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Exact Solutions of the Nonlinear Generalized Shallow Water Wave Equation

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Abstract: In this article, we have employed an enhanced (G′/G)-expansion method to find the exact solutions first and then the solitary wave solutions of the nonlinear generalized shallow water wave equation. Here we have derived solitons, singular solitons and periodic wave solutions through the enhanced (G′/G)-expansion method. The solutions obtained hereby reveal the richness of explicit solitons and periodic solutions to the applied equation. It has been shown that the enhanced (G′/G)-expansion method is effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics.

Mathematics subject classification: 35K99 . 35P05 . 35P99

Key words: Enhanced (G′/G)-expansion method . nonlinear generalized shallow water wave equation . solitary waves . soliton . exact solution . NLEEs

INTRODUCTION

NLEEs are at work in various fields of mathematics, physics, chemistry, biology, engineering and in numerous other applications. Exact solutions of NLEEs play an important role in the proper understanding of qualitative features of many phenomena and processes in various areas of natural science. Exact solutions of nonlinear equations graphically demonstrate and allow unscrambling the mechanisms of many complex nonlinear phenomena such as spatial localization of transfer processes, multiple or absent steady states under various conditions, existence of peaking regimes and many others. Even those special exact solutions that do not have a clear physical meaning can be used as test problems to verify the consistency and estimate errors of various numerical, asymptotic and approximate analytical methods. Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving NLEEs. It is significant that many equations of physics, chemistry and biology contain empirical parameters or empirical functions. Exact solutions allow the researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore, investigating exact traveling wave solutions is becoming increasingly attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have successfully been developed by diverse groups of mathematicians and physicists. Prominent among them are, the modified simple equation method [1-5], the tanh-function method [6, 7], the Exp-function method [8-11], the Jacobi elliptic function method [12], the (G′/G)-expansion method [13-24], the transformed rational function method [25, 26], the multiple Exp-function method [27, 28], the enhanced (G′/G)-expansion method [29].

Various ansatze have been proposed for seeking traveling wave solutions of nonlinear differential equations. The choice of an appropriate ansätze is of great importance in the direct methods.

Recently, Wang *et al*. [15] have introduced a simple method which is called the (G′/G)-expansion method to look for traveling wave solutions of nonlinear evolution equations, where G = G(ξ) satisfies the second order linear ordinary differential equation $G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0$, where λ and μ are arbitrary constants and

$$u(\xi)=\alpha_m\left(\frac{G'}{G}\right)^m+\cdots\cdots$$

be the traveling wave solution of NLEEs. By means of this method they have solved the KdV equation, the

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mKdV equation, the variant Boussinesq equations and the Hirota-Satsuma equations.

Guo *et al*. [24] have introduced an another method so called extended (G′/G)-expansion method where G = G(ξ) satisfies the second order linear ordinary differential equation:

$$G'' + \mu G = 0$$

where

$$G' = \frac{dG(\xi)}{d\xi},\ G'' = \frac{d^2G(\xi)}{d\xi^2},\ \xi = x - Vt$$

V is a constant and

$$u(\xi) = a_0 + \sum_{i=1}^{n}\left(a_i(G'/G)^i + b_i(G'/G)^{i-1}\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}\right)$$

be the traveling wave solution. They proposed extended (G′/G)-expansion method to construct travelling wave solutions of Whitham-Broer-Kaup-Like equations and coupled Hirota-Satsuma KdV equations.

For further references of the (G′/G)-expansion method see the articles [13-24].

The objective of this article is to present an enhanced (G′/G)-expansion method to construct the exact solitary wave solutions for NLEEs in mathematical physics via the nonlinear generalized shallow water wave equation.

The article is arranged as follows: In section 2, the enhanced (G′/G)-expansion method is discussed. In section 3, the method is applied to the nonlinear evolution equation pointed out above; in section 4, results and discussion; in section 5 comparisons and in section 6 conclusions are given.

METHODOLOGY

In this section, we describe the enhanced (G′/G)-expansion method for finding traveling wave solutions of NLEEs. Suppose that a nonlinear partial differential equation, say, in two independent variables x and t, is given bellow

$$\Re(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \cdots\cdots\cdots) = 0 \qquad (2.1)$$

where u(ξ) = u(x,t) is an unknown function, $\Re$ is a polynomial of u(x,t) and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [29]:

Step 1: Combining the independent variables x and t into one variable $\xi = x \pm \omega t$, we suppose that

$$u(\xi) = u(x,t),\ \xi = x \pm \omega t \qquad (2.2)$$

The traveling wave transformation Eq. (2.2) permits us to reduce Eq. (2.1) to the following ODE:

$$F(u, u', u'', \cdots\cdots\cdots) = 0 \qquad (2.3)$$

where F is a polynomial in u(ξ) and its derivatives, while

$$u'(\xi) = \frac{du}{d\xi},\ u''(\xi) = \frac{d^2u}{d\xi^2}$$

and so on.

