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The (G`/G)-expansion Method a Special Case of the Generalized tanh-function Type Method

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Abstract: In this paper a generalized tanh-function type method is proposed by using the idea of the transformed rational function method. We shown that the (G`/G)-expansion method is a special case of the generalized tanh-function type method, so the (G`/G)-expansion method is included by the transformed rational function method. We demonstrate that all solutions obtained by the (G`/G)-expansion method are not new and were found by the generalized tanh-function type method. In comparison with all solution expressions obtained by the generalized tanh-function type method, we find that solution expressions obtained by the (G`/G)-expansion method are very cumbersome.

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INTRODUCTION

Direct searching for exact solutions of nonlinear partial differential equations (NLPDEs) plays an important role in the study of nonlinear physical phenomena and becomes one of the most exciting and extremely active areas of research investigation. In the past several decades, many effective methods for obtaining exact solutions of NLPDEs have been presented, such as inverse scattering method [1], Darboux and Bäcklund transformation [2, 3], Hirota's bilinear method [4], Lie group method [5], variational iteration method [6], Adomian decomposition method [7] and so on. Most recently, Prof. Ma and Lee [8] proposed a new direct method called the transformed rational function method to solve exact solutions of nonlinear partial differential equations. The transformed rational function method provides a more systematical and convenient handling of the solution process of nonlinear equations, unifying the tanh-function method, homogenous balance method, Jacobi elliptic function expansion method, F-expansion method, Exp-function method. Its key point is to search for rational solutions to variable-coefficient ordinary differential equations transformed from given partial differential equations.

In this paper using the idea of the transformed rational function method, a generalized tanh-function type method is introduced to solve exact traveling solutions of NLPDEs. We also show that the (G`/G)-expansion method [9-15] is a special case of the method, so the transformed rational function method includes the (G`/G)-expansion method. We demonstrate that all solutions obtained by the (G`/G)-expansion method are not new and were found by the generalized tanh-function type method. In comparison with the solution expressions obtained by the generalized tanh-function type method, we find that the solution expressions obtained by the (G`/G)-expansion method are very cumbersome.

The rest of the paper is organized as follows. In Section 2, using idea of the transformed rational function method, a generalized tanh-function type method for finding travelling wave solutions of NLPDEs was introduced. In Section 3, we show that the (G`/G)-expansion method is a special case of the generalized tanh-function type method. In Section 4, some conclusions are given.

THE GENERALIZED TANH-FUNCTION TYPE METHOD

A detailed describe of the transformed rational function method can be found in Ma and Lee's paper [8].Using the idea of the transformed rational function method, a new approach called the generalized tanh-function method is

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proposed. The key point of the new approach is based on the assumptions that the solutions can be expressed by Laurent polynomial (a special case of rational functions) and that solution variable satisfies Riccati equation. In the following we give the main steps of the generalized tanh-function type method for finding travelling wave solutions of NLPDEs

For simplicity, let us consider the nonlinear partial differential equation with two independent variables x and t, is given by

$$P(u, u_x, u_t, u_{xt}, u_{xx}, \ldots) = 0 \tag{2.1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved

Step 1: Using the transformation $u = u(x,t) = u(\xi),\ \xi = x - kt$, we reduce the Eq. (2.1) to an ODE

$$Q(u, u', u'', u''', \ldots) = 0 \tag{2.2}$$

Step 2: Suppose that the solution of ODE (2.2) can be expressed by a finite series in Y as follows:

$$u = \sum_{k=-m}^{n} a_k Y^k \tag{2.3}$$

where $Y = Y(\xi)$ satisfied Riccati equation

$$Y' = aY^2 + bY + c,\ a \neq 0 \tag{2.4}$$

where $Y' = dY(\xi)/d\xi$. $a_{-m}, \cdots, a_n$, k, a, b and c are constants to be determined later and non-negative integers m and n can be determined by considering the homogeneous balance between the highest order derivative and nonlinear terms appearing in ODE (2.2).

