

## Enhancement of GPS Single Point Positioning Accuracy Using Referenced Network Stations

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**Abstract:** Positioning using the Global Positioning System (GPS) can be performed through two main techniques which are the Relative Positioning (RP) and Single Point Positioning (SPP). It is well known that the obtainable accuracy using RP is higher than that obtained using SPP. However, it implies the implementation of more than one GPS receiver. Accordingly, SPP is much cheaper and easier than RP. Many researches were devoted to the topic of increasing the accuracy of the SPP through classical techniques like increasing the length of the processed session, implementing the precise orbital information instead of the broadcasted one and the effect of turning off the Selective Availability (SA) in May 2000. Here in, the maximum achievable SPP accuracy (using classical techniques) was investigated using GPS real data. A certain test point was assigned in the core of an area of about 120 km x 92 km. Results proved that an accuracy in the order of 3m can be achieved using a session of 3.5 hours by implementing the precise orbital data for the case when the SA turned off. To increase the above achievable SPP accuracy, a new technique is introduced here called Referenced Network Stations (RNS) technique. This technique is based on the establishment of a reference network in the considered area, whose points are precisely determined. The considered reference network used here consists of 9 points with interspacing ranging from 25- 60 km. The SPP of the 9 reference points are determined every 3 minutes along one complete day. Then, the coordinate discrepancies are computed for the 9 reference points, every 3 minutes and stored in one data bank. Thus, the coordinate discrepancies are estimated at the test point, using the constructed data bank and then added to SPP coordinates of the test point to get the final solution for its position. Nine different prediction models were tested to reach at the most reliable one for estimating the coordinate discrepancies at the test point. Results showed that the weighted mean model, which raises the distances to the 7th power, is the superior one for this purpose. A positional accuracy at the test point of only 31 cm was achieved, by the developed RNS technique; using 3 minutes of GPS synchronized data at both reference points and test point. To generalize the application of the RNS technique, its ability to implement non-synchronized GPS data at the reference points and the test point was investigated. The performed tests had proved that the two most critical factors are the unification of the time (through the day) not the date itself between the reference points and the test point as well as the unification of the state of the SA at all points. Processing of 3 minutes of GPS data, collected at the same time but on different dates at both test and reference points, can yield an accuracy of 36 cm under the condition that the SA was at the same state in both campaigns.

**Key words:** GPS • Single point positioning • Positional accuracy • Reference Network Stations

### INTRODUCTION

The Global Positioning System (GPS) is rapidly replacing most of the traditional surveying techniques. This is due to the great flexible conditions of operation for the system like its ability to work 24 hours per day, un-necessity of inter-visibility and un-limitation for the separation between surveyed points... etc. e.g. [1].

On the other hand, the achievable positional accuracy using GPS is widely ranging from sub centimeter up to several tens of meters [2]. Such huge difference in positional accuracy is mainly function of two main factors which are the type of the used GPS receiver and the adopted observational technique [3]. Basically, there are two main types of GPS receivers which are handheld receivers and geodetic grade receivers.

Handheld GPS receivers observing Coarse Acquisition (C/A) code are generally much lower in positional accuracy and cheaper than geodetic-grade receivers. Positional accuracy of handheld C/A code observing receivers are at the tens of meters level [4], while on the other hand, geodetic-grade receivers collecting phase measurements produce results with sub centimeter level accuracy [5]. In the case of using GPS geodetic-grade receivers, two main techniques can be followed which are the Single Point Positioning (SPP) and Relative Positioning (RP). The SPP technique is meant by the process of determining the absolute 3-D coordinates of any point using stand-alone GPS receiver at such point. On the other hand, the RP is concerned with the determination of the differences in coordinates, between two or more points, using synchronized GPS observations at all concerned points [6]. This superiority in positional accuracy is due to the fact that the RP technique makes use of the spatial correlation of systematic errors between stations to estimate or reduce their effects in determining the highest positional accuracy [7].

Based on the above discussion it can be stated that, the SPP technique is much easier and has lower operational cost than the RP technique (as it implies the use of only one GPS receiver). However, RP has a higher accuracy significantly than SPP. This makes it of great importance in order to increase the achievable positional accuracy rather than SPP. Many researches were devoted to the task of investigating the different ways of increasing the accuracy of the SPP technique. Such researches investigated the effect of some observational and computational parameters on the achievable SPP accuracy like the used session length, the geometry of the used satellite constellation (expressed by the DOP), the effect of turning off the SA...etc. Such studies can be found in several researches [2, 8-11]. Among all these works, the highest achievable positional accuracy using SPP was in the order of 2 to 3 meters. In this study, many trials will be performed to increase the accuracy of SPP. At first, the classical approaches of increasing the SPP accuracy will be used through the investigation of the effect of three different parameters on the accuracy of the SPP. Such three parameters are the type of the used orbital information, the session length of the data capture process and effect of turning the SA off. The final output of these three trials will be considered as the maximum achievable SPP accuracy via classical approaches. At this stage, the new thought approach to increase the accuracy of the SPP will be outlined and tested using the same used test point. Finally, the different parameters affecting the efficiency of the proposed approach will be analyzed.

