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Free Convective Flow of Visco-Elastic Fluid in a Vertical Channel with Dufour Effect

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Abstract: The objective of the present study is to analyze the effect of free convection and mass transfer on the flow of an elastico-viscous fluid (Walters B' model) in a vertical channel. The diffusion thermo effect has been considered on the fully developed laminar flow with uniform plate temperature and concentration. The novelty of the present study is to carefully examine the role of elastic parameter on the flow. The result of present study has been compared with the previous findings reported without elasticity, mass transfer and diffusion thermo effect. The Laplace transform technique has been applied to solve the equations governing the flow phenomenon. The validity of our result is assumed on the fact that our result is good agreement with the previous authors.

Key words: Visco-elastic · Diffusion thermo effect · Mass flux · Sustentation · Laplace transform

INTRODUCTION

N many transport processes and industrial applications, transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. Unsteady natural-convection of heat and mass transfer is of great importance in designing control systems for modern free convection heat exchangers. Soundalgekar [1] studied the effect of mass transfer and free convection currents on the flow past an impulsively started infinite vertical plate and observed that the presence of foreign gasses in the flow domain leads to reduce the shear stress and rate of mass transfer significantly.

In nature, flow occurs due to density differences caused by temperature as well as chemical composition gradients (Gebheart and Peru, [2]). Therefore, it warrants the simultaneous consideration of temperature difference as well as concentration difference when heat and mass transfer occurs simultaneously. It has been found that an energy flux can be created not only by temperature gradients but by composition gradients also. This is called Dufour effect. If, on the other hand, mass fluxes are created by temperature gradients, it is called the Soret effect.

The Soret and the Dufour effects have been found to be useful as the Soret effect is utilized for isotope separation and in a mixture of gases of light and medium molecular weight, the Dufour effect is found to be of considerable order of magnitude such that it cannot be neglected (Eckert and Drake, [3]). In view of importance of the diffusion-thermo effect Kafoussias and Williams [4] have studied the effects of thermal diffusion and diffusion-thermo on mixed free and forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Jha and Singh [5], Hajji and Worek [6], Kafoussias [7] and Alam *et al.* [8,9] have contributed significantly to this field of study. Recently, Beg *et al*. [10] studied chemically reacting mixed convective heat and mass transfer along inclined and vertical plates considering Soret and Dufour effect. More recently, Jha and Ajibade [11] have studied the heat and mass transfer aspect of the flow of a viscous incompressible fluid in a vertical channel considering the Dufour effect. Dursunkaya and Worek [12] have studied the diffusion-thermo and thermal diffusion effects in transient and steady natural convections from a vertical surface.

MHD flow with thermal diffusion and chemical reaction finds numerous applications in various areas such as thermo nuclear fusion, liquid metal cooling of

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nuclear reactions and electromagnetic casting of metals. Chambra and Yang [13] have worked on thermal diffusion of a chemically reactive species in a laminar boundary layer flow. Aberu *et al*. [14] have studied the boundary layer flows with Dufour and Soret effects. Osalusi *et al*. [15] have worked on mixed and free convective heat and mass transfer of an electrically conducting fluid considering Dufour and Soret effects. Effect of Hall current and chemical reaction on MHD flow along an exponentially accelerated porous flat plate with internal heat absorption/generation was studied by Rath *et al*. [16].

As the non-Newtonian fluid flow such as polymers etc plays a vital role in the applications of engineering and technology. The aim of the present study is to bring out the effect of non-Newtonian parameter particularly elastic parameter on the flow characteristics. The flow of non-Newtonian flow model, (Waltes B') has been considered in a vertical channel with uniform temperature and concentration. The consideration of soret and Dufour effect has resulted in coupling the concentration and energy equations

The present study warrants the necessity of suggesting the measures for improving the performance of related thermal system (working in Newtonian as well as non-Newtonian flow environment) and to critically analyze the Dufour effect on heat and mass transfer applications.

