

Spatial Variability of Imprecise Values of Rainfall Erosivity Index

¹Nazila Khorsandi, ²Mohammad Hossein Mahdian,
³Ebrahim Pazira and ⁴Davood Nikkami and ⁵Hadi Chamheidar

¹Member of Young Researchers Club, Takestan Branch, Islamic Azad University, Takestan, Iran
²Agricultural Research, Education and Extension Organization, Ministry of Jihad-e-Agriculture, Iran
³Department of Soil Science, Science and Research Branch, Islamic Azad University, Tehran, Iran
⁴Research Institute for Water Scarcity and Drought in Agriculture and Natural Resources, Tehran, Iran
⁵Department of Soil Science, Shoushtar Branch, Islamic Azad University, Shoushtar, Iran

Abstract: Access to rainfall intensity data is limited in many locations. Therefore, rainfall erosivity index could be estimated from readily available parameters that cause uncertainty in erosivity data. In this study fuzzy logic applied on the imprecise values of rainfall erosivity index and then spatial variability of it investigated by kriging interpolation method for preparation the EI_{30} map. Among different erosivity indexes/parameters based on rainfall amount, only modified Fournier index (FI_{mod}) was shown high correlation with EI_{30} in 11 synoptic stations. A local model was used for estimating EI_{30} from FI_{mod} in other 66 stations without rainfall intensity data. In these 66 stations the EI_{30} values were fuzzified. Number of five gaussian membership function for elevation as input variable and the EI_{30} as output variable were defined. The erosivity index values were defuzzified by centeroid method. Also, the ratio of nugget to sill of semivariogram (0.23) confirmed the strong spatial correlation of EI_{30} at distance of 630 km from unknown locations. The minus values of MBE related to kriging indicated underestimated the EI_{30} . The mean absolute error (MAE) of kriging with crisp values than the fuzzified values were shown a decline of 11, 3 and 4 percent. The output maps of all interpolation methods following similar decreasing trend from west to east of area with the highest erosivity ($1450 MJ mm ha^{-1} h^{-1} y^{-1}$) in the west.

Key words: EI_{30} • Fuzzy logic • Kriging • Modified Fournier index

INTRODUCTION

Rainfall erosivity is a basic key of Universal Soil Loss Equation (USLE) [1] and its Revised (RUSLE) [2] for prediction soil loss [3]. The previous studies on comparison of different rainfall erosivity indices in runoff-sediment plots of the northern of Iran, have been shown that EI_{30} has the most correlation with output sediment [4]. Rainfall erosivity, EI_{30} , is product of kinetic energy (E) and maximum 30-min rainfall intensity (I_{30}).

In many parts of the world, access to the rainfall intensity data is limited. Therefore, the EI_{30} could be estimated for more places, from rainfall amount parameters or indexes by regression model. A number of studies have presented relationships between EI_{30} and indexes based on rainfall amount. In this related, Hoyos *et al.* [5] estimated the annual EI_{30} from rainfall amount, by two

equations for wet and dry seasons in a tropical watershed of Colombian Andes. Also, in another study the models based on $rain_{10}$ - day_{10} for estimation the monthly EI_{30} have been used by Shamshad *et al.* [6].

These imprecise EI_{30} values from regression model can be handling via fuzzy sets [7]. The fuzzy logic creates mathematical models based on a fuzzy rule system. Accurately rainfall erosivity determination could be influence estimation of soil erosion. There are not found studies on the estimation of EI_{30} via fuzzy logic. But, in other research areas of hydrology the fuzzy rule system was investigated. In this relation, some researches have shown that fuzzy logic could be improved the results of prediction soil erosion [8, 9]. Also, Mahabir *et al.* [10] were concluded that fuzzy logic provides reliable water supply forecasts. In another research Kisi *et al.* [11] were reported that neuro-fuzzy models improved the results of estimation suspended sediment.

The map of rainfall erosivity is required for estimating soil erosion at the regional scale [12]. In order to generate the sparse data of erosivity index over a certain area, it is necessary to use of interpolation methods. Kriging is one of the usual interpolation methods. It is on the base of regionalized theory.

