Lithospheric Thinnings and the Thickenings caused by Instability of Deformation under the Influence of Internal Pressure and Forces of Inertia of Rotation

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**Abstract:** The mechanism of local changes of thickness of a lithosphere as a result of instability of deformation of an ellipsoidal lithospheric cover of Earth under the influence of the internal pressure and volume forces of inertia of rotation is found. The main stressed-deformed state of an elastic and viscous ellipsoid of rotation is considered. The equation of elastic balance and the main ratios are defined in degenerate elliptic coordinates. The axisymmetric task about the stressed state of an ellipsoid of rotation is solved at expansion under the influence of uniform pressure on its internal surface. The stressed-deformed state of an expanding ellipsoid of the rotation subject to action of volume forces of inertia of rotation is investigated. Stability of deformation is investigated by a Leybenzon-Ishlinsky method. The main stressed and deformed state is considered at an invariable form of border of a body and revolted taking into account turns of elements of borders of a body in the course of transition to an adjacent form of balance. Asymmetric forms of the indignations leading to loss of stability of an ellipsoid of rotation are defined. The common decision of the equations of balance is defined through the biharmonic functions expressed by means of tesseral spherical functions. Components of indignations are expressed through three any constants which are found from the corresponding boundary conditions. Exponential growth of components of indignations in time, accompanied by oscillatory changes takes place.

**Key words:** Earth • lithosphere • thickness • tectonic • stress • instability

**INTRODUCTION**

Works [1-9] from a position of mechanics of a deformable solid body are devoted to research of tectonic development of Earth. Here on elastic, viscoelastic and viscoplastic models of a lithospheric cover of Earth global and local regularities of tectonic movements are studied.

The basis of the modern concept of tectonics of lithosphere's plates is made by the following provisions [10-15]:

- The precondition about division of the top part of firm Earth into two covers, a lithosphere and an asthenosphere, significantly differing viscous properties;
- The lithosphere is subdivided into limited number of the plates, seven large and as much the small;
- Divergent, convergent and transform borders between plates define nature of mutual movements of plates;
- Movements of lithosphere's plates submit to laws of spherical geometry;
- The seduction completely compensates spreading;
- The reason of movement of plates in mantle convection.

The most part of earthquakes, volcanic eruptions and orogeny processes occurring on a planet is dated for area of borders between plates. Thus concentration of epicenters of the strongest earthquakes on the globe in rather accurately limited belts defines outlines of borders of lithosphere's plates.

The problem of delimitation of lithosphere's plates by mechanic-mathematical methods is unresolved and actual.

**Methods:** The mechanism of emergence of global tectonic breaks on which there is a splitting of a lithospheric cover into lithosphere's plates, is investigated by mathematical methods of the theory of stability of deformable systems.

**Mechanic-mathematical model:** The mechanic-mathematical model of process of emergence of global tectonic breaks is presented by local changes of
thickness of a lithosphere as a result of loss of stability of deformation of an ellipsoidal lithospheric cover of Earth under the influence of the internal pressure and volume forces of inertia of rotation. The lithospheric cover is rigidly linked to an adjacent continuous ellipsoid of rotation. The material of a lithosphere is modeled by a viscoplastic body and an asthenosphere material-a viscous body [16-21]. Stability of deformation is investigated by a Leybenzon-Ishlinsky method [22, 23]. The main stressed and deformed state is considered at an invariable form of border of a body and revolted taking into account turns of elements of borders of a body in the course of transition to an adjacent form of balance.

RESULTS AND DISCUSSIONS

The main stressed-deformed state of an elastic and viscous ellipsoid of rotation is investigated. The equation of elastic balance and the main ratios are defined in degenerate elliptic coordinates of \( s, \mu, \varphi \)

The ellipsoid rotates round its pivot-center symmetry with a constant angular speed \( \omega \) and is under the influence of the uniform pressure \( q \) attached to its surface in the positive direction to a normal. The balance equations in movements look like:

\[
\frac{1}{1-2v} \text{grad} \text{div} \mathbf{\varpi} + \nabla^2 \mathbf{\varpi} = \frac{1}{G} \text{grad} \Phi
\]

where

\[
\Phi = -\frac{1}{2} \frac{\omega^2 r^2}{g^2} : \text{Potential of centrifugal forces} \\
\mathbf{\varpi} : \text{Movement vector}
\]

\[
\mathbf{u} = \frac{1}{2\eta t} \exp(\text{i} \omega t) \sum_{m=1}^{3} \mathbf{f}_m(s, \mu) C_m
\]

\[
\mathbf{u}_n = \frac{1}{2\eta t} \exp(\text{i} \omega t) \sum_{m=1}^{3} \mathbf{g}_m(s, \mu) C_m
\]

