

Stochastic Model of the Thermal Regime and Heat Consumption of Residential Buildings for Heating

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Abstract: Regulating of the thermal regime of indoor areas assumes, on the one side, calculation of combined influence of continuously changing thermal influences, on the other side, - controlling the parameters for compensating the disturbances. During standardization of the thermal regime, the stochastic character of the integrated influence factors for heat consumption in a building should not be taken into consideration. Within this framework, the authors analyze disturbing and regulating influences for the thermal regime and heat consumption in a building. For the measured characteristics of heat supply during the heating season, stochastic relations are investigated between heat consumption and disturbing influences. The authors also used the method of correlation analysis for establishing stochastic relations between variables. According to the results of studying this problem, the authors offer a stochastic model for calculating and evaluating the thermal regime and heat consumption in residential buildings.

Key words: Residential buildings · Thermal regime · Heat consumption

INTRODUCTION

Accurate calculation of heat consumption in heating systems for a particular object is a practically impossible task. Heat consumption in a building is conditioned by a large number of volatile parameters [1-5]:

- thermal-insulating parameters of barrier structures in the building;
- outdoor air temperature variations, wind velocity and direction;
- solar radiation;
- operating regime of heated rooms;
- thermal emissions from people and equipment;
- excessive heating surface of heating devices;
- capabilities of natural ventilation system etc.

The thermal regime in the building is conditioned by combined influence of continuously changing disturbing influences and controlling influences aimed at

compensating them [6-7]. However, during standardization, the characteristics of the micro-climate are given without their relations with each other, whereas they exert integrated influence [8].

MATERIALS AND METHODS

Correlated analysis methods were used for establishing stochastic relations between the variables: “difference between temperatures of indoor and outdoor air” - “heat consumption”; “heat carrier consumption” - “heat consumption”; “difference between temperatures of indoor and outdoor air” - “heat carrier consumption”. The relation between them was evaluated as high or as significant and direct.

The Main Part Disturbing and Regulating Influences: Disturbing influences can be divided into outdoors (meteorological) and indoor ones. Outdoor thermal influences include: changes of outdoor air temperature,

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wind velocity and direction, intensity of solar radiation, air humidity. Indoor thermal influences include household thermal emissions (from household electric and lighting devices; kitchen stoves; pipelines of hot water supply and directly from the consumed hot water; people in the room), influence of natural ventilation systems etc.

The regulating (correcting) influences aimed at stabilization of the thermal regime in the room within the set limits, or changing it over time according to a certain program, include the temperature and consumption of the heat carrier circulating in the heating system.

The main factor determining the operating mode of heating systems is the outdoor air temperature changes. The difference between temperatures of indoor and outdoor air, i.e., the temperature charge causing heat transfer through barrier structures in buildings, also changes. This transfer takes place by conductive heat exchange through barrier structures of the building; radiant and convective heat exchange on their surfaces. Besides, temperature difference causes the thermal flow for heating the infiltrating outdoor air.

By their dynamic parameters, the thermal losses (caused by outdoor air temperature changes) are divided into quick (through low-thermal-capacity barriers) and slow (through high-thermal-capacity barriers).

The investigation data show that the temperature and density of the thermal flow through the outer surface of the building largely depend on time and barriers with different thermal-technical characteristics depend on time in different ways. The lowest values of temperature of outdoor air and outer barrier surface correspond to the maximal values of the specific thermal flow. But, during certain intervals of time, a reverse thermal situation is possible, when, with all other factors unaltered, the layer with a lower thermal resistance has, on the contrary, a lower temperature, than the layer with a higher thermal resistance [9].

Examining stochastic relations between heat consumption of a building for heating and disturbing influences The necessity to use the mathematical approach while examining the thermal regime is conditioned by the nature of the events and processes related to it [10].

