

## Strength and Deformability of Reinforced Concrete Structures in Service

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**Abstract:** There is given methodology of evaluating the strain-stress state of reinforced concrete structures of buildings and constructions in service taking into account the operation conditions and technological impacts.

**Key words:** Reinforced concrete • Mode of operation • Physical and mechanical properties • Reinforcement

### INTRODUCTION

The reinforced concrete theory must describe the strain-stress state in unified terms and evaluate the load-carrying ability of structures in all stages of a building's or construction's lifespan on the basis of quantitative assessment of stress and temporal factors. The basic premises of calculating the reinforced concrete structures in service and evaluating their structural safety are formulated in [1].

The strain resistance of section, exposed to corrosion, is determined by the depth of damage to the concrete. The problem is solved with regard to the assumed premises:

- The intensity of aggressive influence on the structure is accepted as permanent in time;
- The strain resistance of section restores as the layers under study are taken farther from the surface of contact with aggressive medium or from the layer deprived of the strain resistance completely;
- The quantitative assessment of corrosive damage is accepted on the base of V.M. Bondarenko's suggestions [2] with the use of Guldberg-Waage's entropy model including strength and deformation properties;
- The extent of reinforcement steel corrosive damage is evaluated by reduce of cross-section area.

The relation between stresses and deformations is accepted as follows:

$$\varepsilon(t) \cdot f[\varepsilon(t)] = \sigma(t) \cdot \left[ \frac{1}{E_m^0(t)} + A(t, t_0) \right] \quad (1)$$

where  $\varepsilon(t) \cdot f[\varepsilon(t)] = \kappa \cdot e^{m(t)\varepsilon(t)}$

from which

$$\sigma(t) = \frac{k \cdot \varepsilon(t) \cdot e^{m(t)\varepsilon(t)}}{\frac{1}{E_m^0(t)} + A(t_0, t)} \quad (2)$$

According to [2] the temporal secant deformation modulus is:

$$E_{Bp}(t) = \frac{\sigma(t)}{\varepsilon(t)} = \frac{k \cdot e^{m\varepsilon(t)}}{\frac{1}{E_m^0(t)} + A(t_0, t)} \quad (3)$$

The formula (2) allows describing the «stress-deformation» diagram for concrete at all load conditions from the unified methodological perspectives.

Let us accept the relation between stresses  $\sigma_s$  and deformations  $\varepsilon_s$  of reinforcement bars without physical yield line at uniaxial tension in the form, suggested by N.I.Karpenko [3]:

$$\varepsilon_s = \frac{\sigma_s}{E_s v_s} \quad (4)$$

where  $v_s$  – the secant modulus alteration coefficient.

$$v_0 = 1, \sigma_s = \sigma_{s,u}, \hat{\epsilon}_s = \epsilon_{s,u}, \quad (5)$$

$$\hat{v}_s = \frac{\hat{\sigma}_s}{E_s \hat{\epsilon}_s} \quad (6)$$

where  $\sigma_s$  – fracture stress;  $\epsilon_{su}$  – percent elongation at fracture;  $\eta$  – stress level; at  $\sigma_s = \sigma_{s,el}$   $\eta=0$ , at  $\sigma_s > \sigma_{s,el}$

$$\eta = \frac{(\sigma_s - \sigma_{s,el})}{(\hat{\sigma}_s - \sigma_{s,el})} \quad (7)$$

There is considered a vertically-aligned reinforced concrete element with arbitrary form of section. To obtain resolving equations let us consider the most general case of stress state – eccentric compression, a particular case

of which is flexure. It is also assumed that zero stress axis of the element's normal section coincides with zero deformation axis.

Let us determine the strain-stress state and evaluate the load-carrying ability of a corrosion-damaged reinforced concrete beam. The element under study is subjected to force  $N_H$ , impressed with eccentricity  $e_H$ . In Figure 1 there are shown potential cases of stress and strain distribution in the element's section. The first case is typical for elements with small eccentricity of the external load; in other cases the eccentricities increase constantly, which results in expanding the tensile zone of the concrete.

The external force, acting on the considered element, is carried by undamaged concrete ( $N_b, M_b$ ), the concrete, exposed to aggressive influence ( $N_{cr}, M_{cr}$ ) and compressed or tensile (less compressed) reinforcement metal ( $N_{sc}, M_{sc}$  and  $N_s, M_s$ ).

