poorly applicable to select the most preferable computing cluster.

In this paper, we present an approach to selection of complex computing cluster using the PAKS-M modified version of the multi-stage technology PAKS (abbreviation of Russian words: Consequent Aggregation of Classified Situations) [7]. At the first stage of this technology, many initial attributes of the analyzed objects are aggregated step by step into composite criteria or a single integral index based on DM’s preferences or expert’s knowledge. At the second stage, the grades of composite criteria scales are formed using several different methods of verbal decision analysis and/or combinations of the methods. Constructing grades of composite criterion scale is considered as the ordinal classification problem, where the initial combinations of object characteristics are the classified alternatives and the verbal grades of aggregated criterion represent the decision classes. In the PAKS-M
version [8], several hierarchical trees of criteria, which are obtained with different approaches, are built. And each combination of initial characteristics is assigned into some classes correspondent to different options of the composite criterion grades. At the third stage, the most preferable object is selected by several methods of group multicriteria choice.

The construction of various hierarchical trees in order to reduce the criteria space allows us to simplify a multicriteria comparison of cluster configurations, diminish difficulties of problem solution, select the most preferable system of criteria and analyze the obtained results.

**Construction of Hierarchical Criteria Trees:** In order to compare clusters and select the best one, we built several hierarchical criteria trees that aggregate the initial characteristics of cluster. The choice of criteria is informal procedure that depends on the DM preferences and/or knowledge of expert.

When constructing such criteria tree, the initial attributes have to be determined first of all. Based on special studies [9], the following groups of indicators has been selected as the initial characteristics of computing clusters.

TS. Technical specifications of computing module (a core clock; word size of CPU core; a number of threads; a number of cores; RAM memory space supported with a processor; a number of processors in a module; RAM memory space of module; a presence of GPU; disk memory space of module; a presence of optical data storage in a module).

CC. Computational characteristics of cluster (a number of modules in the cluster; a rate of exchange between modules; a presence of built-in input and output equipment; a presence of UPS; software characteristics of the cluster; a possibility to upgrade hardware and software of the cluster).

SC. Structural characteristics of cluster (cluster sizes: height, depth, width; a cluster mass; electrical noise immunity).

OC. Operational characteristics of cluster (a power consumption; a noise level; a heat generation; operational conditions: temperature, humidity; mean time between failures).

CP. The cluster performance.
CM. The cost of manufacturing cluster.

A verbal rating scale with two or three grades has been formed for each initial attribute. For example, the cluster performance is assessed as CP$^0$ – high (>2000 Gflops), CP$^1$ – medium (2000-500 Gflops), CP$^2$ – low (<500 Gflops). The cost of manufacturing cluster may be CM$^0$ – high, CM$^1$ – medium, CM$^2$ – low.

A number of the final criteria and aggregation degree of initial attributes are important when constructing hierarchical criteria trees for the selection problem. Some of initial attributes can be considered as the final criteria. Other characteristics are aggregated into composite criteria.

In our example, we built four types of hierarchical criteria trees with different aggregation degrees of initial characteristics of computing clusters. The first type of tree consists of five final criteria: CP. The cluster performance; CM. The cost of manufacturing cluster; SC. Computational characteristics of cluster; GC. Generic characteristics of cluster, which combine the computational, structural and operational characteristics of cluster.

In the second type of tree, the criteria CP, CM, CC, SC and OC are considered as intermediate criteria, which have been composed into the single integral index QC. Quality of cluster. In the fourth type of tree, the criteria CP, CM, SC and OC are composed into the single integral index QC. Quality of cluster.

