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A Class of Steepest Descent Method with Fixed Range of Step Size

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Abstract: Step size played an important role in steepest descent method. In solving non-convex optimization problem, the coriolis phenomena will occurred and affect the convergence, even by using the established step size procedure. To avoid the coriolis phenomena, an addition range of step size has been fixed into the steepest descent algorithm. The results showed that the modified algorithm guaranteed the convergence of the steepest descent method to the solution.

Key words: Steepest Descent Method · Coriolis Phenomena · Non-Convex Optimization

INTRODUCTION

The studies of solving non-convex optimization problems are more on the determination of its global solution to the multi-modal objective functions. Early studies of the global optimization technique of solving multi-modal functions have been reported in two volumes of book named *"towards global optimization"* in year 1975 [1] and 1978 [2]. We have found that there are several well-known and effective techniques, such as multi-start method (in probabilistic approaches) and filled function method (in deterministic approaches) which have employed the local deterministic optimization technique in their local search phase in order to solve the non-convex optimization problems [3]. Therefore, the effectiveness of the local optimization technique to determine the local solution of the region of attraction played an important role in solving the non-convex optimization problems.

The local deterministic optimization techniques have been proved as effective techniques for solving convex optimization problems. Among the established local deterministic optimization techniques, steepest descent method is one of the well known fundamental, effective and low cost approach [4]. However, the step-size selection of the steepest descent method always plays the important role on its approximate convergent. Therefore, several step-size selection procedures on this method have been proposed in the previous studies. One of the well-known procedure proposed by Barzilai and Borwein [5] have been proved to be R-superlinearly convergent for convex quadratic in two-dimension space.

In year 2006, Goh and Ismail [6] had proposed a Newton-like exact line search as an alternative procedure to determine the step size for steepest descent method. In another paper published by both author [7] have shown that the Newton-like exact line search can give better step size and more efficient in solving non-convex optimization problems. Beside that, they also showed the failure of BB methods in the determination of the local solution for some selected non-convex problems which been tested in the same paper.

However, the attempt to use the initial point to test the effectiveness of the steepest descent method using Newton-like exact line search, often fail to obtain a small non-negative step size. Therefore, an *coriolis* phenomena which been mentioned by Vrahatis [8] will happen and affect the method to be fail in converge to the current local minimizer.

In this paper, it is not being discussed issues to obtain the global solution for the non-convex optimization problems. However, more concern were on how the *coriolis* phenomena incidence and the cause of failure for the steepest descent method which using the Newton-like exact line search as step-size selection procedure converge to the current local minimum. The modification of the Newton-like exact line search procedure was

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attempted to avoid the *coriolis* phenomena by fitting in an additional fixed range of step size. We have found that the steepest descent method by using the improved procedure can successfully converge to current local minimum point.

This paper is organized as follows. In Section 2, elaborations on the Newton-like exact line search procedure were given to determine the step size for steepest descent method. In Section 3, the *coriolis* phenomena will be shown by using Newton-line exact line search as the step-size selection procedure. The improvement has been done on Newton-like exact line search procedure to avoid the *coriolis* phenomena to happen and the improved procedure was shown in Section 4. Section 5 contained numerical results which reflect the effectiveness of the improved algorithm. The conclusion which ends this paper is being discussed in Section 6.

Steepest Descent Method: One of the well known basic and fundamental method to determine the local solution of convex optimization is the steepest descent method which is known as gradient, saddle-point or Cauchy's method. The steepest descent method is the most important and effective procedures for minimization of real-valued functions defined on R^n. This method is designed by Cauchy in 1847 [9]. The iterations are made according to the following equation:

$$x_{k+1} = x_k + \lambda_k d_k \tag{2.1}$$

Which the step size λ_k is obtained using exact line search

$$\lambda_k = \underset{\lambda>0}{\operatorname{argmin}} f(x_k + \lambda d_k) \tag{2.2}$$

$$\lambda_k = \min_{\lambda>0}\{\lambda \mid \nabla f(x_k + \lambda d_k) = 0\} \tag{2.3}$$

respectively where by the search direction is $d_k = -\nabla f(x_k)$ [9, 10]. Nevertheless, to solve some complicated (non-convex) optimization problems, it is difficult to compute the step size λ_k by using (1.2) and (1.3) in practical computational. At times it is impossible to compute it [6, 11, 12].

