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A Multistage Homotopy Perturbation Method for Solving Coullet System

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Abstract: In this article, a multistage homotopy perturbation method is employed to solve a system of nonlinear differential equations, namely Coullet system. Numerical results are compared to those obtained by the fourth-order Runge-Kutta method to illustrate the preciseness and effectiveness of the proposed method. It is shown that the proposed method is robust, accurate and easy to apply.

Key words: Coullet system . a multistage homotopy perturbation method . Runge-Kutta method

INTRODUCTION

In this paper, we consider the Coullet system [1, 2]

$$\frac{dx}{dt} = y, \tag{1}$$

$$\frac{dy}{dt} = z, \tag{2}$$

$$\frac{dz}{dt} = cz + by + ax + dx^3 \tag{3}$$

where x,y,z are the state variables, and a,b,c and d are real constants. If the parameters are taken as a = 0.8, b = -1.1, c = -0.45 and d = -1.0 the system (1)-(3) exhibits chaotic dynamics.

The motivation of this paper is to extend the application of the analytic Homotopy Perturbation Method (HPM) to solve the Coullet system (1)-(3). The HPM was first proposed by Chinese mathematician He [3-7]. The essential idea of this method is to introduce a homotopy parameter, say p, which takes the values from 0 to 1. When p = 0, the system of equations usually reduces to a sufficiently simplified form, which normally admits a rather simple solution. Eventually at p = 1, the system takes the original form of the equation. The HPM has been employed to solve a large variety of linear and nonlinear problems [7-15].

Very recently, Chowdhury *et al*. [16], Chowdhury and Hashim [17], Hashim *et al*. [18] and Hashim and Chowdhury [19] were the first to successfully apply the Multistage Homotopy Perturbation Method (MHPM) to the chaotic Lorenz system, Chen system, bioreaction model and a class of systems of ODEs. In this paper we are again interested in the accuracy of the MHPM for Coullet system capable of exhibiting chaotic behavior. We shall call this technique as the multistage HPM (for short MHPM). Comparison with the classical fourth-order Runge-Kutta (RK4) shall be made.

The paper is organized as follows: A brief review of HPM and MsHPM are given in Section 2 and 3, respectively. The application of the proposed numerical scheme is illustrated in Section 4. The conclusions are then given in the final Section 5.

HOMOTOPY PERTURBATION METHOD

To illustrate the Homotopy Perturbation Method (HPM) for solving non-linear differential equations, He [7, 8] considered the following non-linear differential equation:

$$A(u) = f(r), r \subset \Omega \tag{4}$$

subject to the boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, r \in \Gamma \tag{5}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, Γ is the boundary of the domain Ω and $\partial/\partial n$ denotes differentiation along the normal vector drawn outwards from Ω. The operator A can generally be divided into two parts M and N. Therefore, (4) can be rewritten as follows:

$$M(u) + N(u) = f(r), r \in \Omega \tag{6}$$

He [7, 8] constructed a homotopy $v(r,p)$: $\Omega\times[0,1]\to\mathbb{R}$ which satisfies

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$$H(v,p) = (1-p)[M(v) - M(u_0)] + p[A(v) - f(r)] = 0 \quad (7)$$

which is equivalent to

$$H(v,p) = M(v) - M(u_0) + pM(v_0) + p[N(v) - f(r)] = 0 \quad (8)$$

where $p\in[0,1]$ is an embedding parameter, and u_0 is an initial approximation of (4). Obviously, we have

$$H(v,0) = M(v) - M(u_0) = 0, \quad H(v,1) = A(v) - f(r) = 0. \quad (9)$$

The changing process of p from zero to unity is just that of H (v,p) from M (v)-M (u_0) to A (v)-*f* (r). In topology, this is called deformation and is called homotopic. According to the homotopy perturbation method, the parameter p is used as a small parameter, and the solution of Eq. (7) can be expressed as a series in p in the form

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \cdots \quad (10)$$

When $p\to 1$ Eq. (6) corresponds to the original one, Eqs. (8) and (9) become the approximate solution of Eq. (4), i.e.,

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots \quad (11)$$

The convergence of the series in Eq. (11) is discussed by He in [7, 8].