Step 3: By substituting (2.3) along with (2.4) into ODE (2.2) and collecting all terms with the same order of Y, ODE (2.2) will yield a system of algebraic equation with respect to $a_{-m}, \ldots, a_n$, k, a, b, c because all the coefficient of Y^k have to vanish.

Step 4: With the aid of Mathematica or Maple, one can determine $a_{-m}, \ldots, a_n$, k, a, b and c by solving the system of algebraic equation of Step.3.

Step 5: Substituting $a_{-m}, \ldots, a_n$, k and the solutions of Riccati equation (2.4) into (2.3) we can obtain the solutions of the nonlinear partial differential equation (2.1)
Using the following formula of indefinite integrals, we can obtain general solution of Riccati equation (2.4)
The formula of indefinite integrals. Let $a \neq 0, \Delta = b^2 - 4ac$, then we have the following results:

Case 1: when $\Delta = b^2 - 4ac > 0$

$$\int \frac{dx}{ax^2 + bx + c} = \frac{1}{\sqrt{\Delta}} \ln\left|\frac{2ax + b - \sqrt{\Delta}}{2ax + b + \sqrt{\Delta}}\right| = \frac{-2}{\sqrt{\Delta}} \operatorname{arctanh}\left(\frac{2ax + b}{\sqrt{\Delta}}\right) \text{ or } \frac{-2}{\sqrt{\Delta}} \operatorname{arccoth}\left(\frac{2ax + b}{\sqrt{\Delta}}\right)$$

Case 2: When $\Delta = b^2 - 4ac < 0$

$$\int \frac{dx}{ax^2 + bx + c} = \frac{2}{\sqrt{-\Delta}} \arctan\left(\frac{2ax + b}{\sqrt{-\Delta}}\right) \text{ or } \frac{-2}{\sqrt{-\Delta}} \operatorname{arccot}\left(\frac{2ax + b}{\sqrt{-\Delta}}\right)$$

Case 3: when $\Delta = b^2 - 4ac = 0$

$$\int \frac{dx}{ax^2+bx+c} = \frac{-2}{2ax+b}$$

Using above the formula of indefinite integrals, the Riccati equation (2.4) has the following general solutions:
If $\Delta = b^2\text{-}4ac>0$,

$$Y+\frac{b}{2a} = -\frac{\sqrt{\Delta}}{2a}\tanh[\frac{\sqrt{\Delta}}{2}(\xi+\xi_0)] \text{ or } -\frac{\sqrt{\Delta}}{2a}\coth[\frac{\sqrt{\Delta}}{2}(\xi+\xi_0)] \tag{2.5a}$$

If $\Delta = b^2\text{-}4ac<0$

$$Y+\frac{b}{2a} = \frac{\sqrt{-\Delta}}{2a}\tan[\frac{\sqrt{\Delta}}{2}(\xi+\xi_0)] \text{ or } -\frac{\sqrt{-\Delta}}{2a}\cot[\frac{\sqrt{\Delta}}{2}(\xi+\xi_0)] \tag{2.5b}$$

If $\Delta = b^2\text{-}4ac = 0$,

$$Y+\frac{b}{2a} = -\frac{C_1}{a(C_1\xi+C_2)} \tag{2.5c}$$

where ξ_0, C_1, C_2 are constants. By using formula

$$\tanh(\alpha+\beta) = \frac{\sinh\alpha\cosh\beta+\cosh\alpha\sinh\beta}{\cosh\alpha\cosh\beta+\sinh\alpha\sinh\beta}$$

$$\tan(\alpha+\beta) = \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

We can change (2.5a) and (2.5b) into the following result respectively
(iv) If $\Delta = b^2\text{-}4ac>0$

$$Y+\frac{b}{2a} = -\frac{\sqrt{\Delta}}{2a}\tanh[\frac{\sqrt{\Delta}}{2}(\xi+\xi_0)] = -\frac{\sqrt{\Delta}}{2a} * \frac{A_1\sinh[\frac{\sqrt{\Delta}}{2}\xi]+A_2\cosh[\frac{\sqrt{\Delta}}{2}\xi]}{A_1\cosh[\frac{\sqrt{\Delta}}{2}\xi]+A_2\sinh[\frac{\sqrt{\Delta}}{2}\xi]} \tag{2.6a}$$

