

Calculation of Parameters of Buildings in Seismic Insulation System with Non-Linear Characteristics

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Abstract: Seismic-protection measures make it possible to significantly decrease economical losses. Seismic shock absorption systems and seismic insulation of foundations and buildings can generally increase the reliability of structures. Calculations by Ja. M. Ajzenberg showed that relative horizontal seismic shifts of floors in seismically insulated buildings are significantly lower than in non-insulated buildings. Therefore, seismically insulated buildings are better protected from destruction during earthquakes. A method for calculating elastic oscillations of buildings in a seismic insulation system is used. Examples of calculations are presented so as to show the efficiency of the presented method and the necessity to consider these oscillations for high-rise buildings.

Key words: Seismic-insulating pier • Elastic oscillations • High-rise building • Generalized method of master coordinates • Horizontal absolute accelerations • Relative shifts

INTRODUCTION

Systems for seismic insulation (SSI), an idea that spawned early in the last century, got into wide use in construction starting from 1970-1980. At the moment, there are more than 100 patented structures for seismic insulation of buildings and facilities.

As a rule, a SSI consists of various combinations of seismic-insulating piers (SIP). A serious problem during design works for facilities on elastic piers is the difficulty to ensure their strength during significant mutual shifts between seismically insulated parts of the foundation. This was the reason for wide use of cinematic piers for seismic-insulating foundations. However, if no additional shock-absorbing elements are provided for, we can be sure that foundations on cinematic piers are not safe: during long-period earthquakes with the magnitude of more than 8, the building can collapse from the piers. It should be noted that traditional seismic-insulating appliances, including SIP, have a serious drawback: they separate the integral system “building-foundation” into parts, which weakens the system in general for the sake of

seismic insulation of its part. This gives rise to mutual shifts between the insulated and non-insulated parts. That is why shock-absorbers are installed to limit these movements and dissipate the energy of the seismic impact. The seismic insulation system should apply to the entire integral system, but not to one of its parts.

Problems and methods for creating various types of SIP are examined in a number of works and publications. The most prominent role in solving the problems of seismic insulation was played by G. McVerry [1], J. Kelly [2-5], R. Skinner [6-7], W. Robinson, [8], A. Martelli [9-10], M. Higashino, Sh. Okamoto, T. Saito [11], A. Chopra [12], G. Warburton [13]. Among Russian scientists, we can mention O.A. Savinov [14], Ja. M. Ajzenberg [15], A.M. Uzdin [16], Ju.D. Čerepinskij [17], A.M. Maslennikov [18].

MATERIALS AND METHODS

Usually, in order to investigate the dynamics of the system “protected building-seismic insulation”, we use the calculation scheme presented in Fig. 1. This scheme is

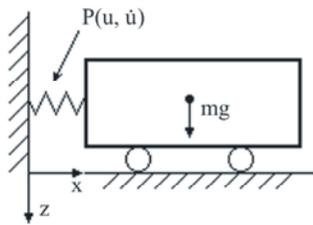


Fig. 1: A simple dynamic model of a seismically insulated building

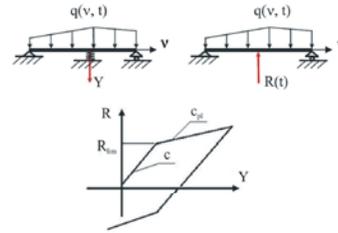


Fig. 3: Two-pier linear-elastic beam with a non-linear intermediary pier

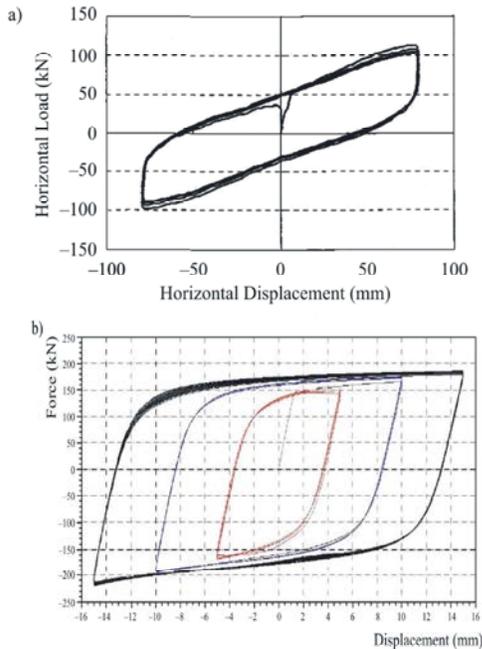


Fig. 2: Power characteristics: a - of rubber-metal pier of the SI series, b- of hysteresic steel shock-absorber made by "FIP Industriale"

based on the assumption that the protected object (PO) shifts relatively to the moving foundation (due to the action of the SSI) as a solid body.