Step 2: We suppose that Eq.(2.3) has the formal solution

$$u(\xi) = \sum_{i=-n}^{n}\left(\frac{a_i(G'/G)^i}{(1+\lambda(G'/G))^i} + b_i(G'/G)^{i-1}\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}\right) \qquad (2.4)$$

where G = G(ξ) satisfies the equation

$$G'' + \mu G = 0 \qquad (2.5)$$

in which $a_i, b_i\ (-n \le i \le n; n \in N)$ and λ are constants to be determined later and $\sigma = \pm 1, \mu \ne 0$.

Step 3: The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq.(2.1) or Eq.(2.3). Moreover, we define the degree of u(ξ) as D(u(ξ)) = n which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q,\ D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s(n+q) \qquad (2.6)$$

Therefore we can find the value of n in Eq. (2.4), using Eq. (2.6).

Step 4: We substitute Eq. (2.4) into Eq.(2.3) using Eq. (2.5) and then collect all terms of same powers of $(G'/G)^j$ and

$$(G'/G)^j\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}$$

together, then set each coefficient of them to zero to yield an over-determined system of algebraic equations, solve this system for a_i, b_i, λ and ω.

Step 5: From the general solution of Eq. (2.5), we obtain when $\mu<0$,

$$\frac{G'}{G}=\sqrt{-\mu}\tanh(A+\sqrt{-\mu}\xi) \qquad (2.7)$$

and

$$\frac{G'}{G}=\sqrt{-\mu}\coth(A+\sqrt{-\mu}\xi) \qquad (2.8)$$

Again, when $\mu>0$,

$$\frac{G'}{G}=\sqrt{\mu}\tan(A-\sqrt{\mu}\xi) \qquad (2.9)$$

and

$$\frac{G'}{G}=\sqrt{\mu}\cot(A+\sqrt{\mu}\xi) \qquad (2.10)$$

where A is an arbitrary constant. Finally, substituting $a_i, b_i (-n\le i\le n; n\in N)$, λ, ω and Eqs. (2.7)-(2.10) into Eq. (2.4) we obtain traveling wave solutions of Eq. (2.1).

APPLICATION

In this section, we will exert enhanced (G'/G)-expansion method to solve the nonlinear generalized shallow water wave equation in the form

$$u_{xxxt}+\alpha u_x u_{xt}+\beta u_t u_{xx}-u_{xt}-\gamma u_{xx}=0 \qquad (3.1)$$

where α, β and γ are nonzero constants and u(x,t) is the amplitude of the relative wave mode.

The traveling wave transformation equation $u(x,t)=u(\xi)$, $\xi=x-\omega t$ transform Eq.(3.1) to the following ordinary differential equation:

$$-\omega u^{iv}-(\alpha+\beta)\omega u'u''+(\omega-\gamma)u''=0 \qquad (3.2)$$

On integrating Eq. (3.2) with respect to ξ once, we obtain

$$C-\omega u'''-\frac{1}{2}(\alpha+\beta)\omega(u')^2+(\omega-\gamma)u'=0 \qquad (3.3)$$

where C is a constant of integration. Considering the homogeneous balance between the highest-order derivative term u''' and the nonlinear term $(u')^2$ from Eq. (3.3) yields 2(n+1) = n+3 which gives n = 1.