Where

$$A_1 = \cosh[\frac{\sqrt{\Delta}}{2}\xi_0],\ A_2 = \sinh[\frac{\sqrt{\Delta}}{2}\xi_0]$$

be constants.
If $\Delta = b^2\text{-}4ac<0$

$$Y+\frac{b}{2a} = \frac{\sqrt{-\Delta}}{2a}\tan[\frac{\sqrt{\Delta}}{2}(\xi+\xi_0)] = \frac{\sqrt{-\Delta}}{2a} * \frac{-B_1\sin[\frac{\sqrt{-\Delta}}{2}\xi]+B_2\cos[\frac{\sqrt{-\Delta}}{2}\xi]}{B_1\cos[\frac{\sqrt{-\Delta}}{2}\xi]+B_2\sin[\frac{\sqrt{-\Delta}}{2}\xi]} \tag{2.6b}$$

Where

$$B_1 = -\cos[\frac{\sqrt{-\Delta}}{2}\xi_0],\ B_2 = \sin[\frac{\sqrt{-\Delta}}{2}\xi_0]$$

be constants

We notice that the expressions (2.6a) and (2,6b) are more cumbersome than (2.5a) and (2.5b) and solutions expression of the (G`/G)-expansion method are just from (2.6a) and (2.6b).

THE (G`/G)-EXPANSION METHOD: A SPECIAL CASE OF THE GENERALIZED TANH FUNCTION TYPE METHOD

Recently, Wang *et al*. [9] introduced a new method called the (G`/G)-expansion method to look for travelling wave solutions of NLPDEs. Later, the (G`/G)-expansion method is used by some authors, such as in [10-15]. The

G`/G)-expansion method is based on the assumptions that the travelling wave solutions can be expressed by a polynomial in (G`/G), that is

$$u = \sum_{k=-m}^{n} a_k (G`/G)^k$$

and that $G = G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$G'' + \lambda G' + \mu G = 0 \tag{3.1}$$

where

$$G` = dG(\xi)/d\xi, G`` = d^2G(\xi)/d\xi^2$$

The degree of the polynomial can be determined by the homogeneous balance method. The coefficients of the polynomial can be obtained by solving a set of algebraic equations resulted from the process of using the method.

In here we show that the (G`/G)-expansion method is a special case of the generalized tanh-function type method. It is not difficult to find if we make the transformation Y = (G`/G) then equation (3.1) is equivalent to the following special Riccati equation

$$Y' + Y^2 + \lambda Y + \mu = 0 \tag{3.2}$$

where λ and μ are constants. In the (G`/G)-expansion method, Y = (G`/G) uses the result from (2,6a) and (2.6b), so the (G`/G)-expansion method is a special case of the generalized tanh-function type method.

Remark 1: In Riccati Equation (2.4), if $b = 0$, then the generalized tanh-function type method degenerate the tanh method. The idea of the tanh-function type method can go back to [16].

Remark 2: In Riccati Equation (2.4), if $a = -1$, $b = -\lambda$, $c = -\mu$, the generalized tanh-function type method is equivalent to (G`/G)-expansion method.

CONCLUSION

The generalized tanh-function type method can be viewed as an application of the transformed rational function method. In fact, using the idea of the transformed rational function method, many methods to solve NLPDEs can be obtained. In this letter we show that the (G`/G)-expansion method is a special case of the generalized tanh-function type method, so the (G`/G)-expansion method is considered as a special deformation application of the transformed rational function method. It is worthy noticing that the generalized tanh-function type method is more concise and straightforward to seek exact solutions of nonlinear partial differential equations (NLPDEs) and can be applied to many other NLPDEs in mathematical physics.

It is well known that exact solutions play a vital role in understanding various qualitative and quantitative features of nonlinear phenomena. There are diverse classes of interesting exact solutions, such as traveling wave solutions and soliton solutions, but it often needs specific mathematical techniques to construct exact solutions due to the nonlinearity present in dynamics [17, 18]. The tanh-function type method is one of efficient ways to search for exact traveling solutions to nonlinear partial differential equations. Most very recently, Prof. Ma *et al*. [19-21] presented a new method which was used to the linear superposition principle to construct multiple exponential traveling wave solutions to some nonlinear partial differential equations. The method provide a new mathematical tool to seek for exact solutions for nonlinear equations.