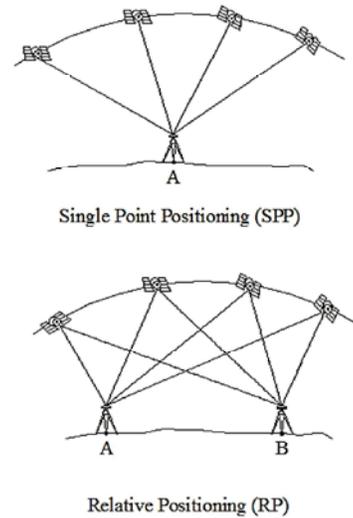


Fig. 1: Concept of SPP and RP

This is to pick out the most suitable conditions of performing the introduced approach.

**Single Point Positioning (SPP) Versus Relative Positioning (RP):** As mentioned above, SPP involves the use of only a single GPS receiver at one station location to collect data from multiple satellites in order to determine the location of the receiver, whereas the Relative Positioning (RP) employs two GPS receivers, simultaneously tracking the same satellites, to determine the difference in coordinates between the two receivers[12]. Figure 1 depicts the main idea of both SPP and RP techniques.

The collected GPS observations are contaminated by many biases like the satellite and receiver clock errors, tropospheric and ionospheric delays, multipath...etc. These biases are taking place in both SPP and RP techniques with the same manner and same magnitude. However, the effect of these biases on the resulted positional accuracy is completely different between such two techniques. This is due to the fact that the RP technique implies a differencing process for the collected GPS observations at the considered two stations. This differencing process leads to a great reduction in the value of the remaining biases, which leads in great reduction of the effect of different GPS biases. Based on the above discussion it can be stated that, the collected GPS data using both SPP and RP techniques are affected by GPS biases by the same amount. However, the effect of such biases is greatly reduced in the case of RP. A summary for the magnitude of different GPS biases for both SPP and RP are listed in Table 1 [10].

Table 1: Values of different GPS biases for both SPP and RP techniques

Bias	Bias magnitude (m)	
	SPP	RP
Satellite clock	3	0.0
Receiver clock	400	0.0
Tropospheric delay	1.8	0.2
Ionospheric delay	8.2	0.4
Orbital error	2.7	0.0
Multipath	0.6	0.6
Selective Availability	30.0	0.0
Receiver noise	0.3	0.3

By observing Table 1, it can be stated that, many GPS biases are totally removed when implementing the RP technique, whereas other biases are greatly reduced. The amount of reduction of the biases is inversely proportional to the length of the processed baseline [13]. In other words, the RP technique can result in better positional accuracies when applied for relatively short baselines. On the other hand, RP can lead to bad results in the case of long baselines. This fact can be considered as another main motivation behind undertaking the current research to increase the reliability of GPS positional accuracy using SPP to be adopted instead of the RP in the case of long baselines.

**Increasing the Accuracy of SPP Using Classical Techniques:** Many trials were performed to increase the accuracy of GPS SPP. These trials are different among each other in studying different parameters that affect the accuracy of SPP. However, they proved that neither the used ionospheric model nor the tropospheric model affect the accuracy of SPP [14]. Moreover, the performed trials proved that the accuracy of SPP is affected mainly by three different parameters which are the kind of the used orbital data, the length of the data acquisition session and the cancellation of the Selective Availability (SA). Therefore, before going through the new thought technique of increasing the accuracy of SPP, these three factors should be investigated first to explore the highest possible accuracy of SPP using these classical approaches. This highest possible accuracy of SPP can then be considered as a reference for the new thought approach.

**Description of the Used Data:** The used GPS data consisted of ten (10) GPS points, covering an area of about 120 km x 92 km. These points are located in Sinai and the western side of Suez Gulf. Figure 2 depicts the location and distribution of the used GPS network.