MULTISTAGE HOMOTOPY PERTURBATION METHOD

For large t, HPM is not good result to approximate solution of some differential equation. To guarantee validity of approximation solution for large t, the studies at [12-15], a new approach called the MsHPM is mentioned. According to this approach, the solution from $[t_0,t)$ to be reproduced by subdividing this interval into $[t_0,t)$, $[t_1,t_2)$,…, $[t_{j-1},t_j = t)$ and a recursive formula of (12) to be applied on each subinterval [11-14]:

The initial approximation in each interval is taken from the solution in the previous interval,

$$u_{i,0}(t) = u_i(t^*) = c_i^* \quad (12)$$

where t_i^* is the left-end point of each subinterval and c_i^* is denoted as the initial approximations for i = 1,2,…,m.

By knowing the first initial conditions, one would be able to by appling the inverse linear operator for all unknowns $u_{i,n}(t)$, (i = 1,2,…,m; n = 0,1,…) as follow:

$$L^{-1}(.) = \int_{t^*}^{t} (.)dt. \quad (13)$$

In order to carry out the iteration in every subinterval of equal length

$$\Delta(t), [t_0,t), [t_1,t_2), \dots, [t_{j-1}, t_j - t)$$

we need to know the values of the following:

$$u_{i,0}^*(t) = u_i(t^*) = c_i^*, i = 1,2,\dots,m. \quad (14)$$

This information is typically not directly attainable, but through the initial value $t^* = t_0$, we could derive all the initial approximations. This is done by taking the previous initial approximation from the nth-iterate of the preceding subinterval given by (5), i.e.

$$u_{i,0}^*(t) \cong u_{i,n}(t^*), i = 1,2,\dots,m \text{ and } t^* \in (t_0,t_1) \quad (15)$$

APPLICATIONS

HPM solution: In this section, we will apply the homotopy perturbation method to nonlinear ordinary differential systems (1).

According to homotopy perturbation method, we derive a correct functional as follows:

$$v_1' - x_0' + p(x_0' - v_2) = 0,$$
$$v_2' - y_0' + p(y_0' - v_3) = 0,$$
$$v_3' - z_0' + p(z_0' - cv_3 - bv_2 - av_1 - dv_1^3) = 0, \quad (16)$$

the initial approximations are as follows:

$$v_{10}(t) = x_0(t) = x(0) = P_1,$$
$$v_{20}(t) = y_0(t) = y(0) = P_2,$$
$$v_{30}(t) = z_0(t) = z(0) = P_3, \quad (17)$$

and

$$v_1 = \sum_{j=0}^{\infty} v_{1j}p^j, v_2 = \sum_{j=0}^{\infty} v_{2j}p^j, v_3 = \sum_{j=0}^{\infty} v_{3j}p^j. \quad (18)$$

where $v_{i,j}$, i = 1,2,3, j = 1,2,3,… are functions yet to be determined. Substituting Eqs.(17) and (18) into Eq. (16)

and arranging the coefficients of "p" powers, we have unknowns $v_{i,j}(t)$. In order to obtain the unknowns $v_{i,j}$, (t), i,j = 1,2,3, … We must construct and solve the system which includes nine equations with nine unknowns, considering the initial conditions $v_{i,j}$, (0) = 0, i,j = 1,2,3,… if the n-terms approximations are sufficient, we will obtain:

$$x(t) = \lim_{p \to 1} v_1(t) = \sum_{k=0}^{n} v_{1,k}(t),$$

$$y(t) = \lim_{p \to 1} v_2(t) = \sum_{k=0}^{n} v_{2,k}(t),$$

$$z(t) = \lim_{p \to 1} v_3(t) = \sum_{k=0}^{n} v_{3,k}(t), \qquad (19)$$

MsHPM solution: According to MsHPM, we choose the initial approximations as

$$v_{10}(t) = x_0(t) = x(t^*) = P_1^*,$$

$$v_{20}(t) = y_0(t) = y(t^*) = P_2^*,$$

$$v_{30}(t) = z_0(t) = z(t^*) = P_3^*. \qquad (20)$$

To carry out the iterations in every subinterval of equal length, we take the values of the following,

$$P_1^* = x(t^*) \cong \varphi_x(t^*),$$

$$P_2^* = y(t^*) \cong \varphi_y(t^*),$$

$$P_3^* = z(t^*) \cong \varphi_z(t^*). \qquad (21)$$

where

$$\varphi_x(t) = \textstyle\sum_{k=0}^{n} v_{1,k},\ \varphi_y(t) = \sum_{k=0}^{n} v_{2,k},\ \varphi_z(t) = \sum_{k=0}^{n} v_{3,k}.$$

Here

$$x(0) = v_{10}(0) = 0.1, y(0) = v_{20}(0) = 0.41,$$
$$z(0) = v_{30}(0) = 0.31$$

for the three-component model.

The accuracy of the HPM is demonstrated against Maples built-in fourth-order Runge Kutta procedure RK for the solutions of Coullet system. The domain is divided using $\Delta t = 0.01$ comparing with RK4 with step size h = 0.001. Figure 1 presents the comparison

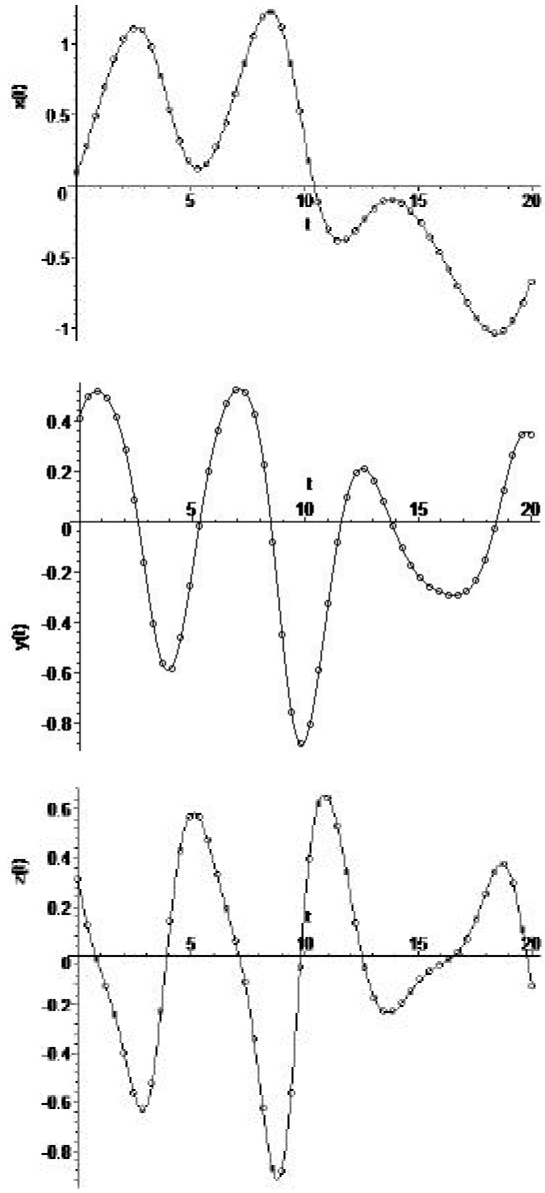

Fig. 1: Local changes of x,y,z for 5-term MsHPM (line) with $\Delta t = 0.01$ and RK4 (circle) with h = 0.001

between HPM solution and RK4 solution. We can see the good agreement for HPM solution with RK4 solution. The phase portray of the Coullet system is given in Fig. 2. It is clear that this is chaotic attractor for Coullet system. Also, Fig. 3 shows the chaotic attractors for the Coullet system (1)-(3) using the HPM solution.The difference between 5-term HPM with Δt = 0.01 and RK4 with h = 0.001 is given in Fig. 4. Figure 4 shown that the HPM have higher accuracy of the solution since in the x, y and z axis we have error until 10^{-3} (i.e., the solution via the new method has agreement with the purely numerical until 3 digit).