Therefore, several inexact line search procedures have been introduced, such as Armijo condition [13], Goldstein condition [14] or Wolfe condition [15]. It is easy to show that the steepest descent method with those condition is always convergent and theoretically the method will only terminate after a stationary point is found [16, 17].

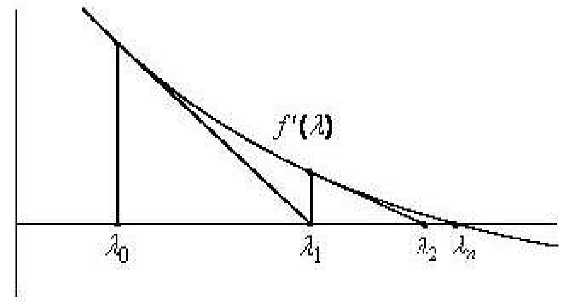

Fig. 2.1: Newton-Raphson's idea

However, even though several inexact line search were proposed and the (2.2) and (2.3) were difficult to use in computing the step size, previous studies on line searches found that only the exact line search gave the greatest possible reduction to the objective function along the search direction [4, 18]. Therefore, Goh and Ismail [6] have used the approximation of Newton-Raphson's idea (Figure 2.1) to obtain the exact step size λ_k and proposed a Newton-like exact line search procedure as show in Algorithm 2.1.

Algorithm 2.1
procedure: Compute. $\lambda_k(\lambda_0, \varepsilon \in R^1 : \lambda_k)$
! This procedure computes λ_k

1. $i = 0$

2. $\lambda_{i+1} = \lambda_i - \dfrac{\varphi'(\lambda_k)}{\varphi''(\lambda_k)}$

3. **while** $\| \lambda_{i+1} - \lambda_i \| \geq \varepsilon$, **do**

3.1 $i = i + 1$

3.2 $\lambda_{i+1} = \lambda_i - \dfrac{\varphi'(\lambda_k)}{\varphi''(\lambda_k)}$

4. $\lambda_k = \lambda_{i+1}$.

5. return.■

Where

$$\varphi'(\lambda_k) = \frac{df(x_k + \lambda_k d_k)}{d\lambda_k} \in R^1 \text{ and } \varphi''(\lambda_k) = \frac{d^2(x_k + \lambda_k d_k)}{d\lambda_k} \in R^1$$

respectively.

The Algorithm 2.1 and several well known step size procedures was implemented into steepest descent

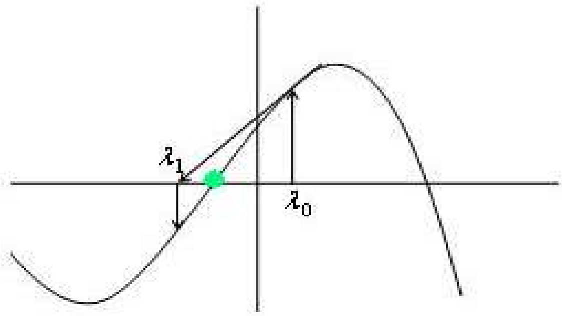

Fig. 3.1: Movement of Newton-like exact line search–Negative Step Size

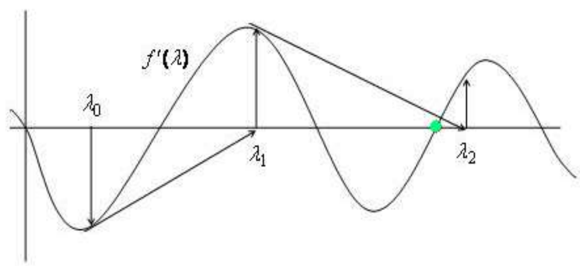

Fig. 3.2: Movement of Newton-like exact line search–Large Step Size

method and tested on several selected optimization problems (convex and non-convex). The comparison which was done by Goh and Ismail [7] found that the Newton-like exact line search can produce a better step size which help the steepest descent method to approximate the solution with least number of iteration.

Weakness of the Newton-like Exact Line Search: The Algorithm 2.1 has been successfully solved several complicated optimization problems with the least number of iteration as compared to those well known step size selection procedure. However, our attempt to select the initial point which is far from the local minimizer (close to the nearest maximizer), failed to obtain a small non-negative step size which is the basic requirement of exact line search procedure as mentioned in (2.2) and (2.3).