Hence for n = 1, the solution of Eq.(3.3) reduces to

$$\begin{aligned}u(\xi)=&\frac{a_{-1}(1+\lambda(G'/G))}{(G'/G)}+a_0+\frac{a_1(G'/G)}{1+\lambda(G'/G)}\\&+b_{-1}(G'/G)^{-2}\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}\\&+b_0(G'/G)^{-1}\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}\\&+b_1\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}\end{aligned} \qquad (3.4)$$

where $G = G(\xi)$ satisfies Eq. (2.5). Substitute Eq. (3.4) along with Eq. (2.5) into Eq.(3.3). As a result of this substitution, we get a polynomial of $(G'/G)^j$ and

$$(G'/G)^j\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}$$

From these polynomials, we equate the coefficients of $(G'/G)^j$ and

$$(G'/G)^j\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}$$

and setting them to zero, we get an over-determined system that consists of thirty algebraic equations. Solving these algebraic equations for a_i, b_i, λ and ω, with the aid of symbolic computer software Maple, we have the following five sets of results:

Set 1: $C=0,\omega=\frac{\gamma}{\mu+1},\lambda=0,a_{-1}=0,a_0=a_0,a_1=\frac{6}{\alpha+\beta},b_{-1}=0,b_0=0,b_1=\pm\frac{6}{\alpha+\beta}\sqrt{\frac{\mu}{\sigma}}$.

Set 2: $C=0,\omega=\frac{\gamma}{16\mu+1},\lambda=0,a_{-1}=\frac{12\mu}{\alpha+\beta},a_0=a_0,a_1=\frac{12}{\alpha+\beta},b_{-1}=0,b_0=0,b_1=0$.

Set 3: $C=0,\omega=\frac{\gamma}{4\mu+1},\lambda=\lambda,a_{-1}=0,a_0=a_0,a_1=\frac{12(\mu\lambda^2+1)}{\alpha+\beta},b_{-1}=0,b_0=0,b_1=0$.

Set 4: $C=0,\omega=\frac{\gamma}{4\mu+1},\lambda=\lambda,a_{-1}=-\frac{12\mu}{\alpha+\beta},a_0=a_0,a_1=0,b_{-1}=0,b_0=0,b_1=0$.

Set 5: $C=0,\omega=\frac{\gamma}{\mu+1},\lambda=\lambda,a_{-1}=-\frac{6\mu}{\alpha+\beta},a_0=a_0,a_1=0,b_{-1}=0,b_0=\pm\frac{6}{\alpha+\beta}\sqrt{\frac{1}{\sigma}},b_1=0$.

Substituting Set 1-Set 5 into Eq. (3.4) along with Eqs. (2.7)-(2.10), we obtain the following families of traveling wave solutions:

Hyperbolic function solutions: When $\mu<0$, we get the following five families of hyperbolic function solutions.

Family 1:

$$u_1(\xi) = a_0 + \frac{6\sqrt{-\mu}}{\alpha+\beta}\left(\tanh(A+\sqrt{-\mu}\xi) \mp I\operatorname{sech}(A+\sqrt{-\mu}\xi)\right)$$

$$u_2(\xi) = a_0 + \frac{6\sqrt{-\mu}}{\alpha+\beta}\left(\coth(A+\sqrt{-\mu}\xi) \mp \operatorname{csch}(A+\sqrt{-\mu}\xi)\right)$$

where $\xi = x - \left(\frac{\gamma}{\mu+1}\right)t$ and $\mu \neq -1$.

Family 2:

$$u_3(\xi) = a_0 + \frac{12\sqrt{-\mu}}{\alpha+\beta}\left(\tanh(A+\sqrt{-\mu}\xi) + \coth(A+\sqrt{-\mu}\xi)\right)$$

where $\xi = x - \left(\frac{\gamma}{16\mu+1}\right)t$ and $\mu \neq -\frac{1}{16}$.

Family 3:

$$u_4(\xi) = a_0 + \frac{12(\mu\lambda^2+1)\sqrt{-\mu}}{\alpha+\beta}\left(\coth(A+\sqrt{-\mu}\xi) + \lambda\sqrt{-\mu}\right)^{-1}$$

$$u_5(\xi) = a_0 + \frac{12(\mu\lambda^2+1)\sqrt{-\mu}}{\alpha+\beta}\left(\tanh(A+\sqrt{-\mu}\xi) + \lambda\sqrt{-\mu}\right)^{-1}$$

where $\xi = x - \left(\frac{\gamma}{4\mu+1}\right)t$ and $\mu \neq -\frac{1}{4}$.

Family 4:

$$u_6(\xi) = a_0 + \frac{12\sqrt{-\mu}}{\alpha+\beta}\left(\coth(A+\sqrt{-\mu}\xi) + \lambda\sqrt{-\mu}\right)$$

$$u_7(\xi) = a_0 + \frac{12\sqrt{-\mu}}{\alpha+\beta}\left(\tanh(A+\sqrt{-\mu}\xi) + \lambda\sqrt{-\mu}\right)$$

where $\xi = x - \left(\frac{\gamma}{4\mu+1}\right)t$ and $\mu \neq -\frac{1}{4}$.

