# Determination of the Aerodynamic Characteristics of Darrieus Wind Turbine System of Troposkino 

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Submitted: Aug 2, 2013; Accepted: Aug 31, 2013; Published: Sep 4, 2013


#### Abstract

Recently, most of wind energy show interest in connection with a number of positive qualities to the vertical-axis Darrieus wind turbine type. Until now exist wind power installations are can be said that the age of 25-30 years, while other types of wind turbines (sailing, propeller) seriously began to study almost half a century ago. The theory of these wind turbines in sufficient detail given in the well-known monograph Fateeva. In this article presents the results theoretical substantiation for one of the types of Darrieus wind turbine system design troposkino. This structural form of apparatus Daria is becoming more and more popular. A general theory, identifies almost all structural and aerodynamic characteristics (attack speed, angle of attack, lift, air resistance, the rotation of the blades, the utilization of wind power, torque and power turbines, etc.) of this unit. Thus, an attempt to lay the basic foundations of the theory of Daria devices troposkino system.


$\underline{\text { Key words: Wind turbine Darrieus • Power • Wind speed • Blade • Reynolds number }}$

## INTRODUCTION

The theory of sailing, propeller wind turbines in sufficient detail given in the well-known monograph [1]. In recent years the popularity of wind turbines (wind turbines) with a vertical axis of rotation has increased significantly [2-9]. The authors of [10] are developed by their production technology symmetrical airfoils NASA made of composite material. At present mainly manufactured and used units with wind turbine Darrieus with straight blades workers [2-9]. However, studies of these devices as troposkino substantially omitted.

This article is devoted to the development of the foundations of the theory of Darrieus wind turbine troposkino.

## Basic Physical Modelling: Troposkino design

 (Fig. 1) by near-parabolic form, described following equation$$
\begin{equation*}
x=r=r_{m}-\frac{9 z^{2}}{16 r_{m}}=\frac{3}{8} H-\frac{3}{2} \frac{z^{2}}{H}, \tag{1}
\end{equation*}
$$

where $r_{m}=\frac{3}{8} H$ is a maximum rotation radius of the turbine (as per the proposed design), $H$ - height. So, the formula (1) looks like:

$$
\begin{equation*}
r=r_{m}\left[1-\left(\frac{z}{H / 2}\right)^{2}\right] \tag{2}
\end{equation*}
$$

In case of a turbine with straight blades " $r$ " does not depend upon " $z$ " $[3,4]$ and $r_{m}=r_{0}$

A gap area, formed by a symmetrical pair of blades is a frontal area of the turbine rotation surface and equals

$$
\begin{equation*}
F=\frac{4}{3} r_{m} H=\frac{H^{2}}{2} \tag{3}
\end{equation*}
$$

Let's find the length of 2 blades. As know, the $L_{1,2}$ curve length on a $\left[z_{1}, z_{2}\right]$ line is determined by the following formula: $L_{1,2}=\int_{Z_{1}}^{Z_{2}} \sqrt{(d x)^{2}+(d z)^{2}}$. So, the length of both blades equals:


Fig. 1: Darrieus wind turbine with turbine of troposkino system


Fig. 2: Troposkino blade and wind flow interaction scheme
$2 L=4 \int_{0}^{H / 2} \sqrt{1+\left(\frac{d x}{d z}\right)^{2}} d z=4 \int_{0}^{H / 2} \sqrt{1+9 \frac{z^{2}}{H^{2}}} d z=$
$=2\left[\frac{H}{2} \sqrt{1+\frac{9}{4}+\frac{H}{3} \ln \left(\frac{3}{2}+\sqrt{1+\frac{9}{4}}\right)}\right] \approx 2,6 H$.

Define Angular Momentum: Let's find the moment of force received by Darrie two-bladed turbine with troposkino design, replaced by a parabola in our case (2). For this purpose, we define attack rate component vectors at point $M$ first on a wing curved element of incremental length $d S$ The axis system is set in such a way that the wind vector $\vec{U}$ is the same as the $y$ axis and the wind turbine vertical rotation axis with the same of z .