The equation describing the movement of the PO according to Fig. 1 has the form

$$\ddot{u} + \alpha \cdot \dot{u} + f(u, \dot{u}) = -\ddot{x}(t) \quad (1)$$

where x is the coordinate describing the movement of the soil (foundation); $\ddot{x}(t)$ is the accelerogram; u is the coordinate describing the shift of the protected object relative to the foundation; α is the parameter of the internal structural shock-absorbing; $f(u, \dot{u}) = \frac{P(u, \dot{u})}{m}$, $P(u, \dot{u})$ is the power characteristic of SIP, m is the mass of the PO.

The power characteristic is conditioned by elastic and shock-absorbing properties of SIP elements. As a rule, this power characteristic is non-linear. As an example, Fig. 2 shows the power characteristic of rubber-metal SIP and plastic shock-absorber. Such shock-absorbers are often used by SIP so as to ensure the required level of seismic energy dissipation.

The model in Fig. 1 does not consider elastic properties of the PO. Elastic oscillations have a great effect onto the dynamics of tall seismically insulated buildings. If the power characteristic of the SIP is linear, the calculation of the "SSI-PO" system can be done by the method of normal coordinates. However, this method is not suitable for non-linear systems. For system with local non-linearities, a generalization was proposed for the method of master coordinates [19]. This generalized method can be used for calculation of seismically insulated buildings.

The Main Part: Investigation of dynamics of seismically insulated facilities. The physical idea of the proposed calculation method is that, according to the principle of disengagement, the reactions of non-linear elements are considered as known external forces in relation to the linear part of the initial system. Therefore, the initial non-linear system is substituted with a linear system, whose external load has unknown components. In order to calculate this linear system, we can use the method of master coordinates (the method for reduction to own forms). In order to estimate the unknown part of the external load, we form an equation describing the dependency of reactions of non-linear elements from master coordinates of the linear part of the system. As a result, we obtain an integral endless system of regular non-linear differential equations. Its solution is, essentially the solution to the initial task.

The proposed method to compile the required equations can be referred to as a generalized method of master coordinates (GMMC). Let us see how an equation

system for GMMC is formed, on the example of two-pier linear-elastic beam with a non-linear intermediary pier (Fig. 3).

We seek the solution in the form:

$$f(v, t) = \sum_{i=1}^{\infty} x_i(t) \varphi_i(v),$$

where $\varphi_i(v)$ are own forms of the beam without the elastic pier.

Equations for estimating master coordinates are written in the following way [19]:

$$m_i \ddot{x}_i + c_i \dot{x}_i = -\alpha_i R(t) + F_i(t). \tag{2}$$

The initial conditions $x_i(0) = x_{i0}, \dot{x}_i(0) = \dot{x}_{i0}, i = 1, 2, \dots, \infty,$

$$m_i = \int_0^l \mu \varphi_i^2(v) dv; c_i = m_i p_i^2;$$

$$F_i = \int_0^l q(v, t) \varphi_i(v) dv; \alpha_i = F_i(v^*),$$

[μ] is the running mass of the beam, p_i is the i -th own frequency of the two-pier beam, $R(t), v^*$ is the reaction and the coordinate of the intermediary pier.

The equations connecting $R(t)$ and the master coordinates x_i , have the form

$$R(t) = g(y, \dot{y}), \tag{3a}$$

$$y = \sum_{i=1}^{\infty} \alpha_i x_i(t), \tag{3b}$$

where y is the vertical shift of the point [nu]*.

The system of equations (2), (3) provides the solution.

