World Applied Sciences Journal 22 (Special Issue on Techniques and Technologies): 82-86, 2013
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2013.22.tt.22144

# Finding of Service Centers in Gis Described by Second Kind Fuzzy Graphs

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**Abstract:** In this paper the problem of optimal location of service centers is considered by minimax criterion. It is supposed that the information received from GIS is presented like second kind fuzzy graphs. Method of finding of optimal location *k*-centers is suggested. Basis of this method is building procedure of reachability matrix of second kind fuzzy graph in terms of reachability matrix of first kind fuzzy graph. This method allows to solve not only problem of finding of optimal service centers location in transport network but also finding of optimal location *k*-centers with the best vitality degree and selecting of service center numbers. The example of finding optimum allocation of centers in transport networks with the largest degree is considered as well.

**Key words:** Minimax criterion · GIS · Fuzzy graph · Service centers

## INTRODUCTION

Location of facilities on the region map and in transport network is one of the optimal problems. Some variants of these problems are topical for most practical application. For instance, optimal locations of construction units, repair objects, vantage-points have to be found in concordance with some kind of criterion or group of criterions.

Such location problems can be solved with using of fuzzy logic and fuzzy set theory. In this case basic data are set by fuzzy numbers or linguistic variables.

**Fuzzy Data Using to Solve the Location Problems:** The large-scale increasing and versatile introduction of the geographical information system (GIS) is substantially connected with the necessity of perfection of the information systems providing decision-making. The GIS are applied practically in all spheres of human activity. Geographical information technologies have now reached an unprecedented level, offering a wide range of powerful functions such as information retrieval and display, analytical tools and decision support [1, 2]. Unfortunately, geographical data are often analyzed and communicated amid a largely non-negligible uncertainty. Uncertainty exists in the whole process from the geographical abstraction, data acquisition and geo-processing to the usage [3, 4]. One of the tasks solved the GIS is the task of the centers allocation [5]. The search of the optimal placing of hospitals, police stations, fire brigades, important enterprises and services on some sites of a considered territory is reduced to this task. In some cases the criterion of optimality can consist in the minimization of the journey time (or in the minimization of distances) from the service centre to the most remote service station. In other cases the criterion of optimality consists in the choice of such an allocation of the centers that the route from them to any other place of service is passed on the best way by some criterion. In other words the problem is the optimization of "the worst variant» [6]. However, the information represented in the GIS, has an approximate value or insufficiently authentic [7].

Fuzziness concerning problems of servicing centers location can be cause: Information inaccuracy; Parameter inaccuracy; Inaccuracy happens in decision-making process; Inaccuracy is called forth by environmental influence.

Inaccuracy caused by the first reason can consist in imprecise measurement of distance between serving facilities. For example, in case of distance measurement which equal in several kilometer, the measuring accuracy equaling in several centimeters or meters may appear. Further if facilities have complicated forms its parameters are measured difficultly. When measuring distance between facilities of complex form, it is necessary to determine two points among a thousand and only thereafter to carry out the measurement.

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Impossibility of parameter measuring gets connected with the evidence that some characteristics are qualitative and haven't numerical values. For example, quality of a section of a route can't be measured but it will estimate like "good", "bad", "the worst" and etc.

Parameter inaccuracy brought out in characteristic features of objects happens in the event that objects changes its parameters itself (for example, sizes of water body change naturally) or its borders are indistinct (for example, boundaries of forests or meadows).

Inaccuracy of decision-making process happens in cases when data about targets and some parameters aren't determinate exactly or it allows variations in the certain range. Then model of problem described by fuzzy theory will be more appropriate instead of using traditional mathematics. It will lead to more adequate describing of reality and permits to find more suitable decision.

Inaccuracy called forth by environmental influence consists in impact of unforeseen factors upon parameters of objects or its interrelations that changes their values. For instance, passage time between two objects depends on numbers of traffic lights and its condition at the passing moment, weather conditions (rate of motion under a rain, a snow and in a fog is less than in the clear weather) and quantities of vehicles on the road and etc.