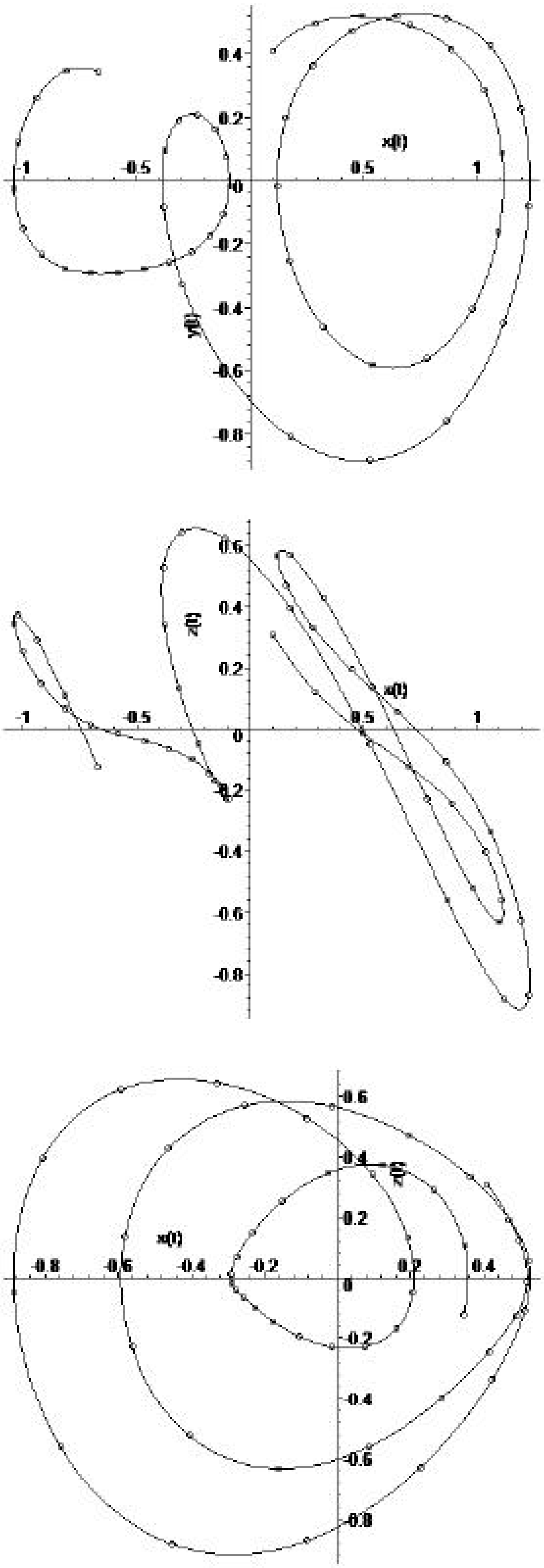

Fig. 2: Chaotic attractors for the system (1)-(3) (5-term MsHPM with $\Delta t = 0.01$)