Direction of the wind turbine rotation with an angular rate $\vec{\omega}$ shall be selected, so that, if you look at the turbine from the positive direction of the axis z , we will see a counterclockwise rotation. Let's review an instantaneous position of blades turned against axis $x$ through an angle $\theta$ (Fig. 1). To facilitate further analysis (Fig. 2), let's take three mutually perpendicular lines. Two of those lines are tangent to blade surface at point $M$. This is a $A A^{\prime}$ tangent to the parabola line and defines blade angularity, i.e. $\gamma$ between line $A A^{\prime}$ and vertical $N N^{\prime}$ and $B B^{\prime}$ - a tangent at point $M$ to a circle, described by radius $r$ by turbine rotation. And, finally, the third $C C^{\prime}$ line presents a normal to the blade surface, focused at point $M$.

The attack rate vector $\vec{V}$ presents a summation of two vectors. One of them is a wind velocity vector component $\vec{U}$, a normal to the blade surface that equals $\vec{U} \sin \theta \cos \gamma$ (Fig. 3), the second one is tangential $B B^{\prime}$ and presents a summation of a linear rotation $(\vec{r} \times \vec{\omega})$ of point $M$ plus wind velocity component, designed for direction $B B^{\prime}(\vec{U} \cos \theta)$. Since line $B B^{\prime}$, along which total velocity is applied $\vec{r} \times \vec{\omega}+\vec{U} \cos \theta$, and line $C C^{\prime}$, along which a normal component of wind velocity $\vec{U} \sin \theta \cos \gamma$ is applied are mutually perpendicular, then their resultant equals

$$
\begin{equation*}
|\vec{V}|=\sqrt{U^{2} \sin ^{2} \theta \cos ^{2} \gamma+(r \omega+U \cos \theta)^{2}} \tag{5}
\end{equation*}
$$

and gives a value of the attack rate of the air flow at point $M$. Hence, the angle of attack is presented by the following formula


Fig. 3: Troposkino blade element for determining the speed and angle of attack.

$$
\begin{equation*}
\operatorname{tg} \alpha=\frac{U \sin \theta \cos \gamma}{r \omega+U \cos \theta} \tag{6}
\end{equation*}
$$

Based on (5) and (6), it is easy to establish a link between the angle of attack and blade angle:

$$
\begin{align*}
& \vec{V} \cos \alpha=\vec{U}(\chi+\cos \theta) \\
& \vec{V} \sin \alpha=\vec{U} \sin \theta \cos \gamma \tag{7}
\end{align*}
$$

A moment, produced by the lift tangential component $\vec{R}_{l}$ equals $M=r R_{\tau}=r\left|\vec{R}_{l}\right| \sin \alpha$. Lift force $\vec{R}_{l}$ is perpendicular to the attack velocity vector and related to the latter by the lift force coefficient. A counterforce is wind resistance to blade motion $\left|\vec{R}_{D}\right|$.

It follows that the moment produced by blade element $d S$ is written as:
$d M_{l}=r \rho \frac{V^{2}}{2} b\left[C_{y} \sin \alpha-C_{x} \cos \alpha\right] d S$,
Substituting values $C_{y}$ and $C_{x}$ for profile NASA-0021 in (8), we obtain
$d M_{l}=\rho \frac{V^{2}}{2} b r\left[\sqrt{2} \pi \sin ^{2} \alpha-\left(0,014+\sin ^{2} \alpha\right) \cos \alpha\right] d S$
The solution (9) divides into 2 separate problems $d M_{l}=d M_{l 1}+d M_{l 2}$
$d M_{l 1}=\sqrt{2} b r \pi \frac{\rho V^{2}}{2} \sin ^{2} \alpha d S$,
where
$d M_{l 2}=-b r \frac{\rho V^{2}}{2}\left(0,014+\sin ^{2} \alpha\right) \cos \alpha d S$.
Angular Momentum by Lifting Force: The first problem (10) may be solved quite easily. Looking at (7), we obtain
$d M_{l 1}=\sqrt{2} \pi b r(z) \frac{\rho U^{2}}{2} \sin ^{2} \theta \cos \gamma d z$,
as $\frac{d z}{d S}=\cos \gamma$.
Applying formula (2), we obtain the following:
$d M_{l 1}=\sqrt{2} \pi b \frac{r_{m}\left(1-\bar{z}^{2}\right) H}{2} \rho \frac{U^{2}}{2} \sin ^{2} \theta \cos \gamma d \bar{z}$,
where $\bar{z}=\frac{z}{H / 2}$.
Dependence $\cos \gamma$ upon axes $z$ is easy to define. For this purpose, we write an equation for a tangent at any parabola point:

$$
\begin{equation*}
\operatorname{tg} \gamma=\frac{d x}{d z}=-3 \frac{z}{H}=\frac{\sqrt{1-\cos ^{2} \gamma}}{\cos \gamma} \tag{14}
\end{equation*}
$$

or
$\frac{1}{\cos \gamma}=\sqrt{1+9\left(\frac{z}{H}\right)^{2}}=\sqrt{1+\frac{9}{4} \bar{z}^{2}}$.