One of the problems related to every interpolation method is uncertainty of original data [13]. This imprecise could be solving by combination of fuzzy logic and kriging interpolation method that create a modification of conventional kriging as fuzzy kriging [14]. There are three fuzzy kriging method bases on input data and types of variogram. Fuzzy kriging type one with crisp and fuzzy data and crisp variogram. Fuzzy kriging type two with crisp data and fuzzy variogram. Also, fuzzy kriging type three with crisp and fuzzy data and fuzzy variogram (14). Comparison of kriging and fuzzy kriging by Rahimi *et al.* [15] was presented that fuzzy logic improved the accuracy of rainfall estimation. Lark [16] was used imprecise data of soil properties for describing a sample grid by fuzzy kriging variance.

As seen, fuzzy logic was applied in many areas. But there are not found any researches about influence of fuzzy logic on imprecise data of rainfall erosivity that are obtained from regression model. Therefore, the purpose of this study was to evaluate of influence of fuzzy models on the accurate of the kriging interpolation method of imprecise EI_{30} values. The results of this study create the best erosivity index map which use in decision-make for evaluation soil erosion.

MATERIALS AND METHODS

Study Area: The study area is located in Khazar watershed between $84^{\circ} 49' - 54^{\circ} 41'$ and $35^{\circ} 36' - 37^{\circ} 19' N$ in Northern Iran (Figure 1). This area is surrounded by the Caspian Sea in the north and a mountain range in the southern Khazar watershed. The mean annual precipitation vary from 1400 mm at coastal areas to 300 mm at Nemarestagh valley. The average elevation is 1300m above sea level.

The climates of this area are humid, Mediterranean sub-humid and semi-arid.

Numbers of 100 stations measure the amount of rainfall, but only 11 of these stations record rainfall intensity.

Data Source: Data on rainfall amounts (daily, monthly and annual), only at 77 stations from 100 stations, which had a minimum record length of 25 years, were used.

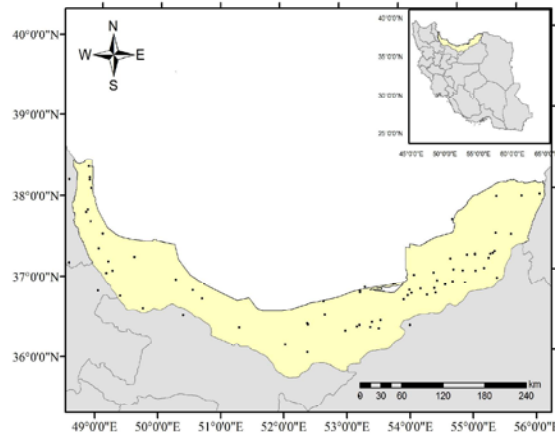


Fig. 1: The location of the study area and stations distribution

These data were collected from the Iran Meteorological office of these 77 stations, only 11 stations recorded rainfall intensity. These rainfall intensity data for a 25-year period were obtained from the Water Resources Management Company.

Before using these data, they were controlled for homogeneity with the Run test.

Rainfall erosivity index: The EI_{30} index was computed for 11 synoptic stations of study area. The kinetic energy (E) was computed for these stations as:

$$E = \sum_{r=1}^k 0.29 [1 - 0.72 \exp(-0.05i_r)] \Delta V_r \quad (1)$$

Where, i_r is rainfall intensity during the time interval (mm min^{-1}) and is rainfall depth for r intervals [17]. Then combining E from equation (1) and maximum intensity for intervals 30 minute produced EI_{30} .

Previous study in this relation was reported that among different the parameters and indices based on the rainfall amounts, only Arnoldus index (FI_{mod}) have been shown high correlation with EI_{30} in these 11 synoptic stations (4). The FI_{mod} or modified Fournier Index computed as:

$$FI_{\text{mod}} = \sum \frac{P_i^2}{P} \quad (2)$$

In which FI_{mod} is the Arnoldus index, P_i is the mean of monthly rainfall amount in mm and P is the mean of annual rainfall amount in mm [18]. It is necessity to generalize appropriate erosivity index (EI_{30}) to 66 stations which are