We need to use the methods of creating mathematical models of heating and ventilation systems with consideration of influences of outdoor disturbing influences recorded in a passive experiment. One of the important tasks is the analysis of dependencies between the investigated variables. The dependency between variables can be either functional or stochastic

(probabilistic). In order to evaluate relations between the investigated variables, when there is stochastic dependency between them, the co-variance and correlation indexes are used [11].

Linear coefficient of correlation r_{xy} :

$$r_{xy} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}, \quad (1)$$

where $\text{cov}(x, y)$ is co-variance of random values X and Y:

$$\text{cov}(x, y) = \overline{(x_i - \bar{x})(y_i - \bar{y})} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}); \quad (2)$$

here \bar{x}, \bar{y} are mean values; n is the sampling size; $[\sigma]_x, [\sigma]_y$ are standard (mean-root-square) deviations:

$$\sigma_x = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}; \quad (3)$$

$$\sigma_y = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2}. \quad (4)$$

The linear coefficient of correlation characterizes the correlation ratio degree, not of any one, but only the linear dependency. If the dependency between the events is non-linear, the linear coefficient of correlation has no meaning and, in order to measure the correlation ratios of relations, the so-called correlation relation is used.

The linear stochastic dependency of random values means that, if one random value grows, the other one tends to grow as well. This tendency can be close to the functional one, to some extent. If random values X and Y are tied in an accurate linear functional dependency $y = ax + b$, then $r_{xy} = \pm 1$. In the general case, when values X and Y are tied in a random functional dependency, the linear coefficient of correlation takes a value in the interval $-1 < r_{xy} < 1$, then the qualitative evaluation of the correlation ratio relations value X and Y can be estimated on the basis of Shaddock's scale.

The below-discussed data on the operating modes, consumptions of heat carrier and heat of the heating system of a residential building were obtained as a result of parameters recording, without intervention into operating modes of the object. As an object of investigation, a residential building in Saint Petersburg was selected. The heat consumption data taken from readings of meters and calculated characteristics of the heating system are presented in the article [12].

Investigations of heat consumption in the building for heating and ventilation showed the following dependencies:

- the thermal flow of the heating system depends on the difference of temperatures between the indoor and outdoor air;
- the thermal flow of the heating system depends on the consumption of heat carrier from the feeding pipeline of the thermal network;
- the consumption of the heat carrier depends on the difference of temperatures between the indoor and outdoor air.

In all these cases, inter-relations between the pairs of temporal ranges. In each of these pairs, one of the processes is dependent and the other one is independent.

Degree of relations between variables and the particular type of dependency between them when they both are random was determined by methods of correlation analysis.

There are stochastic relations between pairs of all the investigated parameters. The character of the detected relations is as follows:

- The relation “difference between temperatures of indoor and outdoor air” - “heat consumption” is high and direct ($r_{xy} = 0.791$), i.e., as the difference of temperatures grows, the heat consumption increases;
- The relation “heat carrier consumption” - “heat consumption” is high and direct ($r_{xy} = 0.881$), i.e., as the heat carrier consumption grows, the heat consumption increases;
- The relation “difference between temperatures of indoor and outdoor air” - “heat carrier consumption” is significant and direct ($r_{xy} = 0.697$), i.e., as the difference of temperatures grows, the heat carrier consumption from the heat supply network increases.

In the relation “difference between temperatures of indoor and outdoor air” - “heat consumption” both variables are random, i.e., each of them depends on the combination of uncontrollable factors. In its turn, the thermal flow of the heating system in a building depends not only on the difference between temperatures of indoor and outdoor air, but also on heat-insulating parameters of barrier structures, operation of the heating system, operating mode of a particular room, presence of heat emitting equipment, people etc. [13].

Heat Consumption in Buildings as a Random Process:
As the temperature regime in a building is a stationary process in time, all the values of indoor air temperature at

the end of every hour are the elements of a statistically uniform series. They can be combined into one sample. The thermal flow for heating in buildings can be treated similarly [14].