Substituting (15) in (13), we obtain
$d M_{l 1}=\sqrt{\frac{2}{2}} \pi H r_{m} b \rho \frac{U^{2}}{2} \sin ^{2} \theta \frac{1-\bar{z}^{2}}{\sqrt{1+9 / 4^{z^{2}}}} d \bar{z}$
and integrating the last value for $\bar{z}$, we obtain the blade rotation moment value
$M_{l 1}=\frac{\sqrt{2}}{4} \pi H r_{m} b \rho U^{2} \sin ^{2} \theta \int_{-1}^{1} \frac{1-\bar{z}^{2}}{\sqrt{1+9 / 4 z^{2}}} d \bar{z}$

Standard integral
$\int_{-1}^{1} \frac{1-\bar{z}^{2}}{\sqrt{1+9 / 4^{2}}} d \bar{z} \approx 1,37$.
Thus, we will have the following for a two bladed turbine:
$2 M_{l 1}=0,685 \sqrt{2} \pi b r_{m} H \rho U^{2} \sin ^{2} \theta$.

Rotation moment mean value per one turbine rotation may be defined by integrating $2 M_{l l}$ from zero to $2 \pi$, dividing by $2 \pi$ :
$M_{l 1}=\frac{1}{\pi} \int_{0}^{2 \pi} M_{l 1} d \theta=0,685 \sqrt{2} b r_{m} H \rho U^{2} \int_{0}^{2 \pi} \sin ^{2} \theta d \theta$
or
$M_{l l}=1,37 \sqrt{2} \pi b r_{m} H \rho \frac{U^{2}}{2}$

Air Resistance of the Rotation Blades: Let us turn to the second problem (11). If to factor out $\sin ^{2} \theta$ and take into account (5) and (7), then expression (11) is as follows:

$$
\begin{align*}
d M_{l 2}= & -b r(z) \frac{\rho U^{2}}{2}\left(1+0,014 \frac{\sin ^{2} \theta \cos ^{2} \gamma+(\chi+\cos \theta)^{2}}{\sin ^{2} \theta \cos ^{2} \gamma}\right) \times \\
& \times \frac{\sin ^{2} \theta \cos ^{2} \gamma(\chi+\cos \theta) d S}{\sqrt{\sin ^{2} \theta \cos ^{2} \gamma+(\chi+\cos \theta)^{2}}} . \tag{20}
\end{align*}
$$

Before turning to a further procedure of solving (20), we first obtain additional relations and links. First of all, please note that in case with troposkino, the turbine specific speed $x$ varies with height
$\chi(z)=\frac{\omega r(z)}{U}$
and this factor makes the problem solving procedure to be difficult (10). There are various ways to circumvent this difficulty, if to take into account that the key element in air resistance to blade motion is a troposkino central section. And this central section covers about $85 \%$ of the blade length. As a matter of fact, from (2) it follows that
$\bar{z}=\frac{z}{H / 2}=\sqrt{1-\bar{r}}$,
where $\bar{r}=\frac{r(z)}{r_{m}}$.
Now we can define $\bar{z}$, for example, by reducing $\bar{r}$ three-fold
$\bar{z}=\sqrt{0,7} \approx 0,85$.
Taking into account (21), dependence (22) may be expressible by turbine element specific speed value $x(z)$
$\bar{z}=\sqrt{1-\bar{\chi}}$,
where $\bar{\chi}=\frac{\chi(z)}{\chi_{m}}, \quad \chi_{m}=\frac{\omega r_{m}}{U}$.
The fact that the key element of turbine operation is a blade central section (85\%), it allows minimizing relation nonlinearity (20). For this purpose, we open the brackets and take out value $x+\cos \theta$ of the root. Then we obtain

$$
\begin{gathered}
d M_{l 2}=-b r(z) \frac{\rho U^{2}}{2}\left[\frac{1}{\sqrt{1+\left(\frac{\sin \theta \cos \gamma}{\chi+\cos \theta}\right)^{2}}}+0,014\left(\frac{\chi+\cos \theta}{\sin \theta \cos \gamma}\right)^{2} \times\right. \\
\times \sqrt{1+\left(\frac{\sin \theta \cos \gamma}{\chi+\cos \theta}\right)^{2}} \sin ^{2} \theta \cos ^{2} \gamma d S
\end{gathered}
$$