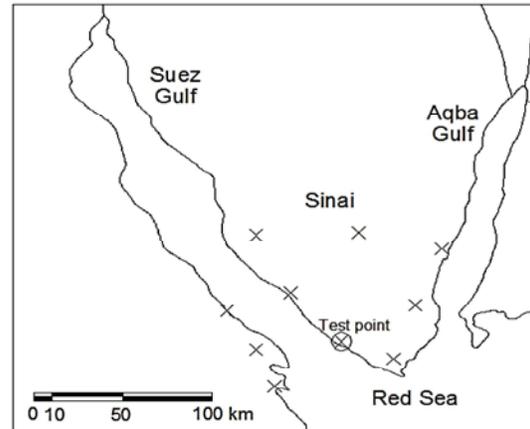


Fig. 2: Used GPS Data

The used GPS network is established by the National Research Institute of Astronomy and Geophysics (NRIAG) and it is observed periodically for the purpose of prediction of earthquake activity. The data of all points are available in Receiver Independent Exchange (RINEX) format throughout different months and years. One GPS point was assigned as a test point (indicated by the circle in Figure 2). The adjusted 3-D coordinates of the 10 points are known beforehand through a network adjustment solution for the whole network.

**Effect of Increasing the Length of the Observation Session:** It is well known that using a longer session of GPS observations will result in a better positional accuracy. In other words, GPS positional accuracy is directly proportional to the adopted GPS session length [3]. However, this increase in positional accuracy is limited, that is increasing the session length beyond a certain value would not increase the achievable positional accuracy. Accordingly, to investigate the effect of increasing the used session length, the 3-D coordinates of the indicated test point in Figure 2 are computed several times by varying the used session length. Every time, the used session length is increased by 15 minutes up to session lengths of 11 hours were explored. The 3-D coordinates of the test point is computed through the LEICA Geo Office software package using broad cast orbital information. The used data are collected on August 4<sup>th</sup>, 1999.

In this context, for each used session length, discrepancies are computed between the known reference coordinates of the test point and the computed coordinates using such session length. These discrepancies are computed as follows:

$$\delta X = X_{ref} - X_{spp} \quad (1)$$

$$\delta Y = Y_{ref} - Y_{spp} \quad (2)$$

$$\delta Z = Z_{ref} - Z_{spp} \quad (3)$$

$$d_p = \sqrt{\delta X^2 + \delta Y^2 + \delta Z^2} \quad (4)$$

Where:

$X_{ref}$ ,  $Y_{ref}$  and  $Z_{ref}$  = The reference coordinates of the test point

$X_{SPP}$ ,  $Y_{SPP}$  and  $Z_{SPP}$  = The 3-D Cartesian coordinates of the test point, estimated in SPP mode

$\delta_x$ ,  $\delta_y$  and  $\delta_z$  = 3-D components of the coordinate discrepancy vector

$\delta_p$  = Positional discrepancy

Based on the above, 44 sets of discrepancies are estimated. The relation between the used session length and the resulted discrepancies is depicted in Figure 3.

Through a quick glance on Figure 3, it can be stated that, increasing the used session length leads to a significant increase in the accuracy of the SPP. This is valid up to a certain limit. Such limit is indicated in Figure 3 by the vertical dashed line (separating regions A and B). Thus, increasing the session length increases the accuracy of the SPP (region A). Yet, this increase in accuracy is observed up to about a session length of 8 hours (the vertical dashed line). Beyond this limit, increasing the session length will not produce any increase in the SPP accuracy (region B). As a conclusion, using the broadcast orbital information, the best achievable positional accuracy of SPP reaches the order of 10 m.

**Effect of Changing Used Orbital Information:** Another attempt is performed here to increase the accuracy of SPP by using the precise orbital information instead of the broadcasted ones. This precise orbital information was downloaded from the IGS site [15]. For convenience, the same criteria, data, session length are used as the previous trial. The obtained results are depicted in Figure 4.

By observing Figure 4 it can be seen that, with the application of the precise orbit instead of the broadcasted orbit, the solution stabled after about six hours (noticed by the position of the vertical dashed line). In addition, a positional accuracy in the order of 4m can be achieved using the precise orbital information.