Nomenclature:

C	=	Dimensionless fluid concentration
C'	=	Dimensional fluid concentration
C_w	=	Fluid concentration on the wall
C_0	=	Initial concentration
D	=	Coefficient of mass diffusion
D^*	=	Dufour number
D_1	=	Dimensional coefficient of the diffusion thermo effect
g	=	Acceleration due to gravity
h	=	Width of the channel
N	=	Sustentation parameter
Nu_0	=	Nusselt number at y =0
Nu_1	=	Nusselt number at y =1
R_c	=	Elastic parameter
P_r	=	Prandtl number
S_c	=	Schmidt number
t	=	Dimensionless time
t'	=	Dimensional time
T	=	Dimensionless fluid temperature
T'	=	Dimensional fluid temperature

Mathematical Analysis: Consider the free convective and mass transfer flow of a visco-elastic fluid in a vertical channel formed by two infinite vertical parallel plates. The convective current is induced by both the temperature and concentration gradient. The flow is assumed to be in the x'-direction, which is taken to be vertically upword direction along the channel walls. and the y'-axis is taken to be normal to the plate that is h distance apart. At the time $t' \leq 0$, the fluid is at rest with the initial temperature T_0 and concentration C_0 For $t'>0.$, the temperature and concentration on the wall $y' = h$ increase and decrease to T_0 and C_w respectively, while the wall $y' = h$ is maintained at T_0 and C_0. The velocity of the fluid on both walls remains $u' = 0$. The governing equations the temperature is governed by concentration, leading to the diffusion - thermo effect.

Under the usual Boussinesq's approximations, the unsteady flow is governed by the following equations,

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial u'}{\partial y'} + g\beta(T' - T) + g\beta(C' - C) - \frac{K}{\rho}\frac{\partial u'}{\partial y' \partial t'} \tag{1}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial C'}{\partial y'} \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \upsilon \frac{\partial T'}{\partial y'} + D\frac{\partial C'}{\partial y'} \tag{3}$$

with the following initial and boundary conditions

$$\left.\begin{array}{lllll} t' \leq 0: & u'(y',t') = 0, & T'(y',t') = T, & C'(y',t') = C & for\ all\ \ y \\ t' > 0: & u'(0,t') = 0, & T'(0,t') = T, & C'(0,t') = C & at\ \ y' = 0 \\ & u'(h,t') = 0, & T'(h,t') = 0, & C'(h,t') = 0 & at\ \ y' = h \end{array}\right\} \tag{4}$$

On introducing the non dimensional quantities:

$$y = \frac{y'}{h},\ t = \frac{t'\upsilon}{h}, u = \frac{u'\upsilon}{g\beta h\,(T - T)}, P = \frac{\upsilon}{\alpha}, S = \frac{\upsilon}{D},\ R = \frac{k}{\rho h}$$
$$T = \frac{T' - T}{T - T}, C = \frac{C' - C}{C - C},\ N = \frac{\beta^*(C - C)}{\beta(T - T)}, D^* = \frac{D(C - C)}{\alpha(T - T)} \tag{5}$$

In equation (1) to (3), the governing equations for the flow in dimensionless form are given by

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial y} + NC + T - R\frac{\partial u}{\partial y\,\partial t} \tag{6}$$

$$S\frac{\partial C}{\partial t}=\frac{\partial C}{\partial y} \quad (7)$$

$$P\frac{\partial T}{\partial t}=\frac{\partial T}{\partial y}+D^*\frac{\partial C}{\partial y} \quad (8)$$

$$\begin{array}{llll} t\le 0: & u(y,t)=0, & T(y,t)=0, & C(y,t)=0 \;\; for\;\; all \;\; y \\ t>0: & u(0,t)=0, & T(0,t)=1, & C(0,t)=1 \;\; at \;\; y=0 \\ & u(1,t)=0, & T(1,t)=0, & C(1,t)= \end{array} \quad (9)$$

Here P_r, the Prandtl number, which is inversely proportional to the thermal diffusivity of the working fluid, D*, the Dufour number, which is the coefficient of the concentration - energy, S_c, the Schmidt number and N, the sustention parameter whose positive values means that convections due to heat and mass transfer support each other and negative values denote the converse. The physical quantities used in equation are defined in the notation.

Solution of the Problem: The solution of the equations (6) to (8) subject to the boundary conditions (9) are obtained by Laplace Transeform technique:

Unsteady Case: The unsteady state of the problem are obtained in two different cases

Case I: $P\neq 1, S\neq 1$
Case II: $P=1, S=1$

Case I: If $P\neq 1, S\neq 1$ the solutions (6) to (8) subject to the boundary conditions given by (9) are solved and the result is given in figures.

Case II: For $P=1, S=1$,the result for the temperature and velocity presented in equations (7) and (8) respectively become undefined, therefore equations (1) to (3) with condition (5) are solved and the numerical result is given in the figures.

The rate of mass transfer (S_h), the rate of heat transfer (N_u) and the skin friction (τ) on the walls of the channel are also obtained and the numerical result is given in Table 1.