We constructed different verbal rating scales for generated composite criteria by applying the technique of tuples stratification [7]. This technique uses cutting the multi-attribute space with parallel hyper-planes. Each layer (stratum) consists of combinations of the unified initial estimates with fixed sum of indexes and represents any grade on the scale of composite criterion. For instance, the grades on the scale of the integral index Quality of cluster that composes three criteria QC=(CP, GC, CM), were formed as follows:

QC$^0$ – advanced cluster (CP$^0$, GC$^0$, CM$^0$), (CP$^0$, GC$^0$, CM$^1$), (CP$^0$, GC$^1$, CM$^0$), (CP$^0$, GC$^1$, CM$^1$), (CP$^1$, GC$^0$, CM$^0$), (CP$^1$, GC$^0$, CM$^1$), (CP$^1$, GC$^1$, CM$^0$), (CP$^1$, GC$^1$, CM$^1$), (CP$^2$, GC$^0$, CM$^0$), (CP$^2$, GC$^0$, CM$^1$), (CP$^2$, GC$^1$, CM$^0$), (CP$^2$, GC$^1$, CM$^1$); QC$^1$ – modern cluster (CP$^0$, GC$^1$, CM$^0$), (CP$^0$, GC$^1$, CM$^1$), (CP$^1$, GC$^0$, CM$^0$), (CP$^1$, GC$^0$, CM$^1$), (CP$^1$, GC$^1$, CM$^0$), (CP$^1$, GC$^1$, CM$^1$), (CP$^2$, GC$^0$, CM$^0$), (CP$^2$, GC$^0$, CM$^1$), (CP$^2$, GC$^1$, CM$^0$), (CP$^2$, GC$^1$, CM$^1$);
When the integral index Quality of cluster composes five criteria $QC=(CP, CM, CC, SC, OC)$, the grades on the scale were formed analogously but in more complicated way.

When constructing hierarchical criteria trees, we built three versions for each type of tree with different combinations of grades on composite criteria scales at some hierarchical levels. In other words, we get three different options of hierarchical criteria system of each type. In our case, we consider such options with different criteria scales as the independent judgments or opinions of different DM or experts who forms the own tree of composite criteria. Thus, we transform the selection of computing cluster into the problem of group multiple criteria choice.

**Multiple Criteria Selection of Computing Cluster:** A selection of the most preferable computing cluster was performed using three methods of group multicriteria choice: the ARAMIS (Aggregation and Ranking Alternatives nearby the Multi-attribute Ideal Situations) method, the method of lexicographic ordering by gradations of criteria assessments and the method of weighted sum of ranks [1, 2, 6]. In our example, we compared three variants of computing clusters $VC_1$, $VC_2$ and $VC_3$, which were evaluated independently by three experts (criteria trees) upon 1, 3 and 5 criteria, respectively.

In the first case of criteria aggregation, the results of evaluation (on 5 final criteria) of the clusters presented as tuples of the verbal estimates are given in Table 1. In order to compare clusters and select the best one, write the expert estimates of clusters as multisets or sets with repeating elements [10]:

$$A_i = \{k_{ds}(x_i^0) x_i^0, \ldots, k_{ds}(x_i^s) x_i^s, \ldots, k_{ds}(x_i^5) x_i^5\}, \ i = 1, 2, 3$$

over the set $X=X_1 U X_2 U X_3$ of all grades on the scales of criteria CP, CM, GC.

The results of evaluation of the clusters (on 5 final criteria) presented as multisets of the verbal estimates and comparisons of the clusters by the different methods are shown in Table 2. According to Table 2, the cluster $VC_1$ is more preferable than the cluster $VC_2$; the cluster $VC_2$ is more preferable than the cluster $VC_1$; $VC_1>VC_2>VC_3$ (by the ARAMIS method); the cluster $VC_1$ is more preferable than the cluster $VC_2$; $VC_2>VC_1>VC_3$ (by the method of lexicographic ordering); the cluster $VC_1$ is more preferable than the clusters $VC_2$ and $VC_3$, which differs poorly: $VC_1>VC_2=VC_3$ (by the method of weighted sum). The final ordering based on the above three arrangements of clusters was built with the Borda procedure and is presented as follows: $VC_1>VC_3>VC_2$. Thus, in the first case of criteria aggregation, the cluster $VC_1$ is more preferable than the clusters $VC_2$ and $VC_3$, which can be considered as roughly equivalent.