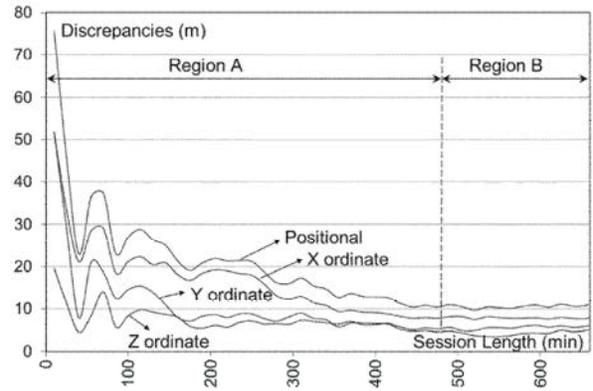


Fig. 3: Effect of the used session length on the accuracy of SPP

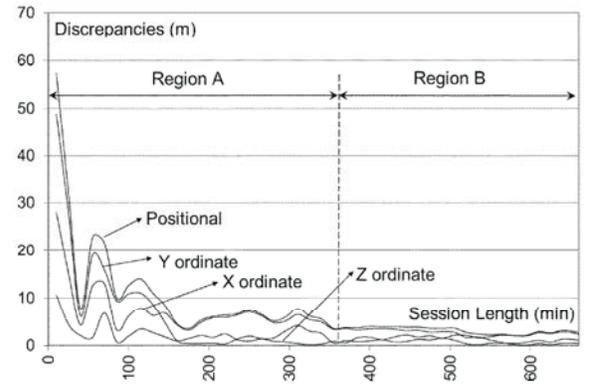


Fig. 4: Effect of the used type of orbital information on the accuracy of SPP

**Effect of Turning off the Selective Availability (SA):**

The Selective Availability (SA) was turned off by the U.S. government on May 1<sup>st</sup>, 2000 [2]. As a result, the accuracy of the SPP is enhanced. To ensure such enhancement, two datasets (collected on August 4<sup>th</sup>, 1999) and on the same date in the next year (after turning the SA off) were processed, while using the same test point, previous criteria and session length, utilizing precise orbital data. Figure 5 summarizes the results of this test, showing here only the positional discrepancies.

Based on Figure 5, it is very evident that, turning the SA off resulted in a significant enhancement in the accuracy of SPP. This enhancement can be noticed in two different forms. First, the solution stabilized after shorter time period, which is about 3.5 hours (denoted by the vertical dashed line). Secondly, after the solution reaches the stability level, the final positional accuracy is raised to the level of about 3m.

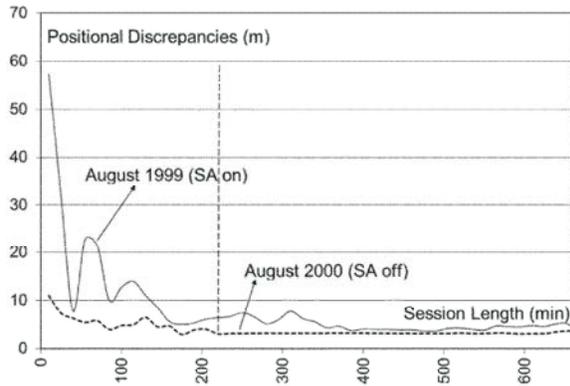


Fig. 5: Effect of turning off the SA on the accuracy of SPP

**Closing Remarks Concerning SPP Accuracy Using Classical Techniques:** The above performed three tests clarify the previously known facts concerning the accuracy of SPP. However, it was of great importance to perform such tests before going through the main investigation of increasing the accuracy of SPP, in order to explore the maximum achieved positional accuracy of SPP using classical techniques. The results of the three tests proved that nowadays, with the absence of the SA, a positional accuracy in the order of 3m can be achieved using stand-alone GPS receiver by adopting precise orbital data whilst using a minimum session of 3.5 hours. Of course, neither this accuracy nor the required session length is suitable for most of GPS users. Consequently, another approach should be established to increase the efficiency of the SPP.

**Increasing the Accuracy of SPP Using Reference Network Stations (RNS) Technique:** As it is well known, the basic reason behind the higher accuracy of relative positioning (RP) when compared to SPP is the minimization of a multitude of biases that affect all GPS observations, basically due to the differencing process between the considered stations in the case of RP. In case of SPP, the 3-D coordinates of the considered station are largely affected by these biases, especially those who do not have a reliable estimation model. Based on the above discussion and previously performed tests it can be stated that, the 3-D coordinates derived using SPP are largely deviated from the correct values. Such deviations can be referenced to the various famous GPS biases (tropospheric and ionospheric effects, orbital errors... etc.). However, if a reliable estimation for the deviations of such 3-D coordinates is performed, the efficiency of SPP technique can then be enhanced significantly.

This is the main idea of the proposed technique, which will be introduced in the next section.