\[
\mathbf{u}_z = \frac{1}{2\eta t} \exp(\text{i} \omega t) \sin \varphi \sum_{m=1}^{3} \mathbf{h}_m(s, \mu) C_m
\]

where \( i \) - imaginary unit, \( f \) - the complex frequency of quasistatic fluctuations, \( t \) - time,

\[
\mathbf{f}_m(s, \mu) = \frac{\sqrt{1+s^2}}{s^2 + \mu^2} \left\{ \frac{\mu(1-\mu^2)}{s^2 + \mu^2} + 4(1-v)\mu g \right\} \left[ P_m^n(s) \right] \left[ P_m^n(\mu) \right] ^n
\]

\[
= \left[ \frac{1}{s^2 + \mu^2} + 4(1-v)\mu^2 \right] P_m^n(s) P_m^n(\mu)
\]

\[
(1-2v)\mu l + s^2 \left[ P_m^n(s) \right] ^n \left[ P_m^n(\mu) \right] ^n
\]

\[
- (1-\mu^2)(s^2 - \mu^2)^2 + 4(1-v)\mu^2
\]

\[
\left[ P_m^n(s) \right] \left[ P_m^n(\mu) \right] ^n
\]

\[
- 2(1-\nu)\mu l + \mu^2 \left[ P_m^n(s) \right] \left[ P_m^n(\mu) \right] ^n
\]

\[
+ \frac{8(1-\nu)\mu l}{l^2 + \mu^2} \left[ P_m^n(s) \right] \left[ P_m^n(\mu) \right] ^n
\]

\[
\mathbf{g}_m(s, \mu) = \frac{\sqrt{1+s^2}}{s^2 + \mu^2} \left\{ \frac{\mu(1-\mu^2)}{s^2 + \mu^2} + 4(1-v)s \right\} \left[ P_m^n(\mu) \right] ^n
\]

\[
+ s \left[ P_m^n(s) \right] \left[ P_m^n(\mu) \right] ^n
\]

\[
+ \frac{1}{s^2 + \mu^2} + 4(1-v)\mu^2
\]

\[
\left[ P_m^n(\mu) \right] ^n
\]

\[
\mathbf{h}_m(s, \mu) = \frac{\sqrt{1+s^2}}{s^2 + \mu^2} \left\{ \frac{\mu(1-\mu^2)}{s^2 + \mu^2} + 4(1-v)s \right\} \left[ P_m^n(s) \right] \left[ P_m^n(\mu) \right] ^n
\]

\[
+ s \left[ P_m^n(s) \right] \left[ P_m^n(\mu) \right] ^n
\]

\[
+ \frac{1}{s^2 + \mu^2} + 4(1-v)\mu^2
\]

\[
\left[ P_m^n(\mu) \right] ^n
\]
\[ f_3(s, \mu) = \frac{\sqrt{1-s^2}}{i \sqrt{s^2 + \mu^2}} \left \{ j \left[ \frac{1-\mu^2}{s^2 + \mu^2} + 4(1-\nu)s \right] \left[ 2sP_n^m(s) \left[ \frac{1}{1-s^2 - \mu^2} \right] \right] P_n^m(\mu) + \left[ \frac{1-\mu^2}{s^2 + \mu^2} + 4(1-\nu)s \right] \right \} \]

\[ + \left[ 1-s^2 - \mu^2 \right] P_n^m(\mu) + \left[ 1-s^2 - \mu^2 \right] \left[ 2sP_n^m(s) \right] P_n^m(\mu) + \frac{8(1-\nu)s(1-s^2 - \mu^2)}{1 + s^2 (1-\mu^2)} P_n^m(\mu) \right \}, \]

\[ \varphi_3(s, \mu) = \frac{\sqrt{1-s^2}}{i \sqrt{s^2 + \mu^2}} \left \{ \left[ 1-s^2 - \mu^2 \right] \left[ \frac{1}{1-s^2 - \mu^2} \right] - 4(1-\nu)s \right \} \]

\[ P_n^m(s) + s(P_n^m(s)) \right \} P_n^m(\mu) \]

\[ + 2s^2 \mu \left[ \left[ 1-s^2 + (1-2\nu)(1-s^2 - \mu^2) \right] P_n^m(\mu) + \mu P_n^m(s) \right] P_n^m(\mu) + \mu P_n^m(s) \right \} P_n^m(\mu) \]

\[ + \left[ 1-s^2 + 2s(1-2\nu)(1-s^2 - \mu^2) \right] P_n^m(\mu) + \mu P_n^m(s) \right \} P_n^m(\mu) \]

\[ - 2(1-\nu)s(1+s^2) \left[ \frac{8(1-\nu)s(1-s^2 - \mu^2)}{1 + s^2 (1-\mu^2)} P_n^m(\mu) \right], \]