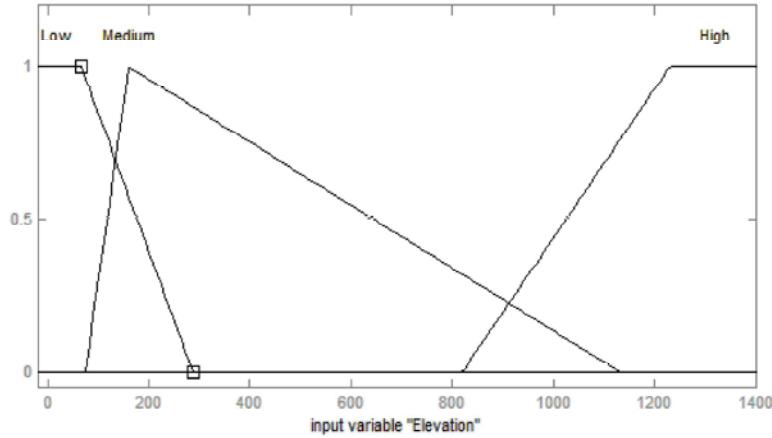


Fig. 2: Membership functions of input variable (elevation)

lack of rainfall intensity. For reaching this purpose in previous investigation [4], the regional regression model was used for estimating EI_{30} in whole of stations of area:

$$EI_{30} = -223.20 + 214.548 FI_{mod} \quad (r^2 = 0.79, P < 0.01) \quad (3)$$

Fuzzification Methodology: In 66 stations without rainfall intensity data, the EI_{30} values that estimated by equation (3), were fuzzified. First, the fuzzification was excluded with crisp values for elevation as input variable and EI_{30} as output variable.

Any values of elevation and EI_{30} are belonging to a function with a degree of membership. It can between zero (no membership) and one (definite membership). For this purpose, the type and number of membership functions and degree of membership were determined for each input and output variables according to Figure 2. Each fuzzy set defined by: $A=[a, b, c]$, that the function equation for this gaussian model is according equation (4):

$$\mu_{Ai}(x) = \exp\left(-\frac{(C_i - x)^2}{2\sigma_i^2}\right) \quad (4)$$

Where C_i and σ_i are the centre and width of the i th fuzzy set Ai , respectively.

As seen in Figure (2), the linguistic terms were defined for each of membership functions including five categories of "Very Low" (VL), "Low" (L), "Medium" (M), "High" (H) and "Very High" (VH).

The rule base was explained for relating inputs and outputs variables based on Mamdani method (19). Since the elevation and EI_{30} relationship has a direct proportional feature, the five rules were designed:

- R1 : IF elevation is very low, THEN EI_{30} is very low.
- R2 : IF elevation is low, THEN EI_{30} is low.

- R3 : IF elevation is medium, THEN EI_{30} is medium.
- R4 : IF elevation is high, THEN EI_{30} is high.
- R5 : IF elevation is very high, THEN EI_{30} is very high.

The membership functions for elevation and EI_{30} are presented in Figure 2.

Finally, defuzzification, fuzzy sets converted to crisp numbers. Various defuzzification methods are existing. In this study, defuzzification was done with gravity center method:

$$\hat{x} = \frac{\int_a^b x \cdot \mu(x) \cdot dx}{\int_a^b \mu(x) \cdot dx} \quad (5)$$

That is given a fuzzy set with membership degree x defined on the interval $[a, b]$ of variable x . These values of EI_{30} were used to kriging.

Kriging Method: It is necessary to the rainfall erosivity data following the normal distribution. The normality of data was controlled by kolmogorov-smirnov test [20]. Semivariogram was used for evaluation spatial correlation of rainfall erosivity index by GS+ software. The semivariance that quantified spatial variations for all possible pairing of data is calculated by:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \{Z(x_i) - Z(x_i + h)\}^2 \quad (6)$$

Where $y(h)$ is the semivariance at each lag (separating distance), h , $N(h)$ is the number of point pairs separated by the giving lag and $z(x_i)$ and $z(x_i + h)$ are the results of measurements at locations x_i and $x_i + h$, respectively.

The best model was fitted to semivariogram functions [21] and its range, sill and nugget were optimized with the use of cross-validation.

Finally, the validation of interpolation method on fuzzified and non-fuzzified data was evaluated by means cross-validation and correspond the trend of the map with nature. The cross validation was carried out by mean absolute of error (MAE) and mean biased of error (MBE):

$$MAE = \frac{1}{n} \sum |Z^* - Z| \quad (7)$$

$$MBE = \frac{1}{n} \sum (Z^* - Z) \quad (8)$$

Where, Z^* is the estimated value, Z is the observed value and n is the station number.

RESULTS AND DISCUSSION

Fuzzification of Rainfall Erosivity Index: Different environmental factors may be influenced rainfall erosivity index, such as elevation, latitude and longitude. But only elevation is correlated significantly with EI_{30} . Therefore, in 66 stations that EI_{30} estimated by regression model, the membership functions of elevation as input variable and EI_{30} as output variable are shown in Figures 2, 3. The number of five gaussian membership function was defined for each subset of input and output variable.