Let us study the thermal regime and heat consumption in a building as a random process, running in time [15]. A number of values of thermal flows for heating a building Q_1, Q_2, \dots, Q_n can be represented as a series of observed values conditioned by the dependency

$$Q_{tr} = M(Q) + \Delta Q_{tr} \quad (5)$$

where Q_{tr} is the measured value of the thermal flow for heating, W; $M(Q)$ is the mathematical expectation of the thermal flow, W; ΔQ_{tr} is the deviation of the thermal flow value from the mathematical expectation.

The mathematical expectation of the thermal flow for heating $M(Q)$ can be assumed as equal to the calculated value defined by the basic factors taken into consideration for calculations (heat-insulating parameters of barrier structures in a building, temperature time-schedule of the heat supply network, temperature of the outdoor air). Therefore, it is a determinate value [12].

The thermal balance of a residential building as a whole and each heated room in particular is found from the formula:

$$Q = Q_{bar} + Q_i - Q_{hh}, W \quad (6)$$

where Q_{bar} are the thermal losses through the barriers of the building (room), W; Q_i is the thermal flow for heating the outdoor air in the amount of infiltration or the sanitary norm, W; Q_{hh} is the total heat influx from the indoor heat sources (household heat emissions), W.

Heat losses through barrier structures in rooms [13, 16]:

$$Q_{bar} = F_{bar} (t_{ind} - t_{outd})(1 + \Sigma\beta)n/R, W \quad (7)$$

where F_{bar} is the calculated area of the barrier structures, m^2 ; $[\beta]$ are additional heat losses expressed in a share of the total losses; n is the coefficient assumed in dependence on the position of the outdoor surface in relation to the outdoor air; R is the resistance of the barrier structure to heat transfer, $(m^2 \cdot ^\circ C)/W$.

The thermal flow for heating the infiltrating outdoor air through barrier structures is calculated from the dependency

$$Q_i = 0.278L \times \sum G_i \times c \times (t_{ind} - t_{outd}) \times k, W \quad (8)$$

where G_i is the consumption of the air infiltrating through barrier structures, kg/h; c is the specific mass thermal capacity of the air, which is equal to 1.005 kJ/(kg·°C); t_{ind} , t_{outd} are the calculated temperatures of the air, correspondingly, indoors and outdoor during the heating season, °C; k is the coefficient for calculating the influence of the counter-running thermal flow in structures, $k = 0.7$ for joints of wall panels and three-fold split-type windows, $k = 0.8$ for split-type windows and balcony doors and $k = 1.0$ for single-pane windows, double-pane windows and balcony doors and unshielded openings.

The thermal flow for heating the infiltrating air in rooms of residential buildings with natural induced ventilation not compensated with heated influx air is assumed as equal to the largest of the values obtained by calculations on the dependencies (2.7) and (2.8):

$$Q_i = 0.278L_{nia} \rho \cdot c \cdot (t_{ind} - t_{outd}) \cdot k, W \quad (9)$$

where L_{nia} is the consumption of the discharged air, not compensated by heated influx air; m³/h; [ρ] is the air density in the room, kg/m³.

The thermal emissions Q_{hh} in residential buildings are conditionally considered as thermal emissions from household electric and lighting devices, kitchen stoves, pipelines of hot water supply and directly from the consumed hot water, people in the rooms.

We assume that the heat consumption statistical distribution deviation from the calculated value with the required confidential probability (P_{conf}) can be described by the normal regularity. The characteristics of the statistical distribution of heat consumption deviation are the selective mean deviation $\overline{\Delta Q}$ and mean square deviation S , which are the basis for compiling a graph of the theoretical density of the normal distribution probability; this makes it possible to visually represent the limits of the heat consumption deviations and their probability in the corresponding limits.

The analysis of the measurements results Q_{tr} and calculation of ΔQ_{tr} showed that the heat consumption deviations can be represented as continuous random values distributed by the regularity that is close to the normal one ($P_{conf} = 0.95$) (Fig. 1, 2) [12].