Family 5:

$$u_8(\xi) = a_0 + \frac{6\sqrt{-\mu}}{\alpha+\beta}\left(\coth(A+\sqrt{-\mu}\xi) \mp \operatorname{csch}(A+\sqrt{-\mu}\xi) + \lambda\sqrt{-\mu}\right)$$

$$u_9(\xi) = a_0 + \frac{6\sqrt{-\mu}}{\alpha+\beta}\left(\tanh(A+\sqrt{-\mu}\xi) \mp I\operatorname{sech}(A+\sqrt{-\mu}\xi) + \lambda\sqrt{-\mu}\right)$$

where $\xi = x - \left(\frac{\gamma}{\mu+1}\right)t$ and $\mu \neq -1$.

Trigonometric function solutions: When $\mu>0$, we get the following five families of trigonometric function solutions.

Family 6:

$$u_{10}(\xi) = a_0 + \frac{6\sqrt{\mu}}{\alpha+\beta}\left(\tan(A-\sqrt{\mu}\xi) \mp \sec(A-\sqrt{\mu}\xi)\right)$$

$$u_{11}(\xi) = a_0 + \frac{6\sqrt{\mu}}{\alpha+\beta}\left(\cot(A+\sqrt{\mu}\xi) \mp \csc(A+\sqrt{\mu}\xi)\right)$$

where $\xi = x - \left(\frac{\gamma}{\mu+1}\right)t$ and $\mu \neq -1$.

Family 7:

$$u_{12}(\xi) = a_0 + \frac{12\sqrt{\mu}}{\alpha+\beta}\left(\tan(A-\sqrt{\mu}\xi) - \cot(A-\sqrt{\mu}\xi)\right)$$

$$u_{13}(\xi) = a_0 + \frac{12\sqrt{\mu}}{\alpha+\beta}\left(\cot(A+\sqrt{\mu}\xi) - \tan(A+\sqrt{\mu}\xi)\right)$$

where $\xi = x - \left(\frac{\gamma}{16\mu+1}\right)t$ and $\mu \neq -\frac{1}{16}$.

Family 8:

$$u_{14}(\xi) = a_0 + \frac{12(\mu\lambda^2+1)\sqrt{\mu}}{\alpha+\beta}\left(\cot(A-\sqrt{\mu}\xi) + \lambda\sqrt{\mu}\right)^{-1}$$

$$u_{15}(\xi) = a_0 + \frac{12(\mu\lambda^2+1)\sqrt{\mu}}{\alpha+\beta}\left(\tan(A+\sqrt{\mu}\xi) + \lambda\sqrt{\mu}\right)^{-1}$$

where $\xi = x - \left(\frac{\gamma}{4\mu+1}\right)t$ and $\mu \neq -\frac{1}{4}$.

Family 9:

$$u_{16}(\xi) = a_0 - \frac{12\sqrt{\mu}}{\alpha+\beta}\left(\cot(A-\sqrt{\mu}\xi) + \lambda\sqrt{\mu}\right)$$

$$u_{17}(\xi) = a_0 - \frac{12\sqrt{\mu}}{\alpha+\beta}\left(\tan(A+\sqrt{\mu}\xi) + \lambda\sqrt{\mu}\right)$$

where $\xi = x - \left(\frac{\gamma}{4\mu+1}\right)t$ and $\mu \neq -\frac{1}{4}$.

Family 10:

$$u_{18}(\xi) = a_0 - \frac{6\sqrt{\mu}}{\alpha+\beta}\left(\cot(A-\sqrt{\mu}\xi) \mp \csc(A-\sqrt{\mu}\xi) + \lambda\sqrt{\mu}\right)$$

$$u_{19}(\xi) = a_0 - \frac{6\sqrt{\mu}}{\alpha+\beta}\left(\tan(A+\sqrt{\mu}\xi) \mp \sec(A+\sqrt{\mu}\xi) + \lambda\sqrt{\mu}\right)$$

where $\xi = x - \left(\frac{\gamma}{\mu+1}\right)t$ and $\mu \neq -1$.

Remark: All the obtained results have been checked with MAPLE by putting them back into the original equation and found correct.

RESULTS AND DISCUSSION

From the obtained families, we observe that $\alpha \neq -\beta$, that is α and β cannot take same values in opposite sign. If we set $\lambda = 0$ then solutions in Family 3 and Family 4 become identical and similarly, Family 1 coincides with Family 5 and Family 8 coincides with Family 9.