The linguistic terms were determined for these membership functions including very low, low, medium, high and very high. Then, the fuzzy rules were related input and output variables. These rules including:

- If elevation is very low, then EI_{30} is very low.
- If elevation is low, then EI_{30} is low.
- If elevation is medium, then EI_{30} is medium.
- If elevation is high, then EI_{30} is high.
- If elevation is very high, then EI_{30} is very high.

After that, the centroid approach was used for converting the fuzzy sets of EI_{30} to crisp values. The fuzzy model was calibrated with $R^2 = 0.742$.

Interpolation Methods: In order to investigation the performance of fuzzy logic on interpolation of the EI_{30} values, the fuzzified and measured values of the EI_{30} , also, the estimated and measured values of the EI_{30} were used. Descriptive statistics of rainfall erosivity index on fuzzified and non-fuzzified the EI_{30} values are given in table 1. For non-fuzzified the EI_{30} values, the skewness (0.98) and coefficient of Kolmogorov-Smirnov test ($P < 0.05$) of rainfall erosivity revealed that these data were not following the normality distribution. Serious violation of data from normal distribution may be cause ruin the variogram structure [22]. Therefore, the primary data of non-fuzzified

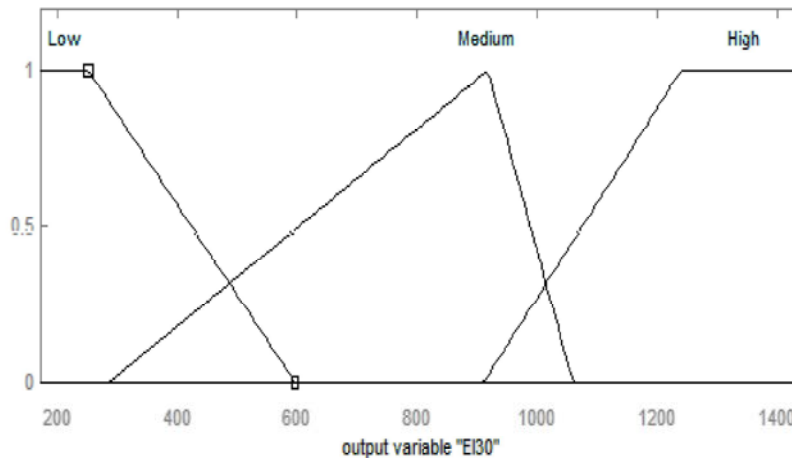


Fig. 3: Membership functions of output variable of EI_{30}

Table 1: The descriptive statistics of rainfall erosivity in the study area.

	Distribution	Min	Max	Mean	Cv ¹	Skewness	Kurtosis
Non-fuzzified EI_{30}	lognormal	2.22	3.16	2.87	8.06	0.29	0.13
fuzzified EI_{30}	lognormal	2.23	3.16	2.88	7	-0.19	-0.11

¹CV means coefficient of variations

Table 2: Parameters of model fitted to semivariogram of EI₃₀ and predicted error of kriging and fuzzy kriging methods

EI ₃₀	Model	Range (km)		(°)	(C ₀ +C)	(C ₀)	$\frac{C_0}{C_0+C}$	Predicted error	
		Minor	Major	Angle	Sill	Nugget	Nugget/sill	Mbe	Mae
Kriging	Gaussian	436	536	55	0.43	0.10	2.3	-1.75	193
Fuzzy Kriging	Gaussian	355	636	50	0.36	0.10	0.26	-1.77	169