Negative Step Size: Based on the movement of the Newton-Raphson's method (Figure 2.1), the movement of step size searching process depended on the tangent of each step size in $f'(x_k + \lambda_k d_k)$. In non-convex optimization problem, the function $f'(x_k + \lambda_k d_k)$ might also be a non-convex function. Therefore, the movement of the step size in the searching process will lead to a negative step size (the green point) as showed in Figure 3.1.

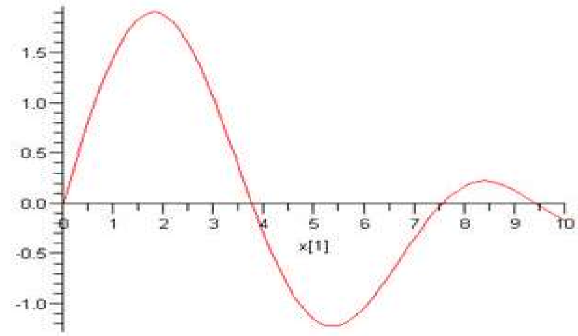

Fig. 3.3 (a) $f(x) = \sin(x) + \sin\left(\frac{2}{3}x\right)$ $x^0 = (3)$

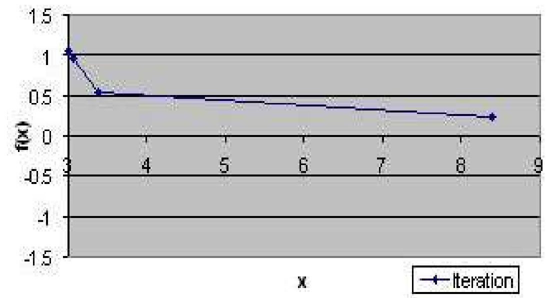

Fig. 3.3(b): Steepest descent method using Newton-Like exact line search (*Coriolis* Phenomena)

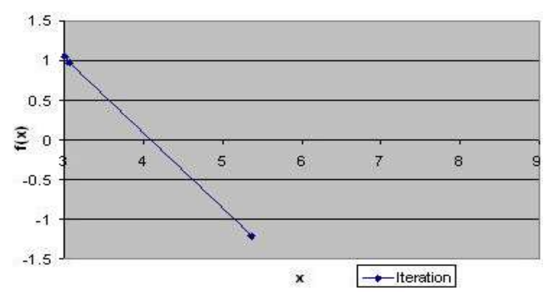

Fig. 3.3(c): Steepest descent method using improved Newton-like exact line search with fixed range of step size

The negative step sizes will directly affect the search direction of the steepest descent method which has been proved as descent direction becomes an ascent direction. Therefore, the negative step size uses in the steepest descent method, a local maximizer was obtained.

***Coriolis* Phenomena:** The negative step size will affect the convergent of the steepest descent method to approximate to the solution as well as a leading the phenomena of *coriolis* to happen and affect the convergent of the steepest descent method. Figure 3.2 showed the movement of the Newton-like exact line search to obtain a large step size (the green point).

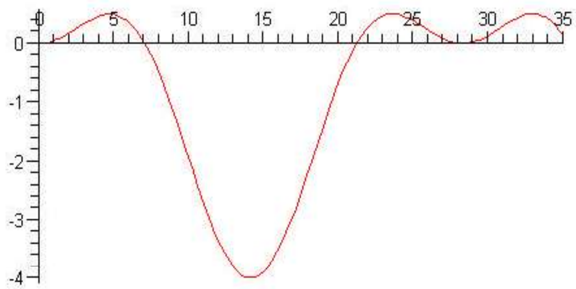

Fig. 3.4(a): $f(x)=\sin\left(\frac{4}{9}x\right)\tan\left(\frac{1}{9}x\right) \quad x^0=(5)$

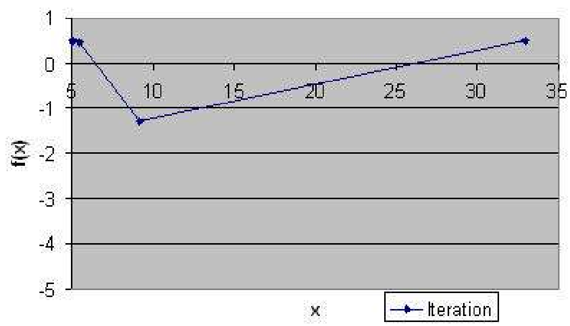

Fig. 3.4(b): Steepest descent method using Newton-Like exact line search (*Coriolis* Phenomena)