If we set $a_0 = 0$, $\alpha = \beta = 1$, $\gamma = -1$, $\mu = -2$, $\lambda = 0$, $A = 0.50$ into $u_2(\xi)$ within the interval $-3 \leq x,t \leq 3$, we get a soliton which is represented in Fig. 1.

Figure 2 is a kink shape wave profile of $u_7(\xi)$ for $a_0 = -1$, $\alpha = \beta = 1$, $\gamma = 1$, $\mu = -1$, $\lambda = 2$, $A = 0$ within the interval $-5 \leq x,t \leq 5$.

For $a_0 = 3$, $\gamma = 2$, $\alpha = \beta = 1$, $\mu = 1$, $\lambda = 2$, $A = 1$ and $a_0 = 0$, $\gamma = 5$, $\alpha = \beta = 1$, $\mu = 1$, $\lambda = 2$, $A = 3$ within the interval $-10 \leq x,t \leq 10$, $u_{10}(\xi)$ and $u_{17}(\xi)$ show the shape of periodic traveling wave solutions represented in Fig. 3 and 4 respectively.

Some of our obtained traveling wave solutions are represented in the following Figures:

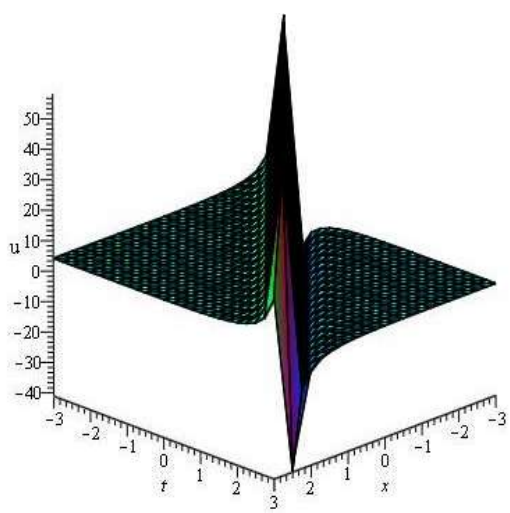

Fig. 1: Shape of $u_2(\xi)$ for $a_0 = 0, \alpha = \beta = 1, \gamma = -1, \mu = -2, \lambda = 0, A = 0.50$ and $-3 \leq x, t \leq 3$

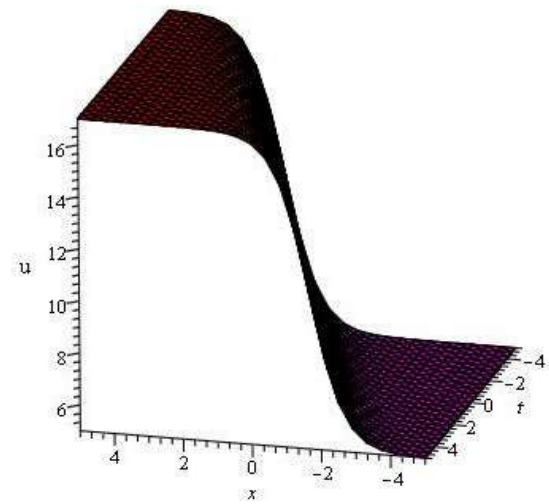

Fig. 2: Profile of $u_7(\xi)$ for $a_0 = -1$, $\alpha = \beta = 1$, $\gamma = 1$, $\mu = -1$, $\lambda = 2$, $A = 0$ and $-5 \leq x, t \leq 5$

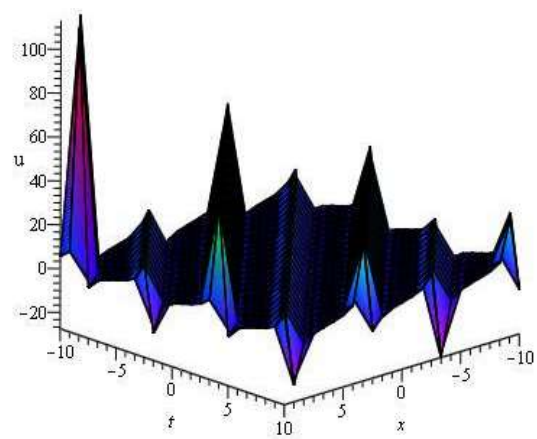

Fig. 3: Profile of $u_{10}(\xi)$ for $a_0 = 3$, $\gamma = 2$, $\alpha = \beta = 1$, $\mu = 1$, $\lambda = 2$, $A = 1$ and $-10 \leq x, t \leq 10$