Unit of C₀ and C₀+C is MJ mm⁻¹ ha⁻¹ h⁻¹ y⁻¹

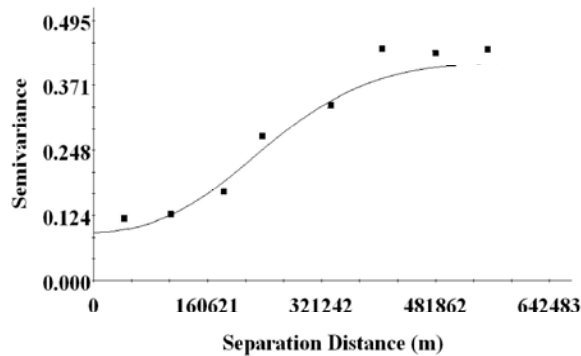


Fig. 4: Semivariogram of rainfall erosivity index

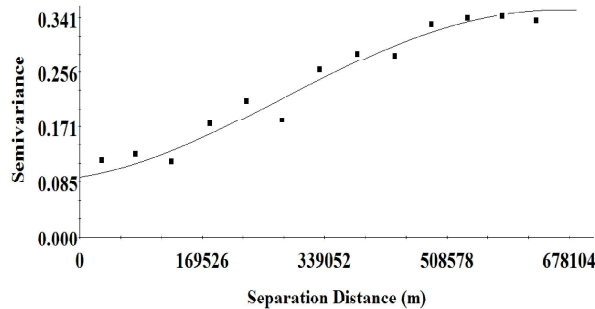


Fig. 5: Semivariogram of fuzzified rainfall erosivity index

rainfall erosivity transformed with lognormal transformation. The Kolmogorov-Smirnov test (P>0.05) for lognormal data verified the normal distribution. Descriptive statistics of the non-fuzzified and fuzzified values of EI₃₀ such as skewness (0.93 and 0.72, respectively) and coefficient of Kolmogorov-Smirnov test (0.47 and 0.49, respectively) was shown that there is not normality in data distribution. Use of log-normal transformation cause distribution of these data near to normal distribution.

Omnidirectional variogram of fuzzified and non-fuzzified values of the EI₃₀ were presented in Figure 4, 5. The Gaussian model as optimal model was fitted to both of semivariogram function by the minimum sum of square of the residual 0.004 and 0.005, respectively. The properties of this model are given in table 2.

The Gaussian model for non-fuzzified and fuzzified data of erosivity explained 97% and 90 % of variations at semivariogram. Also, the ratio of nugget (C₀) to sill (C₀+C) of non-fuzzified and fuzzified values of EI₃₀ was 0.23 and 0.27, respectively (Table 2). According Men *et al.* [20], the $\frac{C_0}{C_0+C} < 0.5$ presents a strong spatial correlation of EI₃₀ values between stations.

There are different types of kriging methods. Ordinary Kriging assumes the constant mean is unknown, but in Simple Kriging it is known exactly. But the Universal Kriging should only be used when there is a trend in data. In this study the semivariogram for every situation of non-fuzzified and fuzzified values have shown a constant sill, so there are not trend in the EI₃₀ values (Figure 4, 5). Also, the constant mean of the rainfall erosivity values is unknown. Therefore, in this study the ordinary kriging method is suitable between different types of kriging.

Furthermore, the semivariance of non-fuzzified and fuzzified values of EI₃₀ has dissimilar trend at azimuth 55° and 50° than other directions. Therefore, the non-fuzzified and fuzzified values of rainfall erosivity showed anisotropy at 55° and 50°. It could be related to heterogeneity of rainfall erosivity at this direction. The major and minor range revealed correlation distances of rainfall erosivity, 536 and 436.64 km for kriging and 636 and 355 for fuzzy kriging, respectively.

Comparative Performance of the Interpolation Methods:

The cross validation results of non-fuzzy kriging and fuzzy kriging method were comparing in this step (Table 3). These results showed that the fuzzy kriging method is more accuracy than non-fuzzy kriging. MAE for fuzzy kriging relative kriging method decreased 12 percentage.

The minus values of MBE in non-fuzzy kriging of rainfall erosivity indicated that they are underestimated methods. While, fuzzy kriging is an overestimated method.