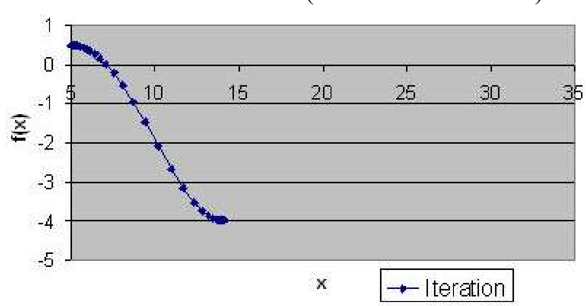

Fig. 3.4(c): Steepest descent method using improved Newton-like exact line search with fixed range of step size

The Algorithm 2.1 tends to obtain negative step size when the initial point of objective function is close to a local maximizer. The Algorithm 2.1 is then modified by changing the initial step size for the step size searching process when a negative step size been obtained. The changing process is repeated until a non-negative step size was determined. However, the outcome of the modification lead to another challenge in the convergent of the steepest descent method, which is the *coriolis* phenomena. There are two group of figure shown as below. Figure 3.3 (a) and Figure 3.4 (a) showed the graph of the objective function for the first and second testing problems which were listed in Section 5.1. The Figure 3.3 (b) and Figure 3.4 (b) showed the *coriolis* phenomena of the movement of steepest descent method by using the Newton-like exact line search which can be modified to force the Algorithm 2.1 to determined non-negative step size; Figure 3.3 (c) and Figure 3.4 (c) showed the convergent of the steepest descent method by using improved Newton-like exact line search with Fixed range of step size (which be discuss in Section 4).

The Improved Newton-Like Exact Line Search: In previous section, the negative step size and *coriolis* phenomena (large step size) affect the convergent of the steepest descent method to the solution. This section will discuss on methods to avoid both of the situations by using fixed range of step size.

The Idea of Improvement: Vrahatis [8] noticed that a 'small' step size has to be chosen to avoid *coriolis* and guarantee the convergent. However, this will lead to slow convergent of the steepest descent method. Therefore, a suitable range to define the 'small' for step size selection must be determined to overcome the weakness of the Newton-like exact line search.

By comparing the results of the Newton-like exact line search with other step size selection procedures [7], we have found that Armijo line search is the well perform procedure among other selected procedure which not including Newton-like exact line search. Besides that, previous researchers had noticed that Armijo line search is one of the most effective and easy to implement in computational compared to others. The Armijo line search rule is described as follows [11, 13].

Given $s > 0$, $\beta \in (0, 1)$ $\sigma \in (0, 1)$ and $\lambda_k = \max\{s, s\beta, s\beta^2, \ldots\}$ such that

$$f(x_k + \lambda_k d_k) - f(x_k) \le \sigma \lambda_k g_k d_k. \quad (4.1)$$

As we can see in the rules described above, we can conclude that the range of the step size λ_k fixed by Armijo line search is as shown in the inequation below:

$$0 < \lambda_k \le s. \quad (4.2)$$

However, previous studies [4,7,8,11-13] on this Armijo line search had suggested the value of $s = 1$ *or* 2. Therefore, in order to improve the Newton-like exact line search procedure, we fit in a fixed range of step size

$$0 < \lambda_k \le 2. \quad (4.3)$$

to the Algorithm 2.1. An initial step size changing procedure was designed and fit in to improve Newton-like exact line search procedure which will be shown in following sub-section.

Improved Newton-Like Exact Line Search Procedure: After fit in the fixed range of step size (4.3) and the initial step size changing procedure in to Algorithm 2.1, the improved algorithm for Newton-like exact line search procedure is as shown in Algorithm 4.1. The number of initial step size changing process, j is limit to 3 times only for each iteration, this is to make sure the changed initial step size still fit in the fixed range (4.3) and to save the computational time and memory usage.

Algorithm 4.1
Procedure: FixedRange $\lambda_k(\lambda_0\ \varepsilon\in R^1 : \lambda_k)$
! This procedure computes λ_k using fixed range of step size selection.