Estimation of value $\beta=\left(\frac{\sin \theta \cos \gamma}{x+\cos \theta}\right)^{2}$ shows that with regard to the troposkino central section, covering $85 \%$ of the blade length, it may be assumed that $\beta \ll 1$. In fact, taking into account (21) and (15), value $\beta$ may be presented as follows
$\beta=\left[\frac{\sin \theta}{\left(1+\frac{9}{4} \bar{z}\right)\left(\cos \theta+\chi_{m} \bar{\chi}\right)}\right]^{2}$
Considering assumption $\bar{z}=0,85$ at $\bar{\chi}=\frac{r}{r_{m}}=0,3$ and taking into consideration the fact that wind turbine operating mode requires $x_{m} \geq 3$ let's define
$\beta=\left[\frac{\sin \theta}{\left(1+0,85 \frac{9}{4}\right)(\cos \theta+3 \cdot 0,3)}\right]^{2}$
From which it follows that, even at the maximum value $\sin \theta=1, \cos \theta=0, \beta \approx 0,1$. In this case we expand $\frac{1}{\sqrt{1-\beta}}$ and $\sqrt{1-\beta}$ in series and restrict ourselves to first two terms of the series
$d M_{l 2}=-b r(z) \frac{\rho U^{2}}{2}\left[1-\frac{1}{2}\left(\frac{\sin \theta \cos \gamma}{x_{m} \bar{x}+\cos \theta}\right)^{2}+0,014\left(\frac{x_{m} \bar{x}+\cos \theta}{\sin \theta \cos \theta}\right)^{2}+0,007\right] \times$
$\sin ^{2} \theta \cos ^{2} \gamma d S=-b r(z) \frac{\rho U^{2}}{2} \times$
$\left[1,007 \sin ^{2} \theta \cos ^{2} \gamma-\frac{1}{2} \frac{(\sin \theta \cos \gamma)^{2}}{2\left(x_{m} \bar{x}+\cos \theta\right)^{2}}+0,014\left(x_{m} \bar{x}+\cos \theta\right)^{2}\right] d S$.
It is obvious that the second term in square brackets, in comparison with other two ones, can be ignored. As a result, the solution is again divided into 2 problems

$$
\begin{equation*}
d M_{l 2}=d M_{l 2}^{\prime}+d M_{l 2}^{\prime \prime} \tag{25}
\end{equation*}
$$

where

$$
\begin{equation*}
d M_{l 2}^{\prime}=-1,007 b r(z) \frac{\rho U^{2}}{2} \sin ^{2} \theta \cos \gamma d z \tag{26}
\end{equation*}
$$

since $\frac{d z}{d S}=\cos \gamma$ and

$$
\begin{equation*}
d M_{l 2}^{\prime \prime}=-0,014 b r(z) \frac{\rho U^{2}}{2}\left(\cos \theta+\chi_{m} \bar{\chi}\right)^{2} d S \tag{27}
\end{equation*}
$$

The solution first task (26) reduces to that already known solution (18) of (12) and has the form
$2 M_{l 2}^{\prime}=-0,577 r_{m} b H \frac{\rho U^{2}}{2}(1-\cos 2 \theta)$.