While considering location problems in imprecise conditions it is said commonly about inaccuracy of parameters measurement of object because of their complex structure and about impact of external factors on parameters taking into account for optimization [8].

We consider that a certain way system has $n$ stations. There are $k$ service centres, which may be placed into these stations. Each centre can serve several stations. The degree of a service station by a centre depends on a cyclic route which connects them. It is necessary for the given number of centers to define the places of their best allocation. In other words, it is necessary to define the places of k centers into n stations so that the «control» of all territory (all stations) is carried out with the greatest possible degree of service.

**Location Options of Service Centers in Second Kind Fuzzy Graph:** For choosing of the best location of service centers we can use the notion of conjunctive strength and the reachability degree of vertex. In this case we will compare all of the movement paths among themselves and find routes with a maximal reachability degree. It can be carried out by using of reachability matrix of second kind fuzzy graph.

Let $\tilde{G}=(\tilde{X},\tilde{U})$ is a second kind fuzzy graph. Here a set $\tilde{X}=\{<\mu_X(x)/x>\}$, $x\in\tilde{X}$ is the fuzzy set of vertices, defined on set $X$, $|X|=n$ and $\tilde{U}=\{<\mu_U(x_i,x_j)/(x_i,x_j)>\}$, $x_i,x_j \in X$ is fuzzy set of directed edges. Here $\mu_x(x)\in[0,1]$ - membership function for vertex $x$, $\mu_U(x_i,x_j)\in[0,1]$ - membership function for edge $(x_i,x_j)$.

A path of second kind fuzzy graph $\tilde{l}(x_i,x_j)$ is called a directed sequence of fuzzy edges from vertex $x_i$ to vertex $x_j$, in which the final vertex of any edge is the first vertex of the following edge [9, 10].

A conjunctive strength of second kind fuzzy graph is defined by expression:

$$\mu_{\tilde{l}}(x_i,x_j)=\underset{<x_k,x_t>\in\tilde{l}(x_i,x_j)}{\&}\mu_U(x_k,x_t)\,\&\,\underset{\substack{x_t\neq x_i\\ x_i\neq x_j}}{\mu_X}(x_t)$$

Let $\tilde{L}(x_i,x_j)$ is a family of second kind fuzzy graph paths from vertex $x_i$ to vertex $x_j$. Then the value

$$\tau(x_i,x_j)=\max_{\tilde{l}\in\tilde{L}}\{\mu_{\tilde{l}}(x_i,x_j)\}$$

defines the reachability degree of vertex $x_j$ from vertex $x_i$.

For location of service centers in second kind fuzzy graph we should consider finding problem of "optimal" situation in the sense that all (residual) vertices are serviced with the greatest vitality degree [11].

Let $k$ is service centers ($k<n$), placed in the vertices of subset $Y$, $|Y|=k$, $Y\subset X$ and $\tau(x_i,x_j)$ is reachability degree of vertex $x_j$ from vertex $x_i$.

Value

$$V_{\tilde{G}}(Y)=\underset{\forall x_j\in X\setminus Y}{\&}\left(\underset{\forall x_i\in Y}{\vee}\tau(x_i,x_j)\,\&\,\tau(x_j,x_i)\right)$$

is vitality degree of second kind fuzzy graph $\tilde{G}$ which serviced by $k$-centers from vertex set $Y$.

Value $V_{\tilde{G}}(Y)$ determines minimax strong connectivity value between each vertex from set $X/Y$ and one center from set $Y$.

Fuzzy set

$$\tilde{V}_{\tilde{G}}=\{<V_{\tilde{G}}(1)/1>,<V_{\tilde{G}}(2)/2>,\ldots,<V_{\tilde{G}}(n)/n>\}$$

defined on vertex set $X$, is called fuzzy set of vitality of graph $\tilde{G}=(X,\tilde{U})$. Fuzzy set of vitality $\tilde{V}_{\tilde{G}}$ determines the greatest vitality degrees of graph $\tilde{G}$ in the event that it is serviced by 1, 2,….., $n$ centers. Here, $\tilde{V}_{\tilde{G}}(n)=1$.