Steady Case: Setting $\frac{\partial()}{\partial t}=0$ in equation (1) to (3), steady state of the problem is obtained as

$$\frac{du}{dy}+NC+T=0 \quad (10)$$

Table 1: Value of Skin Friction at both the lower and upper plate

t	Sc	Pr	D*	N	Rc	τ_0	τ_1
0.4	0.2	0.71	30	4	0	-22.89840479	10.98813751
0.4	0.2	0.71	30	4	0.2	-8.465112063	20.42784769
0.4	0.2	0.71	30	2	0.2	-7.224136571	18.93147876
0.4	0.2	0.71	2	4	0.2	-2.994989901	3.406575923
0.4	0.2	7	30	4	0.2	-7.999530903	3.658187842
0.4	0.6	7	30	4	0.2	-29.54603282	9.017302369
0.4	0.2	7	30	4	0.3	-9.261732298	4.214269272
0.4	0.2	7	30	2	0.3	-5.478277444	2.491026126
0.4	0.2	7	2	4	0.3	-7.883398439	3.557399103
0.2	0.2	7	30	4	0.3	-11.15702451	3.213176252

$$\frac{dC}{dy}=0 \quad (11)$$

$$\frac{dT}{dy}+D^*\frac{dC}{dy}=0 \quad (12)$$

The solution of the equations (10) to (12) subject to boundary conditions (9) are

$$\tilde{C}=1-y \quad (13)$$

$$\tilde{T}=1-y \quad (14)$$

$$\tilde{u}=\frac{N+1}{6}(y-3y+2y) \quad (15)$$

In this case the rate of mass transfer, the rate of mass transfer and the skin friction are given by

$$\tilde{S}h=\tilde{S}h=1 \quad (16)$$

$$\tilde{N}u=\tilde{N}u=1 \quad (17)$$

$$\tilde{\tau}=\frac{N+1}{3} \quad (18)$$

$$\tilde{\tau}=\frac{N+1}{6} \quad (19)$$

RESULTS AND DISCUSSION

The numerical results are exhibited through the graphs to exhibits the effects of pertinent parameters on the free convection and mass transfer of a visco-elastic fluid in a vertical channel with asymptotic wall temperature and concentration with dufour effect.

The equation (6) represents the x-momentum equation of visco-elastic fluid of Walters B' model.

From equation (6) we get

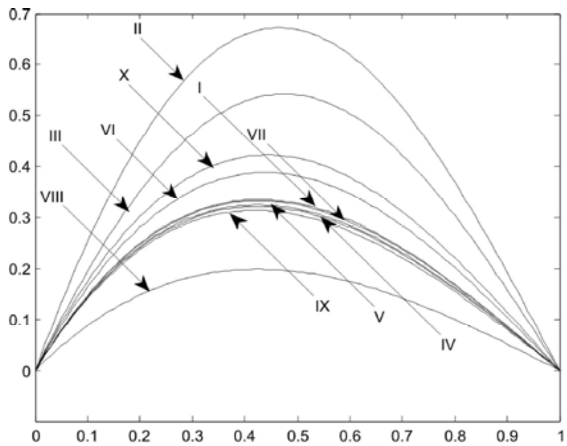

Fig. 1: Velocity profile

Curve	t	Sc	Pr	D*	N	Rc
I	0.4	0.2	0.71	30	4	0
II	0.4	0.2	0.71	30	4	0.2
III	0.4	0.2	0.71	30	2	0.2
IV	0.4	0.2	0.71	2	4	0.2
V	0.4	0.2	7	30	4	0.2
VI	0.4	0.6	7	30	4	0.2
VII	0.4	0.2	7	30	4	0.3
VIII	0.4	0.2	7	30	2	0.3
IX	0.4	0.2	7	2	4	0.3
X	0.2	0.2	7	30	4	0.3

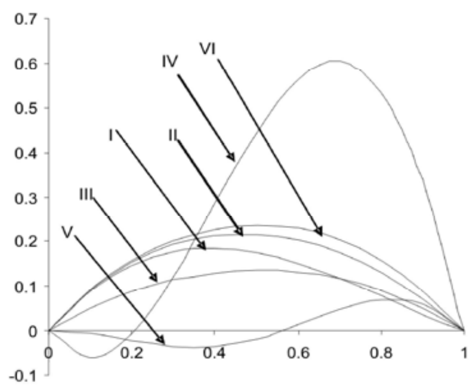

Fig. 2: Velocity profile for Pr=Sc=1

Curve	t	D*	N	Rc
I	0.4	2	4	0
II	0.4	2	4	0.2
III	0.4	2	2	0.2
IV	0.4	30	4	0.2
V	0.2	2	4	0.2
VI	0.4	2	4	0.3

(1) R_c = 0.0 case of Jha and Ajibade [11]

(2) R_c = 0.0 and without mass transfer case of Jha and Singh [5]

We have also studied the present problem in two cases

Case (I) $P \neq 1, S \neq 1$ Case (II) $P = 1, S = 1$.