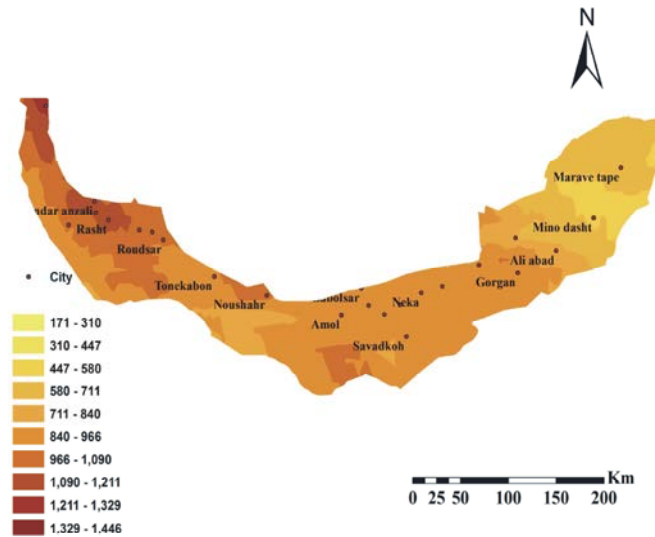


Fig. 6: Kriging Map of rainfall erosivity index

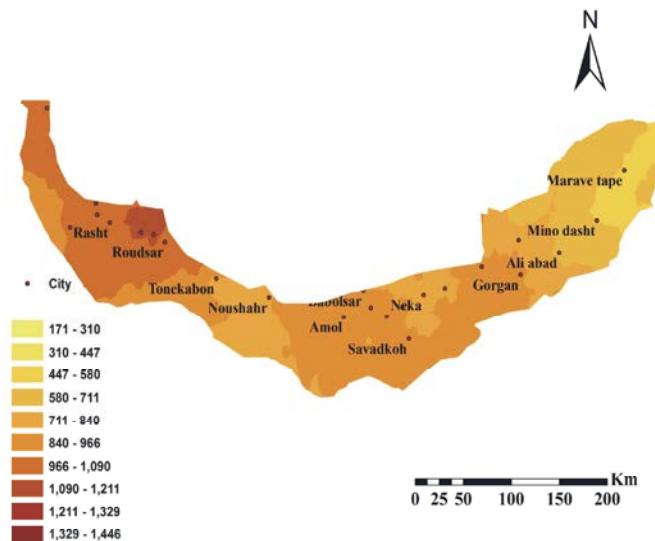


Fig. 7: Fuzzy kriging map of rainfall erosivity index

In all of interpolation maps of the rainfall erosivity index was exhibited a decreasing trend from west to east, it was according to climatic trend from humid to semi-arid (Figure 6, 7). The greatest surface area of watershed allocated to EI_{30} in the range of 858- 973 $MJ\ mm\ ha^{-1}\ h^{-1}\ y^{-1}$ at obtained map of fuzzy kriging.

CONCLUSIONS

The EI_{30} as rainfall erosivity factor of USLE equation is important in erosion control and soil conservation. Computation of EI_{30} based on rainfall intensity requires

long time intensity data at short intervals. But access to the short term intervals rainfall intensity in many parts of the world, especially in Iran, are limited [24]. While usually the data based on rainfall amount are available for longer periods. In this study, EI_{30} was estimated from modified Fournier index based on rainfall amount.

The ratio of nugget to sill of the EI_{30} values indicated strong spatial correlation between the rainfall erosivity index values in study area. Also, the fuzzy kriging method is more precise than kriging interpolation method by minimum MAE (169) and MBE (-1.77) (Table 2).

The highest EI_{30} was observed in the west of region with humid climate. The rainfall erosivity was shown decreasing trend from west (humid climate) to east (semi-arid climate) in the range of 171-1450 MJ mm ha⁻¹ h⁻¹ y⁻¹ (Figure 6, 7). Therefore, the spatial variations of rainfall erosivity become dependent on spatial variability of climate. The result of this study could be used in erosion models and conservation in this area with high rate of erosion.

ACKNOWLEDGMENTS

We are grateful to Iran Meteorological Office and Water Resources Management Company for providing the data sets.