Values $\tilde{v}_{\tilde{G}}(k)$ $(1 \le k \le n)$ signify we can place $k$-centers in the graph $\tilde{G}$ so that there is a path from at least one center to any vertex of graph $\tilde{G}$ and back. Conjunctive strength of the graph will be not less than $\tilde{v}_{\tilde{G}}(k)$.

In the paper [12] was considered method of finding of minimal sets of service centers. These sets have the greatest vitality degree in transport network described first kind fuzzy graph.

Let $Y$ is vertices' subset of first kind fuzzy graph $\tilde{G} = (X, \tilde{U})$ in which service centers is located and vitality degree equals $V$. Therefore one of the two conditions for any vertex $x_i \in X$ can be satisfied:

- Vertex $x_i$ belongs to set $Y$;
- There is vertex $x_i$ that belongs to set $Y$ and inequalities $\tau(x_i, x_j) \ge V$ and $\tau(x_j, x_i) \ge V$ are met.

Using the quantifier form of notation we can get the truth of the following expression [13-15]:

$$(\forall x_i \in X)[x_i \in Y \vee (\exists x_j)(x_j \in Y \,\&\, \tau(x_i, x_j) \ge V \,\&\, \tau(x_j, x_i) \ge V)]$$

Each vertex $x_i \in X$ can be set in correspondence the logical variable $p_i$. This logical variable possesses the value 1 in case of $x_i \in Y$ and 0 if not. Expression $\tau(x_i, x_j) \ge V$ can be set in correspondence the fuzzy variable $\xi_{ij} = \tau(x_i, x_j)$.

So we can get the truth of logical expression:

$$\Phi_V = \underset{i=\overline{1,n}}{\&}(p_i \vee \underset{j=\overline{1,n}}{\vee}(p_j \,\&\, \xi_{ij} \,\&\, \xi_{ij}))$$

Let $\xi_{ii} = 1$ and equality $p_i \vee \underset{j}{\vee} p_i \,\&\, \xi_{ij} \,\&\, \xi_{ji} = \underset{j}{\vee} p_j \,\&\, \xi_{ij} \,\&\, \xi_{ji}$ holds for any $x_i$ then:

$$\Phi_V = \underset{i=\overline{1,n}}{\&} \underset{j=\overline{1,n}}{\vee}(\xi_{ij} \,\&\, \xi_{ji} \,\&\, p_j) \tag{1}$$

Remove brackets in expression (1) and collect using rules of fuzzy capture:

$$\begin{gathered} a \vee a\&b = a,\ a\&b \vee a\&\bar{b} = a,\ \xi' \,\&\, a \vee \xi'' \,\& \\ a \,\&\, b = \xi' \,\&\, a, \text{ if } \xi' \ge \xi'' \end{gathered} \tag{2}$$

Here, $a, b \in \{0,1\}$ and $\xi', \xi'' \in [0,1]$.

Consequently expression (1) will be represented as:

$$\Phi_V = \underset{i=\overline{1,l}}{\vee}(p_{1_i} \,\&\, p_{2_i} \,\&\, \ldots \,\&\, p_{k_i} \,\&\, V_i) \tag{3}$$

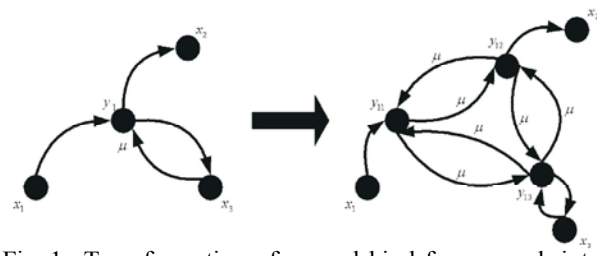

Fig. 1: Transformation of second kind fuzzy graph into first kind fuzzy graph in the case $t$=3