\[ \left \{ \left[ 1-s^2 + \mu^2 \right] \left[ \frac{1}{1-s^2 + \mu^2} \right] - 4(1-\nu)s \right \} \left[ 2sP_n^m(s) + 4s(1-s^2 - \mu^2) \right] P_n^m(\mu) \]

\[ + 2s^2 \mu \left[ \left[ 1-s^2 + (1-2\nu)(1-s^2 + \mu^2) \right] P_n^m(\mu) + \mu P_n^m(s) \right] P_n^m(\mu) + \mu P_n^m(s) \right \} P_n^m(\mu) \]

\[ + \left[ 1-s^2 + 2s(1-2\nu)(1-s^2 + \mu^2) \right] P_n^m(\mu) + \mu P_n^m(s) \right \} P_n^m(\mu) \]

\[ - 2(1-\nu)s(1+s^2) \left[ \frac{8(1-\nu)s(1-s^2 - \mu^2)}{1 + s^2 (1-\mu^2)} P_n^m(\mu) \right], \]

\[ \left \{ \left[ 1-s^2 + \mu^2 \right] \left[ \frac{1}{1-s^2 + \mu^2} \right] - 4(1-\nu)s \right \} \left[ 2sP_n^m(s) + 4s(1-s^2 + \mu^2) \right] P_n^m(\mu) \]

\[ + 2s^2 \mu \left[ \left[ 1-s^2 + (1-2\nu)(1-s^2 + \mu^2) \right] P_n^m(\mu) + \mu P_n^m(s) \right] P_n^m(\mu) + \mu P_n^m(s) \right \} P_n^m(\mu) \]

\[ + \left[ 1-s^2 + 2s(1-2\nu)(1-s^2 + \mu^2) \right] P_n^m(\mu) + \mu P_n^m(s) \right \} P_n^m(\mu) \]

\[ - 2(1-\nu)s(1+s^2) \left[ \frac{8(1-\nu)s(1-s^2 + \mu^2)}{1 + s^2 (1-\mu^2)} P_n^m(\mu) \right], \]

\[ \left \{ \left[ 1-s^2 + \mu^2 \right] \left[ \frac{1}{1-s^2 + \mu^2} \right] - 4(1-\nu)s \right \} \left[ 2sP_n^m(s) + 4s(1-s^2 + \mu^2) \right] P_n^m(\mu) \]

\[ + 2s^2 \mu \left[ \left[ 1-s^2 + (1-2\nu)(1-s^2 + \mu^2) \right] P_n^m(\mu) + \mu P_n^m(s) \right] P_n^m(\mu) + \mu P_n^m(s) \right \} P_n^m(\mu) \]

\[ + \left[ 1-s^2 + 2s(1-2\nu)(1-s^2 + \mu^2) \right] P_n^m(\mu) + \mu P_n^m(s) \right \} P_n^m(\mu) \]

\[ - 2(1-\nu)s(1+s^2) \left[ \frac{8(1-\nu)s(1-s^2 + \mu^2)}{1 + s^2 (1-\mu^2)} P_n^m(\mu) \right]. \]
\[
\mathcal{V}_1(s, \mu) = \frac{m}{(s^2 + \mu^2)\left(1 + s^2\right)\left(1 - \mu^2\right)} \left[ \mu \left(1 + s^2\right) \left(\int P_n^m(s)^2 \right)^2 \right] P_n^m(\mu) + s(1 - \mu^2) P_n^m(s) P_n^m(\mu)
\]

\[
\mathcal{V}_2(s, \mu) = \frac{m}{(s^2 + \mu^2)\left(1 + s^2\right)\left(1 - \mu^2\right)} \left[ \mu \left(1 + s^2\right) \left(\int P_n^m(s)^2 \right)^2 \right] P_n^m(\mu) + s(1 - \mu^2) P_n^m(s) P_n^m(\mu)
\]

\[
\mathcal{V}_3(s, \mu) = \frac{m}{(s^2 + \mu^2)\left(1 + s^2\right)\left(1 - \mu^2\right)} \left[ \mu \left(1 + s^2\right) \left(\int P_n^m(s)^2 \right)^2 \right] P_n^m(\mu) + s(1 - \mu^2) P_n^m(s) P_n^m(\mu)
\]

\[P_n^m(s) \text{ Legendre's attached functions of the 1st sort, } C_1, C_2, C_3 \text{-any constants are defined by boundary conditions.}\]

For a component of indignations of movements, speeds of deformations and stresses of a viscous ellipsoid of rotation exponential growth in time, accompanied by oscillatory changes takes place.