The expressions for the velocity, temperature and mass transfer are presented graphically in Figures 1 to 4 also the skin friction coefficient is given in the table. The solution for steady case indicates that temperature curves are linear in nature.

Fig.1 shows the variation of velocity ($P_r \neq 1$, $S_c \neq 1$) exhibiting the effect of various parameters. The nature of variation is parabolic attaining the maximum values in the middle of the channel. All the parameters characterizing the velocity distribution enhance it. The dominating factor is Suction parameter N. An increase in the value of N contributes maximum to enhance the velocity in case of P_r = 0.71. Now comparing curve VII and VIII it is seen that an increase in P_r leads to decrease in the velocity. Thus it is conclude that dominance of momentum diffusivity (higher Prandtl number fluid) reduces the velocity at all point in the flow domain.

Fig. 2 displays quite interesting features of the velocity distribution exhibiting the flow reversal in the neighborhood of the plate and for low value of time parameter when $P_r \neq 1$, $S_c \neq 1$. The curve IV exhibits the effect of high value of Dufour number, D^* = 30and curve V shows the case of t = 0.2. The high value of Dufour number gives rise to low thermal diffusivity in the layer close to the vertical plate causing the flow reversal at the onset of motion but after the lapse of time thermal diffusivity turns to be high accelerating the fluid particle to posses an asymmetric distribution of velocity. Another point of interest is the role of elasticity property of the fluid. The increasing value of elastic parameter and suction parameter increase the velocity at all points. A few layers near the plate. In the absence of elasticity, R_c = 0.0 (curve I) accelerates the appearance of pick of the profile earlier than the curve II, $R_c \neq 0.0$. Another specialty of the distribution is that no tangible effect is observed for all the parameters near the plate except the case of high value of Dufour number.

Further, the flow instability is marked for low span of time. Thus the present study reveals that high Dufour effect and the low time span are not desirable for attaining the stability of flow. The consideration elasticity of the fluid is beneficial in accelerating velocity in the flow domain.

Fig. 3 shows that effect of temperature distribution for various values of Prandtl number, Dufour number and Schmidt number. Without considering the Dufour effect the temperature equation becomes decoupled and hence independent of concentration variation. The temperature profile exhibits two distinct behaviors for $P_r \neq 1$, $S_c \neq 1$ and

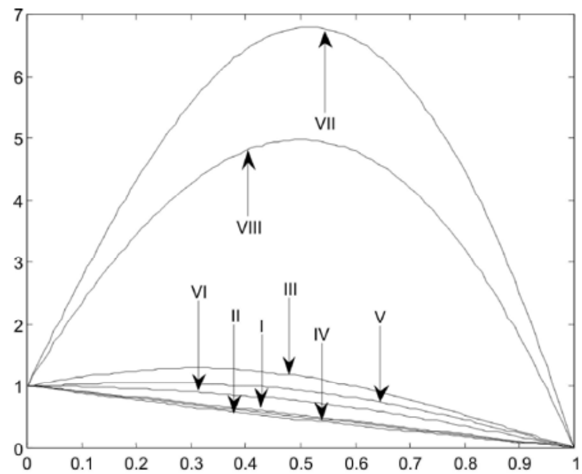

Fig. 3: Temperature Porfile

Curve	t	Sc	Pr	D*
I	0.4	0.2	0.71	30
II	0.4	0.2	7	30
III	0.4	0.2	7	30
IV	0.4	0.2	0.71	2
V	0.2	0.2	0.71	30
VI	0.4	0.6	0.71	30
VII	0.4	1	1	30
VIII	0.2	1	1	30