In the general case, the system with local non-linearities is divided into m of linear systems connected by q non-linear elements. Therefore, the system of equations GMMC has the form

$$m_{ir} \ddot{x}_{ir} + c_{ir} \dot{x}_{ir} = -\sum_{k=1}^q \alpha_{irk} R_k + F_{ir},$$

$$x_{ir}(0) = x_{ir0}, \dot{x}_{ir}(0) = \dot{x}_{ir0};$$

$$i = 1, 2, \dots, n, \dots; r = 1, 2, \dots, m.$$

$$R_k = G_k(y_1, y_2, \dots, y_q; \dot{y}_1, \dots, \dot{y}_q, t),$$

$$y_k = \sum_{\substack{i=1, 2, \dots, \infty \\ r=1, 2, \dots, m}} \alpha_{irk} x_{ir}. \tag{4}$$

With continuous numbering of own forms, the equation (4) can be recorded in the matrix form:

$$M\ddot{X} + CX = -AR + F, \tag{5a}$$

$$X(0) = X_0, \dot{X}(0) = \dot{X}_0;$$

$$R = G(Y, \dot{Y}, t), \tag{5b}$$

$$Y = A^T X \tag{5c}$$

From (5) we obtain

$$M\ddot{X} + CX = -AG[A^T X, (A^T X)'_t, t] + F. \tag{6}$$

Practically, we solve the equations (6) by transferring to the reduced (final) system of equations, i.e. considering the terminal number of own forms $i = 1, 2, \dots, n$.

Own frequencies and forms of the linear parts of the system can be estimated with the help of the terminal-element model. In some cases, numerical-analytical methods can also be used [20-21].

The advantages of such approach are in that, basing on physical considerations, we can select just a few own forms of the linear part of the system for calculation. This limits the dimension of the task to reasonable limits. Moreover, the solution results are easily interpreted physically.

The presented algorithm is implemented in the software set "MicroFe 2005" intended for calculating strength, resistance and oscillations of construction and machinery structures.

With the help of this software set, the influence of elastic oscillations was investigated for the results of dynamic calculations of high-rise buildings; 15-, 20-, 25-level buildings on SSI have been calculated; 2 variants of SSI for such buildings were considered: SSI of rubber-metal piers (RMP) and SSI of pendulum piers (PPPSA) with plastic shock-absorbers [22]. Calculations were done for seismic impacts of magnitude 9. For calculations, impacts with different frequency compositions were used. The results of calculations are presented in Tables 1-3.

Table 1: Results of calculations for a 15-level building

Impact No	RMP		PPPSA	
	Absolute acceleration, m/s ² , top/mid/low level	Relative shift, m top/mid/low level	Absolute acceleration, m/s ² top/mid/low level	Relative shift, m top/mid/low level
1	2.5/2.2/2	0.27/0.25/0.22	2/1.85/1.76	0.14/0.135/0.125
2	2.4/2.1/2	0.27/0.25/0.24	1.8/1.7/1.6	0.12/0.11/0.11
3	0.83/0.79/0.75	0.08/0.07/0.065	1.61/1.6/1.58	0.085/0.08/0.08
4	1.9/1.8/1.9	0.21/0.20/0.18	2.1/1.8/1.8	0.15/0.145/0.145
5	2.1/2/1.9	0.21/0.21/0.20	3/2.7/2.6	0.18/0.17/0.17
6	2.2/2/1.8	0.25/0.24/0.23	3/2.7/2.7	0.18/0.17/0.17
7	1.8/1.5/1.2	0.16/0.159/0.15	2.2/1.8/1.8	0.16/0.15/0.148
8	1.7/1.58/1.4	0.17/0.16/0.16	1.7/1.6/1.55	0.09/0.086/0.084
9	1.2/1.1/1	0.11/0.11/0.10	1.6/1.5/1.5	0.07/0.065/0.06
10	1.6/1.45/1.3	0.163/0.16/0.15	1.45/1.35/1.3	0.058
11	0.8/0.75/0.7	0.07/0.063/0.06	1.5/1.4/1.35	0.06
12	1.6/1.48/1.4	0.16/0.15/0.15	2.2/2.1/2	0.15
Average value for the top floor	1.72	0.18	2.02	0.12