The resultant vector and resultant moment for the corrosion-damaged reinforced concrete beam are defined by the following formulas:

$$N = \int_{F_b} \sigma_b(t_0, t) dF + \int_{F_{cr}} \sigma_{cr}(t_0, t) dF + \sum_{k=1}^i \sigma_{sc,i}(t_0, t) A_{sc,i} + \sum_{k=1}^i \sigma_{si}(t_0, t) A_{si}; \quad (8)$$

$$M = \int_{F_b} \sigma_b(t_0, t)(y_b) dF + \int_{F_{cr}} \sigma_{cr}(t_0, t)(y_{cr}) dF + \sum_{k=1}^i \sigma_{sc,i}(t_0, t) A_{sc,i}(y_{sc,i}) + \sum_{k=1}^i \sigma_{s,i}(t_0, t) A_{s,i}(y_{s,i}). \quad (9)$$

On integrating the equations (8) and (9) for case I of stress and strain distribution in the element (Fig. 1), we get the following formulas:

$$N = \frac{b_{cp} h_{cr}}{6} (\sigma_{cr,0} + 2\sigma_{cr,1} + 2\sigma_{cr,2} + \sigma_0) + \frac{b_{cp}(h-h_{cr})}{8} (\sigma_0 + 2\sigma_1 + 2\sigma_2 + 2\sigma_3 + \sigma_4) + \left[ \epsilon_{cr,0} - \frac{a'}{h} (\epsilon_{cr,0} - \epsilon_4) \right] E_{sc} A'_s \omega_{cr} k_{cr} + \left[ (\epsilon_{cr,0} + \epsilon_4) \frac{a}{h} - \epsilon_4 \right] E_s A_s; \quad (11)$$

$$M = \frac{b_{cp} h_{cr}}{6} (\sigma_{cr,0} + 2\sigma_{cr,1} + 2\sigma_{cr,2} + \sigma_0) \left( y_{u.T} - \frac{h_{cr}}{9} \frac{\sigma_{cr,0} + 6\sigma_{cr,1} + 12\sigma_{cr,2} + 8\sigma_0}{\sigma_{cr,0} + 2\sigma_{cr,1} + 2\sigma_{cr,2} + \sigma_0} \right) - \frac{b_{cp}(h-h_{cr})}{8} (\sigma_0 + 2\sigma_1 + 2\sigma_2 + 2\sigma_3 + \sigma_4) \left( h - y_{u.T} - \frac{(h-h_{cr})}{12} \times \frac{11\sigma_0 + 18\sigma_1 + 12\sigma_2 + 6\sigma_3 + \sigma_4}{\sigma_0 + 2\sigma_1 + 2\sigma_2 + 2\sigma_3 + \sigma_4} \right) + \left[ \epsilon_{cr,0} - \frac{a'}{h} (\epsilon_{cr,0} - \epsilon_4) \right] E_{sc} A'_s \omega_{cr} k_{cr} (y_{u.T} - a') + \left[ (\epsilon_{cr,0} + \epsilon_4) \frac{a}{h} - \epsilon_4 \right] E_s A_s (h - y_{u.T} - a).$$