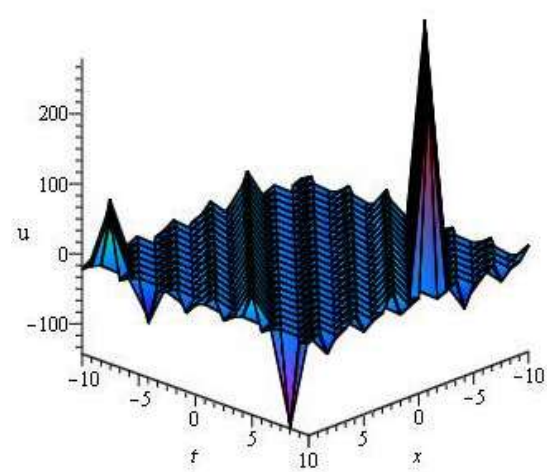

Fig. 4: Profile of $u_{17}(\xi)$ for $a_0 = 0$, $\gamma = 5$, $\alpha = \beta = 1$, $\mu = 1$, $\lambda = 2$, $A = 3$ and $-10 \leq x, t \leq 10$

COMPARISONS

With modified simple equation method: Zayed and Hoda [4] investigated exact solutions of the nonlinear generalized shallow water wave equation by using the modified simple equation method and obtained only two solutions (Appendix). On the contrary by using the enhanced (G′/G)-expansion method in this article we have obtained nineteen solutions. Furthermore, If we set $a_0 = A_0$, $\lambda\sqrt{-\mu} = \pm 1$, $A = 0$ and

$$\sqrt{-\mu} = \frac{1}{2}\sqrt{(1-\gamma)}$$

in our solution $u_6(\xi)$ and $u_7(\xi)$ (in Family 4), we conclude that our results coincide with the results (Eq. (3.18) and Eq. (3.19)) obtained by Zayed and Hoda [4], which conclude that the results derived by Zayed and Hoda [4] are particular cases of our results. Moreover we have derived more new solutions which were not derived in the article of Zayed and Hoda [4] by using modified simple equation method.

CONCLUSIONS

In this paper, an enhanced (G′/G)-expansion method has been successfully applied to find the exact solitary wave solutions for the nonlinear generalized shallow water wave equation. Abundant sets of solutions of a variety of distinct physical structures such as solitons, singular solitons and periodic solutions were formally derived. The study highlights the power of this method for the determination of exact solutions to several nonlinear evolution equations.

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Appendix

Zayed and Hoda [4] examined the exact solutions of the nonlinear generalized shallow water wave equation by using the modified simple equation method. They assumed the solution is of the form,

$$u(\xi) = \sum_{k=0}^{N} A_k \left(\frac{\psi'}{\psi}\right)^k$$

and they obtained the following solutions,

$$u(x,t) = A_0 \pm \frac{12C_1\sqrt{(1-\gamma)}}{\alpha+\beta}\left(\frac{\exp\left(\pm\sqrt{(1-\gamma)}(x-t)\right)}{C_2(1-\gamma)+C_1\exp\left(\pm\sqrt{(1-\gamma)}(x-t)\right)}\right) \quad (3.17)$$

where A_0 is an arbitrary constant. Setting $C_1 = 1$ and $C_2 = \frac{1}{1-\gamma}$ into Eq. (3.17), where $\gamma \neq 1$, Zayed and Hoda [4] found the following Solitary wave solutions:

$$u_{1,2}(x,t) = A_0 \pm \frac{6\sqrt{(1-\gamma)}}{\alpha+\beta}\left\{1 \pm \tanh\left(\frac{1}{2}\sqrt{(1-\gamma)}(x-t)\right)\right\} \quad (3.18)$$

Similarly, setting $C_1 = -1$ and $C_2 = \frac{1}{1-\gamma}$ into Eq. (3.17), where $\gamma \neq 1$, Zayed and Hoda [4] found the following Solitary wave solutions:

$$u_{3,4}(x,t) = A_0 \pm \frac{6\sqrt{(1-\gamma)}}{\alpha+\beta}\left\{1 \pm \coth\left(\frac{1}{2}\sqrt{(1-\gamma)}(x-t)\right)\right\} \quad (3.19)$$

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