CONCLUSIONS

- The thermal regime and heat consumption of heating systems of residential buildings are inter-related by the regression scheme. The regression line is straight.
- The thermal regime and heat consumption in a building can be considered as random processes running in time. The thermal flow for indoor heating is represented as a sum of mathematical expectations of the thermal flow $M(Q)$ and deviation of the thermal flow value from the mathematical expectation ΔQ_i .

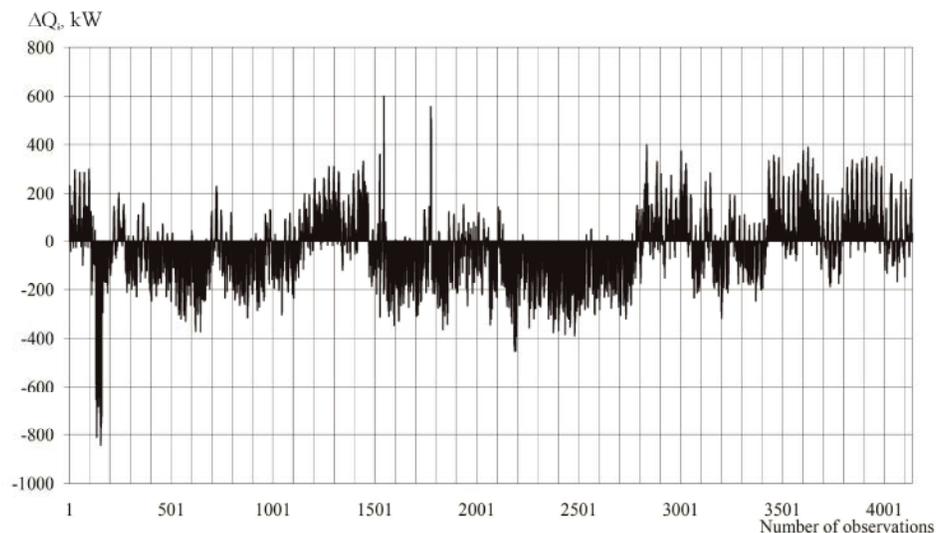


Fig. 1: Dependency of the ΔQ_i thermal flow deviation values from the mathematical expectation during the heating season

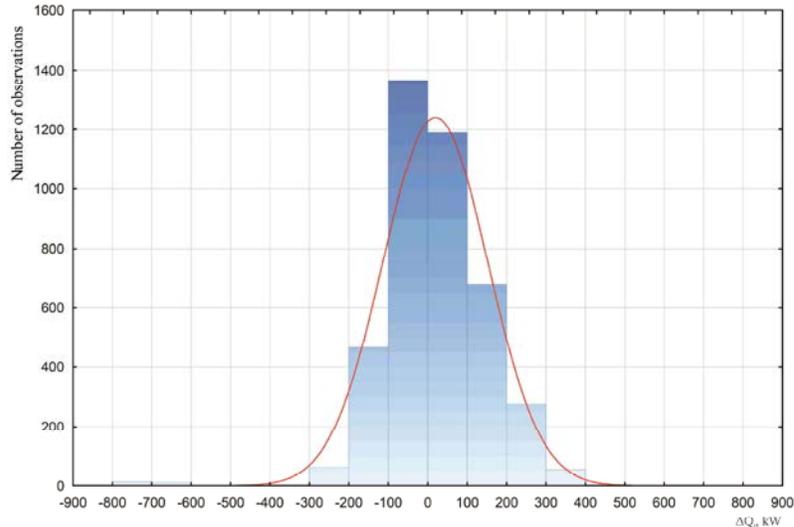


Fig. 2: Histogram of distribution of thermal flow deviation from the mathematical expectation values

- The mathematical expectation of the thermal flow for heating $M(Q)$ can be assumed as equal to the calculated value defined by the basic calculation factors; it is a determinate value. The random factors not considered in calculations introduce an error ΔQ ; due to this, $Q = M(Q) + \Delta Q$.
- The accepted model allows considering the heat consumption as a random value and using it in statistical methods of processing and analysis of the measurements results.

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