In the second case of criteria aggregation, the results of evaluation (on 3 final criteria) of the clusters presented as tuples of the verbal estimates are given in Table 3. In this case, the aggregated multiple criteria estimates of the computing clusters $VC_1$, $VC_2$ and $VC_3$, can be written as the following multisets:

$$A_i = \{k_{ds}(x_i^0) x_i^0, \ldots, k_{ds}(x_i^s) x_i^s, \ldots, k_{ds}(x_i^5) x_i^5\}, \ i = 1, 2, 3$$

over the set $X=X_1 U X_2 U X_3$ of all grades on the scales of criteria CP, CM, GC.

The results of evaluation (on 3 final criteria) of the clusters presented as multisets of the verbal estimates and comparisons of the clusters by the different methods are shown in Table 4. According to Table 4, the cluster $VC_1$ is more preferable than the cluster $VC_2$; the cluster $VC_2$ is more preferable than the cluster $VC_3$; $VC_1>VC_2>VC_3$ (by the ARAMIS method); the cluster $VC_1$ is more preferable than the cluster $VC_2$ and the cluster $VC_3$ is slightly more preferable than the cluster $VC_2$; $VC_2>VC_1>VC_3$ (by the method of lexicographic ordering); the cluster $VC_1$ is more preferable than the clusters $VC_2$ and $VC_3$, which differs poorly: $VC_1>VC_2=VC_3$ (by the method of weighted sum). The final ordering based on the above three arrangements of clusters was built with the Borda procedure and is presented as follows: $VC_1>VC_3>VC_2$. Thus, in the second case of criteria aggregation, the cluster $VC_1$ is more preferable than the clusters $VC_2$ and $VC_3$, The cluster $VC_2$ is more preferable than the cluster $VC_3$.
In the third and fourth cases of criteria aggregation, we have the following results of cluster evaluation (on 1 integral index QC. Quality of cluster): VC₁ is characterized as the advanced cluster, VC₂ and VC₃ – as the modern clusters. The cluster VC₁ is again more preferable than the clusters VC₂ and VC₃.

So, in all cases of criteria aggregation, we obtain qualitatively the same results: the cluster VC₁ is more preferable than the clusters VC₂ and VC₃. We can give the following explanation of these results. In the first case, the cluster VC₁ has the high structural and operational characteristics, the medium cluster performance and cost of manufacturing, but the low computational characteristics. Similarly, in the second case, the cluster VC₁ has the high generic characteristics, the medium cluster performance and cost of manufacturing. The clusters VC₂ and VC₃ have different estimates on criteria, but lower than cluster VC₁. In the third and fourth cases, VC₁ is the advanced cluster, VC₂ and VC₃ are the modern clusters.

In comparison with the clusters VC₂ and VC₃, the cluster VC₁ occupies the first place due to the predominance of high and medium estimates for most of the criteria. Set of estimates on the criteria as a whole, make the clusters VC₂ and VC₃ approximately equivalent.

### RESULTS AND DISCUSSION

The multiple criteria selection of computing cluster refers to the ill-structured problems of unique subjective choice. Despite of the primary complexity of the selection task, due to a large number of cluster characteristics, we could reduce the dimension of the criteria space by an aggregation of initial attributes. We also applied several methods of group multi-criteria decision-making in order to find the most preferable computing cluster that has concrete and understandable properties.

The choice of one and the same cluster as the most preferable by four different ways demonstrates the high reliability of the multi-stage technology PAKS-M. The criteria systems do not contain contradictions and give enough clear explanation of the results. At the same time, a construction of the criteria systems shows that aggregating criteria and final ordering objects depend generally on the DM preferences and expert opinions, which can be caused both different points of view on the problem and life experience. This fact forces us to conduct a comprehensive analysis at all stages of the hierarchical aggregation procedure.
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