**Basic Idea of the Proposed Technique:** The main motivation here is to increase the accuracy of SPP, based on the existence of a reference network surrounding -or in the vicinity of- the point(s) that shall be positioned using SPP. The 3-D adjusted coordinates of the points of this reference network should be known beforehand. After which, all points of such reference network are positioned using SPP. Then, the coordinate discrepancies are computed and stored in a data bank for all the adopted reference stations using equations (1, 2 and 3). At this stage, the coordinate discrepancies or in other word “corrections” are estimated at the considered point(s) using the computed discrepancies at the reference stations. Finally, the final coordinates of the considered point(s) are computed by adding the estimated corrections to the coordinates obtained via classical SPP technique. Based on the above mentioned idea, it can be stated that, the main factor affecting the efficiency of the proposed technique is the reliability of the estimated discrepancies at the considered point(s). In other words, the degree of dependency of these discrepancies at the considered point(s) on those at the reference points is the main key of this technique. The estimation of the coordinate discrepancies at the considered point(s) can be performed using many different prediction techniques. In the following, the different investigated prediction models will be summarized.

**Different Used Prediction Models of Coordinate Discrepancies:** Nine different prediction models are investigated, in order to find out the most suitable model. The used prediction models are the mean, weighted mean, nearest neighbor, search radius, triangulation, polynomial model, logarithmic trend, power trend and exponential trend. Mathematical details of these prediction models can be found in many researches, e.g. [16, 17]. However, all used models are briefly explained in the appendix given at the end of this research. For any tested prediction model, each coordinate along with its corresponding discrepancy is modeled as a separate variable. Consequently, for each one of the applied nine prediction models, three different processing methodologies will be performed corresponding to the three spatial coordinates X, Y and Z. In other words, the X, Y and Z-coordinates of the reference points are fitted with the X, Y and Z-coordinate discrepancies respectively, using the adopted prediction model.

**Application of Different Prediction Models and Analysis of the Results:**

Referring to Figure 2, the used GPS data consists of 10 points. The 3-D Cartesian coordinates of these 10 points are known beforehand. Among these 10 points, only one point is assigned as the test point. Accordingly, in all the sub-sequent analysis, 9 data points will be involved along with one central test point. All the 10 points are processed individually, using SPP technique, considering the longest possible session length (11 hours), while processing precise orbits. The main idea here can be summarized in the following steps:

- Estimation of the coordinate discrepancies at the 9 reference stations using equations 1, 2 and 3.
- Prediction of the three coordinate corrections at the test point, using the corresponding coordinates and discrepancies at the 9 reference points, using the adopted prediction model.
- Estimation of the final coordinates of the test point by adding the predicted corrections to its produced SPP coordinates.
- Accuracy assessment of the applied prediction model by comparing the resulted final coordinates of the test point with its known precise coordinates. This step is controlled by the computation of the resultant positional discrepancy.

The above mentioned nine prediction models are applied. Each one of these nine prediction models were investigated once except three models which are the search radius, weighted mean and polynomial model. For each one these three models, it is applied several times by varying its order or vicinity range. Concerning the search radius model, radii of 30, 40, 50, 60, 70, 80 and 90 kilometers are used. For the weighted mean model, the distances are weighted using powers started from 1 to 10. Finally, for the polynomial model, polynomials of the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> orders were explored. This is mainly done in order to find out the most reliable condition of application for each of these three models. Figures 6, 7 and 8 summarize the results of the search radius, weighted mean and polynomial models, respectively.

By noticing Figures 6, 7 and 8, it can be concluded that, using a search radius of 40 km yields the best results among all examined radii. For the weighted mean technique, weighting the distances by the power 7, gives the most reliable SPP positions for the test point when compared to the other tested powers. Finally, concerning the polynomial model, the 2<sup>nd</sup> order polynomial can be considered as the optimal one that can be used to predict