REFERENCES

1. Wichmeier, W.H. and D.D. Smith, 1978. Predicting rainfall losses- a guide to conservation planning. USDA, Science and Education Administration, Agricultural Research, Agriculture Handbook, Washington DC. 537: 58.
2. Renard, K.G., G.R. Foster, G.A. Weesies, D.K. McCool and D.C. Yoder, 1997. Predicting Soil Erosion by Water: A Guide to Conservation Planning with the Revised Universal Soil Loss Equation (RUSLE). USDA Agr. Handbook. pp: 703.
3. Grauso, S., N. Diodato and V. Verrubbi, 2010. Calibrating a rainfall erosivity assessment model at regional scale in Mediterranean area. *Environ Earth Sci.*, 60: 1597-1606.
4. Khorsandi, N., M.H. Mahdian, E. Pazira and D. Nikkami, 2010. Comparison of rainfall erosivity indices in runoff-sediment plots in northern Iran. *World Applied Sciences Journal*, 10(8): 975-979.
5. Hoyos, N., P.R. Waylen and A. Jaramillo, 2005. Seasonal and spatial patterns of erosivity in a tropical watershed of the Colombian Andes. *Journal of Hydrology*, 317: 177-191.
6. Shamshad, A., M.N. Azhari, M.H. Isa, W.M.A. Wan Hussin and B.P. Parida, 2008. Development of an appropriate procedure for estimation of RUSLE EI_{30} index and preparation of erosivity maps for Pulau Penang in Peninsular Malaysia. *Catena*, 72: 423-432.
7. Salaski, A., 2007. Fuzzy approach to ecological data analysis. In the proceedings of the 8th WSEAS International Conference on Natural Network, Vancouver, British Columbia, Canada. pp: 19-21.
8. Tran, L.T., M.A. Ridgley and L. Duckstein, 2002. Application of fuzzy logic- based on the Revised Universal Soil Loss Equation. *Catena*, 47: 203-226.
9. Nanda, A., A.K. Rath, B. Dinda and R. Rath, 2011. Genetic fuzzy approach for prediction of coastal erosion. *International Journal of Computer Information Systems*, 2(2): 80-93.
10. Mahabir, C., F.E. Hicks and A. Robinson Fayek, 2003. Application of fuzzy logic to forecast seasonal runoff. *Hydrological Processes*, 17: 3749-3762.
11. Kisi, O., T. Haktanir, M. Ardiclioglu, O. Ozturk, E. Yalcin and S. Uludag, 2009. Adaptive neuro-fuzzy computing technique for suspended sediment estimation. *Advances in Engineering Software*, 40(6): 438-444.
12. Angulo-Martinez, M., M. Lopez-Vicente, S.M. Vicente-serrano and S. Begueria, 2009. Mapping rainfall erosivity at a regional scale: a comparison of interpolation method in the Ebro Basin (NE Spain). *Hydrology and Earth system sciences Discussions*, 6: 417-453.
13. Theodossiou, N. and P. Latinopoulos, 2006. Evaluation and optimization of groundwater observation networks using the kriging methodology. *Environ. Model. Softw.* 21: 991-1000.
14. Piotrowski, J.A., F. Bartels, A. Salski and G. Schmidt, 1996. Geostatistical regionalization of glacial aquitard thickness in northwestern Germany based on fuzzy kriging. *Math. Geol.*, 28: 437-452.
15. Rahimi Bandarabadi, S. and B. Saghaian, 2007. Estimating spatial distribution of rainfall by fuzzy set theory. *Iran Water Res.*, 3: 17-18.
16. Lark, R.M., 2000. Designing sampling grids from imprecise information on soil variability, an approach based on the fuzzy kriging variance. *Geoderma*, 98: 35-59.
17. Brown, L.C. and G.R. Foster, 1987. Storm erosivity using idealized intensity distributions. *Transactions of the American Society of Agricultural Engineers*, 30: 379-386.
18. Arnoldus, H.M.J., 1980. An approximation to the rainfall factor in the universal soil loss equation. In *Assessment of Erosion*. Eds. De Boodt, M. and M. Gabriels. Wiley, New York, pp: 127-132.
19. Mamdani, E.H. and S. Assilian, 1999. An Experiment in Linguistic Synthesis with a Fuzzy Logic Controller. *Int. J. Hum. Comput. Stud.*, 51(2): 135-147.

20. Men, M., Z. Yu and H. Xu, 2008. Study on the spatial pattern of rainfall erosivity based on geostatistics in Hebei Province, China. *Front. Agric. China*, 2(3): 281-289.
21. Goovaerts, P., 1999. Using elevation to aid the geostatistical mapping of rainfall erosivity. *Catena*, 34: 227-242.
22. Mcgrath, D., J.E. Zhang and L.T. Qu, 2004. Temporal and spatial distribution of sediment total organic carbon in an estuary river. *J. Environ. Qual.*, 35: 93-100.
23. Wang, G., G. Gertner, V. Singh, S. Shinkareva, P. Parysow and A. Anderson, 2002. Spatial and temporal prediction and uncertainty of soil loss using the revised universal soil loss equation: a case study of the rainfall-runoff erosivity R factor. *Ecological Modeling*, 153: 143-155.
24. Yu, B., G.M. Hashim and Z. Eusof, 2001. Estimating the R-factor with limited rainfall data: a case study from peninsular Malaysia. *Journal of Soil and Water Conservation*, 56: 101-105.