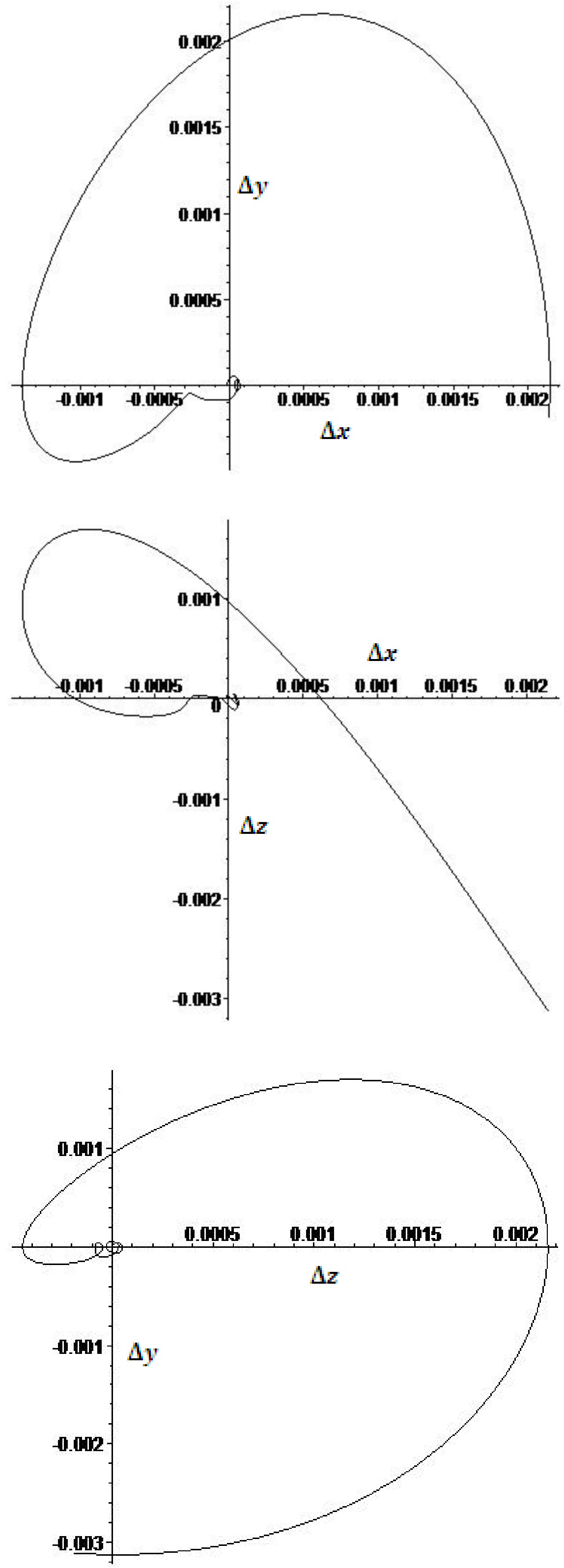

Fig. 3: Difference between 5-term HPM with $\Delta t = 0.01$ and RK4 with $h = 0.001$

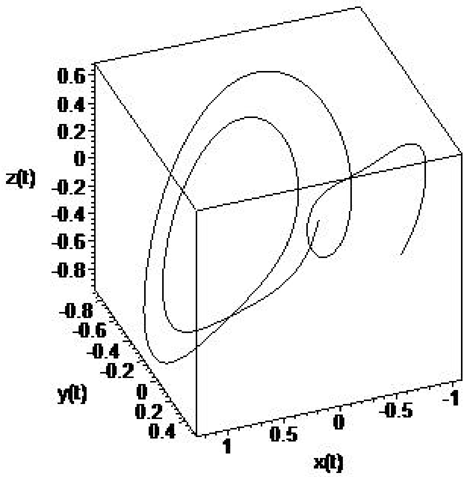

Fig. 4: Phase portray for Coullet system with time span [0,20] using MsHPM

CONCLUSIONS

In this paper, MsHPM is implemented to solve Coullet chaotic system. Higher accuracy solution was obtained via this method. Comparison between MsHPM solution and RK4 solution is discussed and plotted. The solution via MsHPM is continuous on this domain and analytical at each subdomain which is the best in our knowledge

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