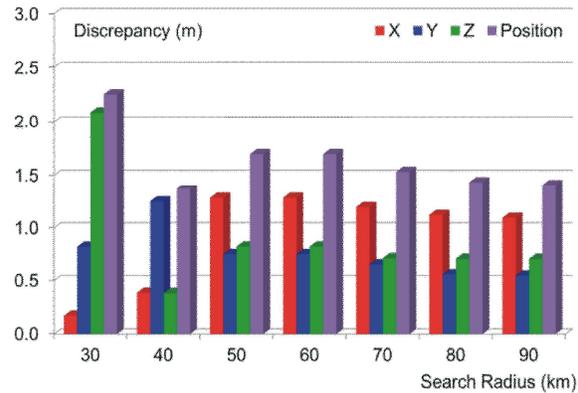


Fig. 6: Results of the search radius prediction model

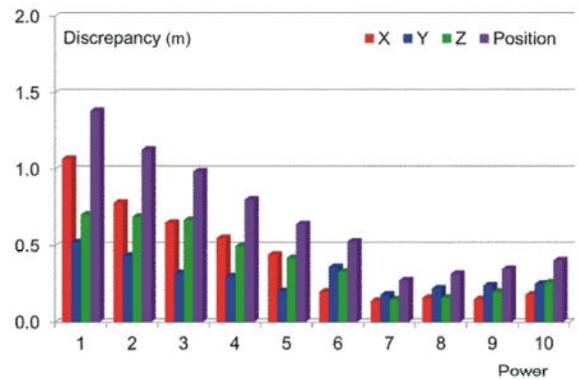


Fig. 7: Results of the weighted mean prediction model

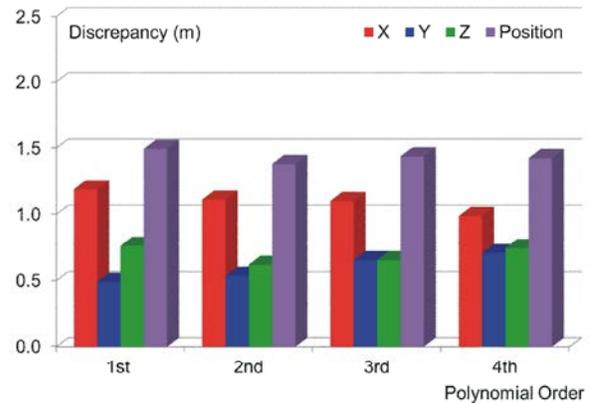


Fig. 8: Results of the polynomial prediction model

the coordinate corrections at the test point. Based on the results from the latter three figures, the 40 km search radius, the power 7 for the weighted mean model and the 2<sup>nd</sup> order polynomials will be assigned to represent the search radius method, the weighted mean and polynomial models, respectively.

Referring again the nine applied prediction model and after selecting the most reliable configuration for the three variant models, the results of all nine models are listed in

Table 2: Summary of the results of all tested models

Prediction Model	Discrepancies (m)			
	X	Y	Z	Position
Nearest Neighbor	0.39	1.24	0.39	1.36
Mean	1.09	0.55	0.70	1.41
Triangulation	0.88	0.26	0.78	1.20
Logarithmic Trend	1.16	0.33	0.76	1.42
Power Trend	1.28	0.2	0.26	1.32
Exponential Trend	1.16	0.31	0.17	1.21
Search Radius (40 km)	0.39	1.24	0.39	1.36
Weighted Mean (Power 7)	0.14	0.18	0.15	0.27
Polynomial (2nd order)	1.11	0.54	0.62	1.38

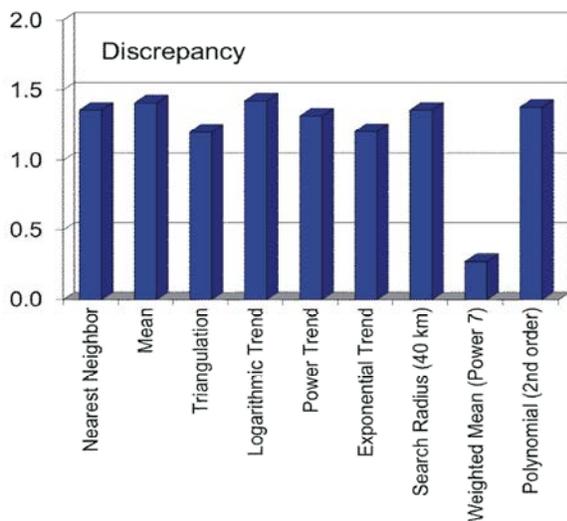


Fig. 9: Positional discrepancies for the nine tested models

Table 2. Also, for simplicity, only the positional discrepancies are depicted in Figure 9 for all nine tested models.

Based on Table 2 and Figure 9, it can be seen that, the application of the weighted mean prediction model (with the power 7) in the estimation of the coordinate discrepancies, using the corresponding discrepancies at the reference points, yields the highest possible positional accuracy. A positional error of few decimeters level is obtained using such technique. Accordingly, the Reference Network Stations (RNS) technique (in its current form) can increase the accuracy of the SPP from 3m (using the classical approaches) to the few decimeters level (0.27m).

**Effect of the Length of Processed Data on the Quality of RNS Technique:** The above achieved sub-meter accuracy for the SPP, using the developed RNS technique, was based on the implementation of the longest available