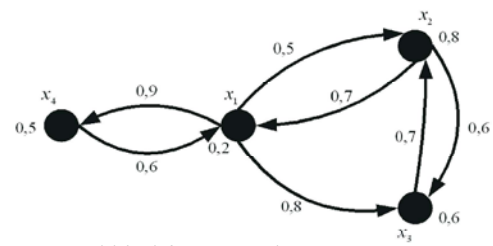


Fig. 2: Second kind fuzzy graph $\tilde{G}$

**Property:** If further simplification in the expression (3) based on rules (2) is not possible then totality of all vertices, conforming to variables, for each disjunctive term $i$ gives vertices subset $Y \subseteq X$ with vitality degree $V_i$ of fuzzy graph $\tilde{G} = (X, \tilde{U})$. Here subset $Y$ is minimal in the sense of its any subset doesn't have this property.

Considering method works with reachability vertex matrix of initial fuzzy graph. It doesn't matter what kind of initial fuzzy graph is taken. So in order to apply the method in case of second kind fuzzy graph we should transform this graph $\tilde{G}$ into first kind fuzzy graph $\tilde{G}'$ like that:

- Let $y_1 \in X$ - vertex of initial second kind fuzzy graph $\tilde{G}$ that is adjacent for $t$ vertices and has a vitality degree $\mu$. Represent vertex $x$ as directed complete $t$-subgraph of first kind with vitality degree of "inside" edges equal $\mu$.
- Connect everyone of the initial "outside" $t$ vertices with one of the vertices of received subgraph.

Example of transformation is presented in Fig. 1:

First kind fuzzy graph $\tilde{G}'$ comes out of this transformation. Meanwhile, transition from graph $\tilde{G}$ to graph $\tilde{G}'$ is biunique. Built reachability matrix $R_C^{(I)}$ of received first kind fuzzy graph $\tilde{G}'$ and pass from it to reachability matrix $R_C^{(I)}$ of initial second kind fuzzy graph $\tilde{G}$. Apply the foregoing method to the matrix $R_C^{(I)}$.

Given procedure is considered by example.

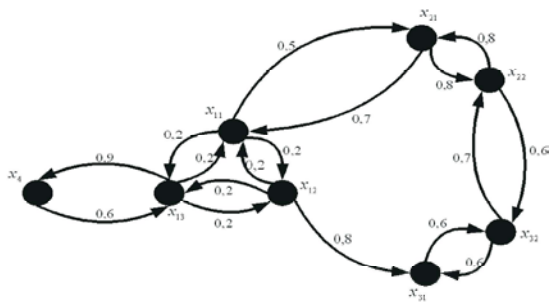


Fig. 3: Complementary first kind fuzzy graph $\tilde{G}'$

**Example:** Let's consider an example of finding of service centers in second kind fuzzy graph $\tilde{G}$ given in Fig.2.

Here vertices of graph are objects of transport network but edges are paths that connect the objects.

Let's find centers of graph supposing that it is located in graph vertices. In other words service centers should be located in objects of transport network.

The adjacent matrix of fuzzy graph $\tilde{G}$ is presented as:

$$R_X = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{vmatrix} 0,2 & 0,5 & 0,8 & 0,9 \\ 0,7 & 0,8 & 0,6 & 0 \\ 0 & 0,7 & 0,6 & 0 \\ 0,6 & 0 & 0 & 0,5 \end{vmatrix} \end{array}$$

For finding of reachability matrix $R_?^{(II)}$ of second kind fuzzy graph $\tilde{G}$ we need built new complementary first kind fuzzy graph $\tilde{G}'$. This graph is presented in Fig.3:

The reachability matrix $R_?^{(II)}$ of fuzzy graph $\tilde{G}'$ can be presented: $R_?^{(1)} = \bigcup_{i=\overline{0,n-1}} R_X^i$. Here, $R_X^0$ - diagonal unitary matrix, $R_X^i$ - $i$ degree of adjacent matrix. Adjacent matrix of first kind fuzzy graph $\tilde{G}'$ takes on form:

Find degree of matrix $R_X^2 = R_X^1 \times R_X^1$, where matrix elements $R_X^2 = \left\| r_{ij}^{(2)} \right\|, i, j = \overline{1,n}$ are determined as: $r_{ij}^{(2)} = \bigvee_{k=\overline{1,n}} r_{ik}^{(1)} \,\&\, r_{kj}^{(1)}$, where $r_{ik}^{(1)}$ and $r_{kj}^{(1)}$ are elements of matrix $R_X^1$:

Similarly find: $R_X^i = R_X^{i-1} \times R_X^1$, for index $i$=3,4,5,6,7.

Diagonal unitary matrix $R_X^0$ for our case equals:

Let's join the matrixes and find matrix $R_?^{(I)} = \bigcup_{i=\overline{0,7}} R_X^i$:

Each marked block of complementary matrix conforms to vertex of initial second kind fuzzy graph. So the greatest values are chosen in these blocks. And reachability matrix $R_?^{(II)}$ of second kind fuzzy graph $\tilde{G}$, given in Fig. 2., will be defined as:

$$R_C^{(II)} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{vmatrix} 1 & 0,6 & 0,8 & 0,9 \\ 0,7 & 1 & 0,6 & 0,2 \\ 0,7 & 0,7 & 1 & 0,2 \\ 0,6 & 0,2 & 0,2 & 1 \end{vmatrix} \end{array}$$

Expression (1) takes on form:

$$\Phi_V = (1p_1 \vee 0,6p_2 \vee 0,7p_3 \vee 0,6p_4) \,\&\, (0,6p_1 \vee 1p_2 \vee 0,6p_3 \vee 0,2p_4) \,\&\, (0,7p_1 \vee 0,6p_2 \vee 1p_3 \vee 0,2p_4) \,\&\, (0,6p_1 \vee 0,2p_2 \vee 0,2p_3 \vee 1p_4) =$$

$$= 1p_1p_2p_3p_4 \vee 0,7p_2p_3p_4 \vee 0,6p_2p_4 \vee 0,6p_3p_4 \vee 0,6p_1 \vee 0,2p_2 \vee 0,2p_3 \vee 0,2p_4.$$

Therefore second kind fuzzy graph $\tilde{G}$, given in Fig.2., has one maximal subset of vertices $Y_1 = \{x_1, x_2, x_3, x_4\}$ (i.e. all graph vertices). If service centers are located in these vertices vitality degree of graph will be equal 1. Also graph has one maximal subset of three vertices: $Y_2 = \{x_2, x_3, x_4\}$. In this case location of service centers in these vertices leads to vitality degree of graph is equal 0,7. Graph has two maximal subset $Y_3 = \{x_2, x_4\}$, $Y_4 = \{x_3, x_4\}$ and its vitality degree will be equal 0,6 in case of service centers location exactly in these vertices. And finally, graph has one maximal subset $Y_5 = \{x_1\}$, so vitality degree of graph will be equal 0,6 by location of service center in vertex $x_1$.

Fuzzy set of vitality of given graph will be of the form:

$$\tilde{V}_{\tilde{G}} = \{<0,6/1>, <0,6/2>, <0,7/3>, <1/4>\}.$$

We can notice that there is no point in locating two service centers in one graph because the greatest vitality degree in this case will coincide with vitality reached by one center.

## CONCLUSION

After realization of transition to characteristics of interrelation between objects and to properties of objects

itself that represented as membership functions we can begin to solve the problem of the "best" location of service centers. Then basic definitions of strong connectivity and fuzzy graph vitality should be used. The present approach of finding of vitality fuzzy set allows us to solve not only problem of *k*-centers optimal location with maximal vitality degree but also problem of selecting of *k*-service center numbers.

## ACKNOWLEDGEMENTS

This work has been supported by the Russian Foundation for Basic Research, Project, 12-01-00032.

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