Table 2: Results of calculations for a 20-level building

Impact No	RMP		PPPSA	
	Absolute acceleration, m/s ² , top/mid/low level	Relative shift, m top/mid/low level	Absolute acceleration, m/s ² top/mid/low level	Relative shift, m top/mid/low level
1	2.8/2.1/1.8	0.30/0.26/0.23	2.3/1.8/1.8	0.18/0.16/0.14
2	1.4/1.1/0.9	0.158/0.13/0.11	1.6/1.1/1.1	0.04/0.032/0.03
3	1.2/0.75/0.85	0.085/0.07/0.06	2/1.2/1.2	0.065/0.058/0.05
4	2.2/1.8/1.6	0.28/0.23/0.20	2.4/1.9/1.8	0.20/0.17/0.16
5	2.5/2/1.8	0.30/0.25/0.20	2/1.4/1.4	0.12/0.09/0.08
6	2.3/1.2/1	0.18/0.16/0.13	3.2/2/1.8	0.18/0.12/0.11
7	2.5/1.6/1.4	0.25/0.20/0.16	2.5/1.6/1.5	0.16/0.15/0.13
8	3/2.4/2	0.38/0.30/0.28	1.6/1.2/1.2	0.08/0.07/0.065
9	1.3/1/0.90	0.14/0.12/0.10	2/1.6/1.6	0.17/0.14/0.14
10	1.6/1.3/1.2	0.20/0.16/0.15	2.1/1.6/1.6	0.16/0.13/0.12
11	1.1/0.6/0.50	0.06/0.055/0.045	1.8/1.3/1.2	0.07/0.06/0.059
12	1.7/1/1	0.16/0.13/0.11	2.2/1.6/1.5	0.13/0.11/1.10
MAA assessment for the top floor	1.97	0.21	2.14	0.13

Table 3: Results of calculations for a 25-level building

Impact No	RMP		PPPSA	
	Absolute acceleration, m/s ² , top/mid/low level	Relative shift, m top/mid/low level	Absolute acceleration, m/s ² top/mid/low level	Relative shift, m top/mid/low level
1	3.8/2/1.7	0.5/0.3/0.2	2.8/1.6/1.65	0.28/0.21
2	2/0.6/1.4	0.06/0.04/0.03	2/1/1.2	0.06/0.04/0.03
3	2/0.6/1.6	0.10/0.08/0.06	2.4/1.1/1.5	0.08/0.055/0.043
4	2.3/1.6/1.6	0.30/0.20/0.15	2.8/1.6/1.8	0.20/0.17/0.12
5	2.2/1.5/1.6	0.30/0.25/0.19	2.3/1.3/1.8	0.20/0.14/0.10
6	3.2/0.8/1.3	0.20/0.14/0.10	3.3/1.8/1.6	0.16/0.14/0.13
7	3/1/1.3	0.25/0.18/0.13	3/1.6/1.6	0.19/0.16/0.16
8	2.5/1.6/1.8	0.38/0.23/0.17	2.9/1.3/1.8	0.20/0.14/0.11
9	1.6/0.7/1.3	0.16/0.10/0.07	2/1.2/1.7	0.10/0.05/0.06
10	1.5/0.8/1.3	0.16/0.10/0.08	2/1/1.3	0.07/0.05/0.04
11	1.6/0.7/1.5	0.08/0.05/0.037	2.2/1/1.2	0.04/0.03/0.025
12	2.3/1.6/2	0.30/0.20/0.15	2.3/1.8/2.1	0.23/0.20/0.15
MAA assessment for the top floor	2.33	0.2325	2.5	0.150833

These tables show horizontal absolute accelerations and relative shifts (relative to the ground) of 3 levels of the buildings: the upper, the middle and the lower; a mathematical anticipation assessment is presented (MAA) for the top floor.

CONCLUSIONS

The present article presents the methods for calculating elastic oscillations of seismically insulated buildings. All the results were obtained using thirty forms of free oscillations (of master coordinates). However, in this case, using for example, ten forms of oscillation, the results practically remain the same.

These examples show that, as tall buildings on SSI are calculated, elastic oscillations must be taken into consideration as they significantly change the results of calculations.

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