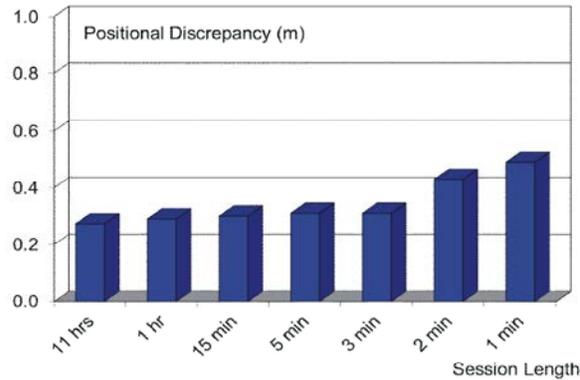


Fig. 10: Effect of the session length on the efficiency of the RNS technique

session length (which is 11 hours). Of course, this time span is very long and it is impractical to consider it in any SPP application. The question arises now is "Can this time span be reduced and to which extent?". To answer the above question, the RNS technique is applied many times, using GPS synchronized data of different lengths, applying the most efficient weighted mean prediction model with 7<sup>th</sup> power along with precise orbital data. Data sessions of 11 hours, 1 hour, 15 minutes, 5 minutes, 3 minutes, 2 minutes and 1 minute are used. Results are depicted in Figure 10.

Based on Figure 10, it can be seen that, in the introduced RNS technique, the effect of decreasing the length of the considered session on the resulted positional accuracy is very slight. This is valid up to a session length of 3 minutes. Considering session lengths shorter than 3 minutes was found to degrade the efficiency of the RNS technique. Based on this, a session length of 3 minutes can be considered as the optimum session length for the proposed technique. Note that data sessions of 6 hours and 3.5 hours were also investigated and possessed almost the same results, which is evident here also by the similar accuracy of 11hours and 1 hour data session.

**Efficiency of the RNS Technique for Non-synchronized Data at Both Reference Points and Test Point:** The introduced RNS technique had proved relatively good results for SPP, even with session lengths of only 3 minutes. These results were obtained using GPS synchronized data at both the reference points and the test point. However, it is impractical to apply the RNS technique in its current form. This is due to the simple fact that if a synchronized GPS data is available at both reference and test points, one should go through a RP

scenario. Therefore, the main interest of the introduced technique can be expressed by its ability of application for non-synchronized data at both the reference points and the test point. Two different types of non-synchronicity between reference points and test point should be distinguished here, which are:

**Type (A)**

**Hour Non-Synchronicity:** In this case, GPS data are collected at the reference points and the test point at different time interval (same day but different hour).

**Type (B)**

**Date Non-Synchronicity:** In this case, GPS data are collected at the reference points and the test point at the same time but in different dates.

Based on the above discussion, the efficiency of the RNS technique should be checked for both types of non-synchronicity. To achieve such a goal, a data bank is constructed for the reference network. Such data bank is constructed using the available 11 hours of GPS data, containing the SPP coordinates of the 9 reference points computed every 3 minutes at the start of each hour, using precise orbital information. Then the 3-D coordinate corrections for the 9 reference points are estimated 11 times for the start of each hour. This data bank will be used now in checking the efficiency of the RNS technique for both types of non-synchronized GPS data in two different tests.

**Efficiency of the RNS Technique in Case of Hour Non-Synchronicity:**

This test is performed to check the reliability of the developed RNS technique when applied on data collected at the reference points at a certain hour and collected at the test point at a different hour (within the same day). The SPP coordinates of the test point are estimated using the optimum session length (which was determined as 3 minutes). These 3 minutes were selected to be the first 3 minutes in the first hour of observations on the same day used for constructing the reference stations data bank (August 4<sup>th</sup>, 1999). Using the established data bank, 11 different sets of coordinate corrections are used in this test, corresponding to the first 3 minutes in each hour of observations. The coordinate discrepancies are estimated at the test point, using the concluded best model (weighted mean with power 7), using each set of corrections. Consequently, 11 different solutions now exist for the test point. Finally, the positional discrepancy at the test point is computed for each corrections set. Results are depicted in Figure 11.