1. $j = 0$

2. SmallStep = false

3. while $j \le 3$ and not SmallStep do

3.1. $\lambda_1 = \lambda_0 \times 5^j$

3.2. $i = 1$

3.3. $\lambda_{i+1} = \lambda_i - \frac{\varphi'(\lambda_k)}{\varphi''(\lambda_k)}$

3.4. while $\| \lambda_{i+1} - \lambda_i \| \ge \varepsilon$, do

3.4.1. $i = i + 1$

3.4.2. $\lambda_{i+1} = \lambda_i - \frac{\varphi'(\lambda_k)}{\varphi''(\lambda_k)}$

3.5. If $0 < \lambda_{i+1} \le 2$,
then

3.5.1. SmallStep = true.

3.6. $j = j + 1$

4. if $\lambda_{i+1} \le 2$,
then

4.1. $\lambda_k = \lambda_{i+1}$.

else

4.2. $\lambda_k = 1$

5. return. ■

Numerical Experiments: In this section, the numerical results of the implementation of the improved procedure were reported by solving several selected non-convex optimization testing problems which listed in following sub-section.

List of testing problems

1. $f(x) = \sin(x) + \sin\left(\frac{2}{3}x\right) \quad x^0 = (3)$

2. $f(x) = \sin\left(\frac{4}{9}x\right)\tan\left(\frac{1}{9}x\right) \quad x^0 = (5)$

3. $f(x) = \cos\left(\frac{3}{5}x\right)\cos(2x) + \sin(x) \quad x^0 = (5)$

4. $f(x) = \cos\left(\frac{2}{5}x\right)\sin\left(\frac{1}{10}x\right) + \cos(x) \quad x^0 = (-6)$

5. $f(x) = \sin\left(\frac{4x}{9}\right)\sin(x) \quad x^0 = (2.5)$

6. six hump came back function

$$f(x_1, x_2) = 4x_1^2 - 2.1x_1^4 + \frac{x_1^6}{3} - x_1x_2 - 4x_2^2 + 4x_2^4, x^0 = (0.5, 0.5)$$

7. Rastrigin Function

$$f(x_1, x_2) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2), x^0 = (1,1)$$

8. The Two dimension Function

$$f(x_1, x_2) = [1 - 2x_2 + c\sin(4\pi x_2) - x_1]^2 + [x_2 - 0.5\sin(2\pi x_1)]^2, c = 0.2, x^0 = (6, -2)$$

9. The Two dimension Function

$$f(x_1, x_2) = [1 - 2x_2 + c\sin(4\pi x_2) - x_1]^2 + [x_2 - 0.5\sin(2\pi x_1)]^2, c = 0.5, x^0 = (0,0)$$

10. The Two dimension Function

$$f(x_1, x_2) = [1 - 2x_2 + c\sin(4\pi x_2) - x_1]^2 + [x_2 - 0.5\sin(2\pi x_1)]^2, c = 0.05, x^0 = (-1,1)$$

Numerical Results: The improved procedure that was discussed in previous section has been implemented into steepest descent method and the algorithms was programmed into visual C++ language. Our testing problems and the initial points used are shown in Section 5.1. For each problem, the limiting number of iteration is set to 100,000, the tolerance $\varepsilon = 10^{-6}$ and the initial step size $\lambda_0 = 0.01$.

Problem	Initial Function Value $f(x^0)$	Minimum Function Value $f(x^*)$	Minimizer	Number of Iteration
1	1.05042	-1.21598	(5.36225)	2
2	0.49365	-4	(14.1372)	74
3	-0.12825	-1.04212	(5.94596)	10
4	1.37653	-1.09906	(-3.06054)	37
5	0.536346	-0.888917	(4.50953)	26
6	0.373958	-1.03163	(-0.0898419,0.712657)	10
7	0.679367	0.179775	(-1.04076,1.04076)	3
8	5	2.507	(5.72207,-1.8806)	6
9	1	0.517454	(0.0420235,-0.0947717)	10
10	1	0.102163	(-0.72998,0.793414)	12

CONCLUSION

This paper showed the weakness of the Newton-like exact line search to compute the step size for steepest descent method especially when the initial point selected is far from the local solution (close to nearest maximizer). The Figure 3.3 (b) and Figure 3.4 (b) showed the effect of the *coriolis* phenomena which can be occurred when a large step size was selected. The Figure 3.3 (c), Figure 3.4 (c) and the numerical results showed in Section 5.2 showed the effectiveness of the improved Newton-like exact line search procedure to overcome the *coriolis* phenomena and obtained a small non-negative step size to guarantee the convergent of the steepest descent method in solving non-convex optimization problems.

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