The stressed-deformed state of a lithospheric cover from an incompressible viscoplastic material directed by the theory of thin momentless covers is investigated. The ratio for extents of lengthening in a lithospheric cover looks like:

\[
\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 1
\]

where

\[
\lambda_1 = \lambda_0^\mu + \varepsilon_\mu^\mu, \lambda_2 = \lambda_0^\phi + \varepsilon_\phi^\phi
\]

\[\lambda_0^\mu, \lambda_0^\phi \text{-extents of lengthening in a lithospheric cover in the indignant condition; } \lambda_0^\mu \text{-extent of lengthening in the meridional direction in the main condition, } \lambda_0^\phi \text{-the same in the direction of parallels and}\]

\[
\lambda_0^\mu = 1 + \varepsilon_\mu^\mu, \lambda_0^\phi = 1 + \varepsilon_\phi^\phi
\]

where \(\varepsilon_\mu^\mu, \varepsilon_\phi^\phi\) - deformation components in the main condition; \(\varepsilon_\mu^\mu, \varepsilon_\phi^\phi\) - the corresponding components of indignations of deformations in a median surface of a lithospheric cover.

From a condition (5) it is found:

\[
\lambda = \frac{1}{\lambda_0^\mu \lambda_0^\phi} = \frac{1}{(\lambda_0^\mu + \varepsilon_\mu^\mu)(\lambda_0^\phi + \varepsilon_\phi^\phi)}
\]

Multiplying numerator and a denominator in a formula (7) by expressions \(\lambda_0^\mu - \varepsilon_\mu^\mu, \lambda_0^\phi - \varepsilon_\phi^\phi\), is received:

\[
\lambda = \frac{\lambda_0^\mu \lambda_0^\phi - (\lambda_0^\mu \varepsilon_\phi^\phi + \lambda_0^\phi \varepsilon_\mu^\mu) + \varepsilon_\mu^\mu \varepsilon_\phi^\phi}{(\lambda_0^\mu - \varepsilon_\mu^\mu)(\lambda_0^\phi - \varepsilon_\phi^\phi)}
\]

Considering a little sizes of indignations of deformations to within sizes of the second order of a smallness, it is found:

\[
\lambda_s = \frac{1}{\lambda_0^\mu \lambda_0^\phi} \left[ 1 - \left( \frac{\varepsilon_\mu^\mu}{\lambda_0^\mu} + \frac{\varepsilon_\phi^\phi}{\lambda_0^\phi} \right) \right]
\]

Then changing on coordinates of a median surface thickness of a lithospheric cover will be:

\[
h' = h \lambda_s = \frac{h}{\lambda_0^\mu \lambda_0^\phi} \left[ 1 - \left( \frac{\varepsilon_\mu^\mu}{\lambda_0^\mu} + \frac{\varepsilon_\phi^\phi}{\lambda_0^\phi} \right) \right]
\]

where \(h\) - thickness of a lithospheric cover in the main condition.

In lithosphere points, where the sum \(\varepsilon_\mu^\mu + \varepsilon_\phi^\phi\) will be positive, takes place cover thinnings, where it is negative-thickenings, where it is equal to zero-thickness of a cover remains invariable.

Deformations in the main condition

\[
\varepsilon_\mu^\mu = \xi_\mu^\mu \cdot \bar{t}, \varepsilon_\phi^\phi = \xi_\phi^\phi \cdot \bar{t}
\]

where \(\bar{t}\) - some interval of time in a vicinity of a point of linearization \(\xi_\mu^\mu\) and \(\xi_\phi^\phi\), don't depend on time.

Components of indignations of deformations \(\varepsilon_\mu^\mu\) and \(\varepsilon_\phi^\phi\) taking into account (9), are defined by formulas:

\[
\varepsilon_\mu = \frac{\xi_\mu}{\bar{t} \cdot \xi_\mu}
\]

\[
\varepsilon_\phi = \frac{\xi_\phi}{\bar{t} \cdot \xi_\phi}
\]

(11)
Thus, for an incompressible viscoplastic lithospheric cover at stability loss local change of thickness of a cover is determined by a formula (9).

The analysis of numerical results shows that at values of parameters of wave formation in the meridional direction \( n = 8 \) and in the longitudinal direction \( m = 2 \) value of internal pressure \( q = 10^8 \text{ atm} \) correspond to the line of the maximum thinnings on which there is a splitting of a lithospheric cover into the lithosphere's plates close to a modern outline of borders of lithosphere's plates (Fig. 1).

**CONCLUSION**

It is established that the main reason of emergence of global tectonic breaks on which there is a splitting of a lithospheric cover into lithosphere's plates, loss of stability of a lithospheric cover of Earth under the influence of the internal pressure and volume forces of inertia of rotation is.

**REFERENCES**