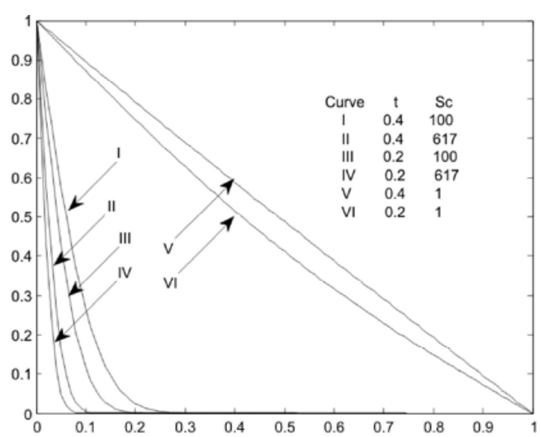

Fig. 4: Conectration porfile

$P_r = 1$, $S_c = 1$ Exhibits the relative importance of heat conduction and viscosity of a fluid i.e. the ratio of momentum diffusivity due to viscosity and thermal diffusivity due to conductivity. Similarly with the case of Schmidt number irrespective of momentum diffusivity and mass diffusivity. The significant rise in temperature is marked as the viscous diffusivity, thermal diffusivity and mass diffusivity favours each other and the longer span of time accelerates the temperature further. When $P_r \neq 1$, $S_c \neq 1$, an increase in Suction parameter as well as dufour number enhance the temperature at all points of the channel.

Thus it is conclude that in case of heavier species (large S_c) i.e. with increasing viscous diffusivity the temperature rises. Further, with large value of Dufour number D*, the fluid temperature rises. Most interestingly time plays a dual role in two distinct cases $P_r = 1$, $S_c = 1$ and $P_r \neq 1$, $S_c \neq 1$. In the former case an increase in time enhance the temperature but in the later case it decreases. Another peculiarity is marked in case of low and high values of P_r. In case of Curve I and II, keeping the values of $t = 0.4$, $S_c = 0.2$ and $D^* = 30$ fixed and increasing the value of P_r from 0.71 to 7.0, it is marked that temperature decreases but in case of curve III and VI. When $t = 0.4$, $S_c = 0.6$ and $D^* = 30$.Similar variation in P_r produces reverse effect. This promises a limiting values of S_c, $0.2 \leq S_c \leq 0.6$ for which reverse trend of temperature variation is marked. Similar analogy can be drawn in case of P_r also, as pointed by Jha and Ajibade [2010] the limiting values of $P_r = 0.2$.

Fig.4 exhibits the concentration profile for various values of Schmidt number (S_c) and time (t). For aqueous medium the $S_c = 617$ takes higher values and hence we have computed for $S_c = 100$ and 617. It is found that in case of unsteady flow an increase in $S_c = 10$ leads to decrease the concentration level at all points but the reverse effect is observed in case of time. Further, it is interesting to note that for $S_c = 1.0$ the concentration level is high in comparison with the case of heavier species $S_c = 100$ and 617 and it increases with the longer span of time.

From Table 1 it is observed that the skin friction at lower plate decreases in magnitude where as at the upper plate it increases in the presence of visco-elastic elements in the fluid but at the time increases reverse effect is observed. This shows that the characteristic of elasticity in the flow behavior depends upon time span. The table further reveals that an increase in Pr i.e. the slow rate of thermal diffusion results in the decrease of skin friction. It is seen that high Prandtl number fluid causes reduction in the skin friction which is desirable. Another interesting point to note that Sustentation parameter, Dufour number and Schimdt number increases the skin friction at both the plate. Thus, the presence of heavier species associated with these two effects results in a counterproductive yielding higher rate of skin friction.

CONCLUSION

- All the parameters characterizing the velocity distribution enhance it.
- Dominance of momentum diffusivity (higher Prandtl number fluid) reduces the velocity at all point in the flow domain.

- The high value of Dufour number gives rise to low thermal diffusivity in the layer close to the vertical plate.
- The flow instability is marked for low span of time.
- An increase in Suction parameter as well as dufour number enhance the temperature at all points of the channel.
- In case of unsteady flow an increase in S_c leads to decrease the concentration level.
- The slow rate of thermal diffusion results in the decrease of skin friction.

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Appendix

$A_1 = D^* S_c / (\mathrm{P_r} - S_c)$, $A_6 = (1 - A_1 - A_3)/(1 - \mathrm{P_r})$,

$A_{12} = -A_3 \sqrt{\mathrm{P_r}} / \sqrt{\pi}$, $A_7 = (N + A_1 - A_3)/(1 - S_c)$,

$A_2 = (N + A_1)/(1 - S_c)$,

$A_{13} = A_4 / \sqrt{\pi}$,

$A_3 = (1 - A_1)/(1 - \mathrm{P_r})$, $A_8 = A_6 + A_7, A_9 = A_2 S_c / 2$,

$A_{14} = A_5 / \sqrt{\pi}$

$A_4 = A_2 + A_3, A_5 = A_4 / 2$,

$A_{10} = A_3 P_r / 2$, $A_{11} = -A_2 \sqrt{S_c} / \sqrt{\pi}$,