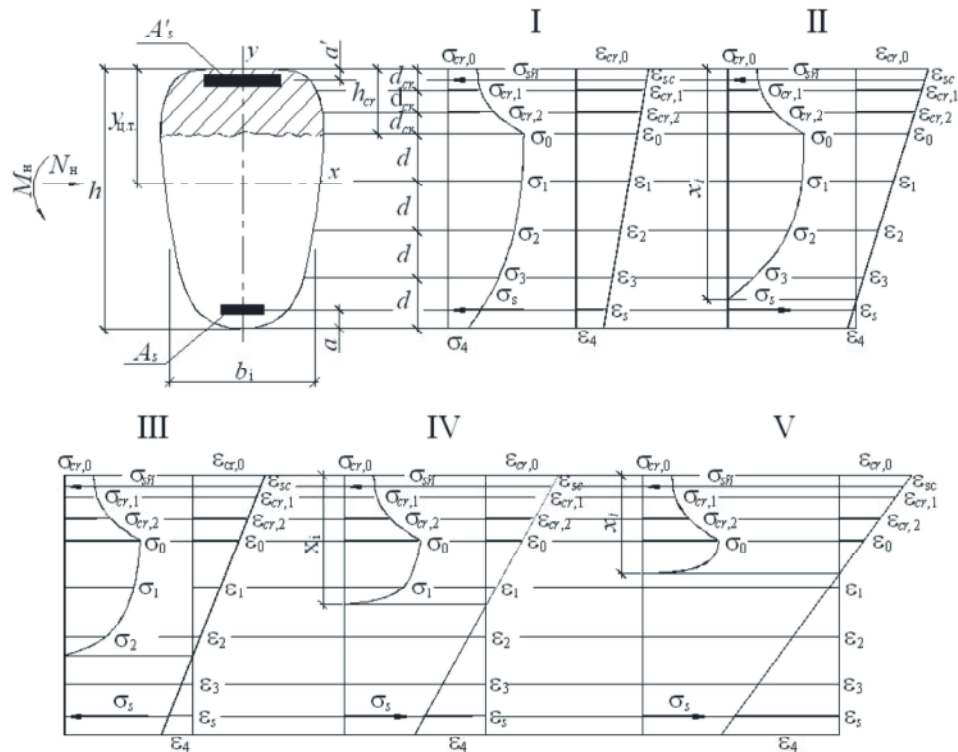


Fig. 1: The stress and strain distribution in the section of an eccentrically compressed reinforced concrete element, exposed to corrosion.

Deformations in the considered sections of damaged and undamaged reinforced concrete are determined through fiber deformation of compressed concrete with regard to the assumed premises for calculating:

$$\begin{aligned}
 \epsilon_{sc} &= \epsilon_{cr,0} \left( \frac{x_i - a'}{x_i} \right)^{n_\epsilon} ; \epsilon_{cr,1} = \epsilon_{cr,0} \left( \frac{x_i - d_{cr}}{x_i} \right)^{n_\epsilon} ; \\
 \epsilon_{cr,2} &= \epsilon_{cr,0} \left( \frac{x_i - 2d_{cr}}{x_i} \right)^{n_\epsilon} ; \epsilon_0 = \epsilon_{cr,0} \left( \frac{x_i - h_{cr}}{x_i} \right)^{n_\epsilon} ; \\
 \epsilon_0 &= \epsilon_{cr,0} \left( \frac{x_i - h_{cr}}{x_i} \right)^{n_\epsilon} ; \epsilon_1 = \epsilon_{cr,0} \left( \frac{x_i - h_{cr} - d}{x_i} \right)^{n_\epsilon} ; \\
 \epsilon_2 &= \epsilon_{cr,0} \left( \frac{x_i - h_{cr} - 2d}{x_i} \right)^{n_\epsilon} ; \epsilon_3 = \epsilon_{cr,0} \left( \frac{x_i - h_{cr} - 3d}{x_i} \right)^{n_\epsilon} ;
 \end{aligned} \tag{12}$$

$$\epsilon_s = \epsilon_{cr,0} \left( \frac{x_i - h + a}{x_i} \right)^{n_\epsilon} ; \epsilon_4 = \epsilon_{cr,0} \left( \frac{x_i - h}{x_i} \right)^{n_\epsilon} , \tag{13}$$

where  $n_\epsilon$  – coefficient with account of warping along the height of the element's section.

Knowing the relation between stresses and deformations in undamaged concrete, in reinforcement metal and equations for defining the stress of concrete, exposed to corrosion, we can determine the strain-stress state of an element in any considered period of time.

The curvature of the element can be determined knowing the deformation of concrete fiber:

$$\frac{1}{\rho} = \frac{\varepsilon_{cr,0}}{x_i} \quad (14)$$

Presenting the deflection curve of the beam with a sinusoid and applying formula (14) for the curvature allow determining the deflection of the beam's height midpoint:

$$f = \frac{l^2}{\pi^2} \frac{\varepsilon_{cr,0}}{x_i} \quad (15)$$

At calculation of eccentrically compressed structures, when deflection is commensurable with the initial eccentricity of load application, we must take into account the influence of a secondary moment, induced by increase of deflection in the process of loading or in time. So the external moment, acting on the element, can be represented as a sum of two moments – the ordinary moment ( $M_0=Ne_0$ ) of external forces and the secondary moment ( $M_d=Nf$ ) of the axial force, appearing due to increase of deflection. The value of the total moment alters both in the process of load increment and at the prolonged acting of it:

$$M = M_0 + M_d \quad (16)$$

By simultaneous solution of these equations (10) - (13), (3.65) and (16) we determine the strain-stress state of an element before strengthening. But the obtained formula is lengthy even for a rectangular-sectioned element, so it's more reasonable to determine the unknowns  $\varepsilon_{cr,0}$  and  $x_i$  by a sequential applications method.

By a similar method we can consider the other cases of strain-stress state of an element (Fig. 1, cases II - V).

In the obtained expressions the descriptions of strain-stress states of eccentrically compressed elements of the deformed concrete and reinforcement metal are represented by fiber deformations of a most compressed face of concrete by formulas (12). The account of the element's deflection influence is done similarly to the first case under consideration.

So, we have got the generalizing equations of evaluating strength and rigidity of reinforced concrete elements with corrosion-damaged concrete and reinforcement metal.

## REFERENCES

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