Problem (27) can be also easily solved. For that, taking into account (2) and (24), we write

$$
\begin{equation*}
\bar{z}=\frac{2 z}{H}=\sqrt{1-\bar{r}}=\sqrt{1-\bar{\chi}} . \tag{29}
\end{equation*}
$$

Then (27) brings to the form of

$$
d M_{l 2}^{\prime \prime}=-0,014 b r_{m} \bar{r} \rho \frac{U^{2}}{2}\left(\cos \theta+\chi_{m} \bar{r}\right)^{2} \frac{d z}{\cos \gamma}
$$

or
$d M_{l 2}^{\prime \prime}=-0,014 \frac{b H}{4} r_{m} \rho U^{2}\left(1-\bar{z}^{2}\right)\left[\cos \theta+\chi_{m}\left(1-\bar{z}^{2}\right)\right]^{2} \sqrt{1+9 / 4 \bar{z}^{2}} d \bar{z}$

Let us open quadric square brackets
$d M_{12}^{\prime \prime}=-0,007 b r_{m} H \rho \frac{U^{2}}{2}\left[\left(\cos \theta+\chi_{m}\right)^{2}\left(1-\bar{z}^{2}\right)-2 \chi_{m} \times\right.$
$\left.\times\left(\cos \theta+\chi_{m}\right) \bar{z}^{2}\left(1-\bar{z}^{2}\right)+\chi_{m}^{2} \bar{z}^{4}\left(1-\bar{z}^{2}\right)\right] \sqrt{1+\frac{9}{4} \bar{z}^{2}} d \bar{z}$
For integrating purpose, let us bring (31) to the form of
$d M_{l 2}^{\prime \prime}=A_{0}\left(A_{1}+A_{2} y^{2}+A_{3} y^{4}+A_{4} y^{6}\right) \sqrt{1+y^{2}} d y$,
where $y=\frac{3}{2} \bar{z}, A_{0}=-\frac{0,007}{3} b r_{m} H \rho U^{2}, A_{1}=\left(\cos \theta+\chi_{m}\right)^{2}$,
$A_{2}=-\frac{4}{9}\left(\cos ^{2} \theta+4 \chi_{m} \cos \theta+3 \chi_{m}^{2}\right)$,
$A_{3}=\frac{16}{81}\left(2 \chi_{m} \cos \theta+3 \chi_{m}^{2}\right), A_{4}=-\frac{64}{729} \chi_{m}^{2}$.
Integration brings to standard integrals
$I_{1}=2 \int_{0}^{3 / 2} \sqrt{1+y^{2}} d y=3,12$;
$3 / 2 \sqrt{0,7}$
$I_{2}=2 \int_{0}^{2} y^{2} \sqrt{1+y^{2}} d y=1,43 ;$
$3 / 2 \sqrt{0,7}$
$I_{3}=2 \int_{0} y^{4} \sqrt{1+y^{2}} d y=2,36 ;$
$3 / 2 \sqrt{0,7}$
$I_{4}=2 \int_{0}^{3 / 2} y^{6} \sqrt{1+y^{2}} d y=2,34$;
The limits of integration are limited by values $\bar{z}=-\sqrt{0,7}$ and $\bar{z}=\sqrt{0,7}$ (see (23)). As a result, we have the following for a two bladed turbine
$2 M_{l 2}^{\prime \prime}=-\frac{0,014}{3} r_{m} b H \rho U^{2}\left[3,12\left(\chi_{m}+\cos \theta\right)^{2}-1,43 \frac{4}{9}\left(\cos ^{2} \theta+4 \chi_{m} \cos \theta+3 \chi_{m}^{2}\right)+\right.$
$\left.+2,37 \frac{16}{81}\left(2 \chi_{m} \cos \theta+3 \chi_{m}^{2}\right)-3,34 \frac{64}{729} \chi_{m}^{2}\right]$

Let us open ordinary brackets and bring together all terms. Then we obtain
$2 M_{l 2}^{\prime \prime}=-\frac{0,014}{3} r_{m} b H \rho U^{2}\left(2,48 \cos ^{2} \theta+4,64 \chi_{m} \cos \theta+2,32 \chi_{m}^{2}\right)$
Join with solution (28)

$$
\begin{align*}
& 2 M_{l 2}=2\left(M_{l 2}^{\prime}+M_{l 2}^{\prime \prime}\right)=-\frac{0,014}{3} r_{m} b H \rho U^{2} \times \\
& \times\left[0,577 \frac{3}{0,014}(1-\cos 2 \theta)+2,48 \cos ^{2} \theta+4,63 \chi_{m} \cos \theta+2,32 \chi_{m}^{2}\right] \tag{32}
\end{align*}
$$