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REFERENCES

1. Ablowitz, M.J. and P.A. Clarkson, 1991. Soliton, Nonlinear Evolution Equations and Inverse Scattering. Cambridge Univ. Press. York.
2. Esteevez, P.G., 1999. Darboux transformation and solutions for an equation in 2+1 dimensions. J. Math. Phys., 40: 1406-1419.
3. Miurs, M.R., 1978. Backlund Transformation, Springer, Berlin.
4. Tam, H.W., W.X. Ma, XB. Hu and D.L. Wang, 2000. The Hirota-Satsuma coupled KdV equation and a coupled Ito system revisite. J. Phys. Soc. Jpn., 69: 45-51.
5. Olver, P.J., 1993. Applications of Lie groups to differential equations. NewYork: Springer.
6. Wazwaz, A.M., 2007. The variational iteration method for solving linear and nonlinear systems of PDEs. Computers and Mathematics with Applications, 54: 895-902.
7. Pamuk, S., 2005. An application for linear and nonlinear heat equations by Adomian's decomposition method. Applied Mathematics and Computation, 163: 89-96.
8. Ma, W.X. and J.-H. Lee, 2009. A transformed rational function method and exact solutions to the 3+1 dimensional Jimbo-Miwa equation, Chaos, Solitons & Fractals, 42: 1356-1363.
9. Wang, M.L., X. Li and W. Wang, 2008. The (G`/G)-expansion method and traveling wave solutions of nonlinear evolutions in mathematical physics. Phys. Lett. A, 372: 417.
10. Zhang, S., J.L. Tong and W. Wang, 2008. A generalized (G`/G)-expansion method for the mKdV equation with variable coefficients. Phys. Lett. A, 372: 2254.
11. Bekir, A., 2008. Application of the (G`/G)-expansion method for nonlinear evolution equations. Phys. Lett. A, 372: 3400.
12. Ganji, D.D. and M. Abdollahzadeh, 2009. Exact traveling solutions of nonlinear evolution equation by (G'/G)-expansion method. J. Math. Phys., 50: 013519.
13. Kudryashov, N.A., 2010. A note on the (G'/G)-expansion method. Appl. Math. Comput., 217 (1): 1755-1758.
14. Zuo, J.-M., 2010. Application of the extended (G`/G)-expansion method to solve the Pochhammer-Chree equations. Appl. Math. Comput., 217 (1): 376-383.
15. Kutluay, S., A. Esen and O. Tasbozan, 2010. The (G`/G)-Expansion Method for Some Nonlinear Evolution Equations. Appl. Math Comput., 217 (1): 384-391.
16. Ma, W.X. and B. Fuchssteiner, 1996. Explicit and Exact Solutions to Kolmogorov-Petrovsk II-Piskunov Equation. Int. J. Non-Linear Mechanics, 31 (3): 329-338.
17. Ma, W.X., 2003. Diversity of exact solutions to a restricted Boiti-Leon-Pempinelli dispersive long-wave system. Phys. Lett. A, 319: 325-333.
18. Hu, H.C., B. Tong and S.Y. Lou, 2006. Nonsingular positon and complexiton solutions for the coupled KdV system. Phys. Lett. A, 351: 403-412.
19. Ma, W.X. and E.G. Fan, 2011. Linear superposition principle applying to Hirota bilinear equations. Computers and Mathematics with Applications, 61: 950-959.
20. Ma, W.X., A. Abdeljabbar and M.G. Asaad, 2011. Wronskian and Grammian solutions to a (3+1)-dimensional generalized KP equation. Applied Mathematics and Computation, 217: 10016-10023.
21. Ma, W.X., Y. Zhang, Y.N. Tang and J.Y. Tu, 2012. Hirota bilinear equations with linear subspaces of solutions. Applied Mathematics and Computation, 218: 7174-7183.