Obtained dependence expresses turbine rotation opposing rotation moment. The turbine total rotation moment consists of an algebraic expression of positive (18) and negative (32) force moments
$2 M_{l}=2\left(M_{l 1}+M_{l 2}\right)=\frac{0,014}{3} r_{m} b H \rho U^{2}\left\{\left[\left(\frac{0,685 \sqrt{2} \pi}{2}-0,577\right) \times\right.\right.$
$\left.\left.\times \frac{3}{0,014}(1-\cos 2 \theta)\right]-1,24(1+\cos 2 \theta)-4,63 \chi_{m} \cos \theta-2,32 \chi_{m}^{2}\right\}$

The average value of operating the turbine rotational moment find by integration (33) from zero to $2 \pi$ and dividing by $2 \pi$ :
$M_{\text {turbine }}=\frac{1}{\pi} \int_{0}^{2 \pi} M_{l} d \theta=\frac{0,028}{3} r_{m} b H \rho \frac{U^{2}}{2}\left(177-2,72 \chi_{m}^{2}\right)$
Power and Coefficient of Using Wind Power by Two-bladed Turbine: Turbine output may be defined by multiplying turbine angle rotation velocity by force of moment
$N_{\text {turbine }}=\omega M_{\text {turbine }}$

Substituting dependence (34) in this equation and introducing $\chi_{m}=\frac{\omega r_{m}}{U}$, we obtain
$N_{\text {turbine }}=\frac{0,028}{3} b H \rho \frac{U^{3}}{2} \chi_{m}\left(177-2,72 \chi_{m}^{2}\right)$

Hence it is easy to determine the utilization of wind energy coefficient $\xi$, if (35) to divide by wind flow rate, going through the turbine swept area $F$
$N_{b}=F \rho \frac{U^{3}}{2}$


Fig. 4: Depending on the utilization of wind energy coefficient is $\xi$ for various types and design of wind turbines, depending upon turbine specific speed $\chi$.

Thus

$$
\begin{equation*}
\xi=\frac{N_{\text {turbine }}}{N_{b}}=\frac{0,028}{3 F} b H \chi_{m}\left(177-2,72 \chi_{m}^{2}\right) \tag{38}
\end{equation*}
$$

From the last formula we define value $x_{m}$ whereby a maximum useful wind energy coefficient can be reached $\xi_{m}$. For this purpose, let us equate first-order derivative $\xi$ by $x_{m}$ to zero
$\frac{d \xi}{d \chi_{m}}=177-8,16 \chi_{m}^{2}=0$.
Hence

$$
\begin{equation*}
\chi_{m}=4,66 \tag{39}
\end{equation*}
$$

Troposkino surface swept area $F$ may be determined from the known formula, applied for rotation bodies
$F=2 \pi \int_{-1}^{1} \bar{r}(\bar{z}) \sqrt{1+\left(\frac{d \bar{r}}{d \bar{z}}\right)^{2}} d \bar{z}$.

Substituting an expression for $\bar{r}(\bar{z})$ from the formula (2), let's define
$F=\frac{3 \pi}{8} H^{2} \int_{0}^{1}\left(1-\bar{z}^{2}\right) \sqrt{1+\frac{9}{4} \bar{z}^{2}} d \bar{z}$.

Let us introduce a new derivative $y=\frac{3}{2} \bar{z}$. Then
$F=\frac{\pi H^{2}}{4} \int_{0}^{3 / 2} \sqrt{1+y^{2}} d y-\frac{\pi H^{2}}{9} \int_{0}^{3 / 2} y^{2} \sqrt{1+y^{2}} d y$.
Let us again return to standards integrals $I_{1}$ and $I_{2}$ (28). Calculating these integrals, we will obtain
$F=0,337 \pi H^{2}$

Substituting expressions (39) and (40) to formula (38), we will determine
$\xi_{\text {max }}=0,337 \pi H^{2}$
Value $\xi_{\text {max }}$ and quantity $x_{m}$, whereby $\xi_{\max }$ is reached, are very close to already known values, provided in Fig. 4. Fig. 4 shows measured values of coefficient $\xi$ for various types and design of wind turbines, depending upon turbine specific speed $\chi=\frac{|\vec{\omega}| r_{0}}{|\vec{U}|}$, taken from [3]

## ACKNOWLEDGEMENTS

This work was supported by grant funding research programs and projects of the Committee of Science RK, Grant number GF, 2012.-2014.

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