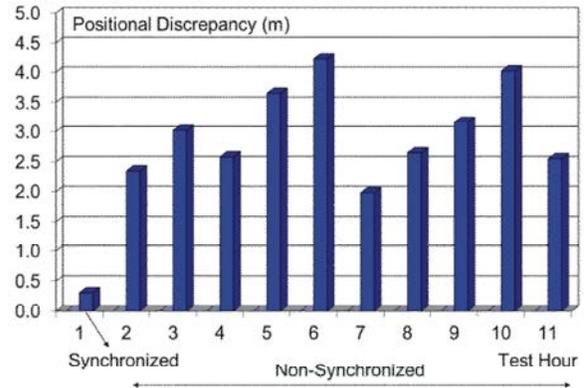


Fig. 11: Efficiency of the RNS technique in case of hourly non-synchronized data

By observing Figure 11, it is very evident that, the developed RNS technique failed to get a reliable SPP solution for the test point when an hourly non-synchronized data is used. This is denoted by the large positional discrepancies obtained in all the tested hours except the first one (positional accuracy=0.31m), which is the only hour having synchronized data. Note that, this test was similarly applied on the first three minutes of the 2<sup>nd</sup>, 3<sup>rd</sup>,....., 11<sup>th</sup> hour of data of the test point and possessed typical results and conclusions. This expected failure can be simply referenced to the complete un-correlation (independency) between the values of different GPS biases at both the reference points and the test point, especially multipath due to the effect of sidereal lag [18]. In other words, for the case of hourly non-synchronized data, different GPS biases will be certainly different in magnitude and behavior at the reference points and the test point. Consequently, the estimation of the coordinate discrepancies at the test point using the corresponding discrepancies at the reference points will not represent the reality. Hence, another approach should be followed to enable the application of the RNS technique for GPS non-synchronized data.

**Efficiency of the RNS Technique in Case of Date Non-Synchronicity:**

Here, the efficiency of the RNS technique will be evaluated for date non-synchronized data at both the reference points and the test point. This means that the used SPP coordinates of the reference points and the test point are computed using the same 3 minutes of observations but on different dates. Specifically, the coordinate corrections at the reference points are extracted from the prepared data bank (constructed on

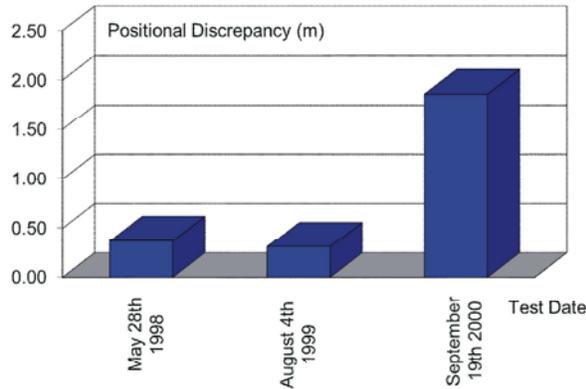


Fig. 12: Efficiency of the RNS technique in case of date non-synchronized data (Using the data Bank of 1999)

August 4<sup>th</sup>, 1999) for the first three minutes. Concerning the test point, its SPP coordinates are computed, using the same three minutes, on three different dates, which are:

- May 28<sup>th</sup>, 1998
- August 4<sup>th</sup>, 1999 (Same date of the reference points)
- September 19<sup>th</sup>, 2000

For every one of the considered three dates, the SPP coordinates of the test point are computed (using the same 3 minutes at the first hour at the reference points). The same used procedures in the previous test are followed to predict the coordinate discrepancies at the test point. Results are summarized in Figure 12.

Based on Figure 12, two very important notices can be observed. First, the RNS technique gets a good result for the data collected on May 28<sup>th</sup>, 1998 and of course August 4<sup>th</sup>, 1999 using the data bank constructed for the reference points using August 4<sup>th</sup>, 1999 data. Only a very slight degradation between such two dates is observed in the resulted positional discrepancy (0.37 m instead of 0.31 m). Such slight change in the resulted accuracy can be referenced to the change in the ionospheric effect on annual basis. On the other hand, the RNS failed for the data of September 19<sup>th</sup>, 2000. A positional discrepancy in the order of 2m can be observed in Figure 12. Such large discrepancy can be interpreted by the fact that the data bank was constructed using the observations of the year 1999 (Where SA was still on), where as the used data at the test point was collected where SA was turned off. Therefore, the constructed corrections data bank cannot be used for any GPS data collected after May 1<sup>st</sup>, 2000 (The date of turning the SA off). Based on the above

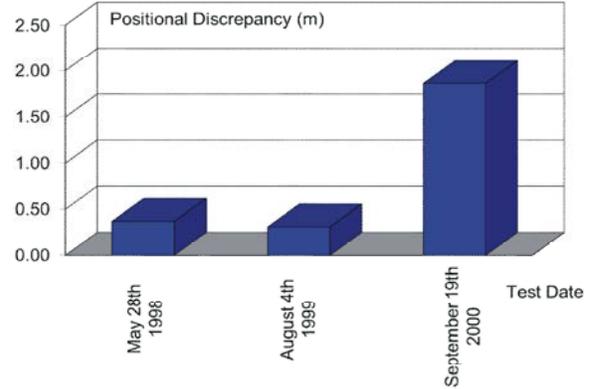


Fig. 13: Efficiency of the RNS technique in case of date non-synchronized data (Using the data Banks of 1999 and 2001)

discussion, another attempt is made to check the validity of the RNS technique for the data collected after turning the SA off. Another correction data bank is constructed for the 9 reference points, using the same procedure. The only change here is that the SPP coordinates involved in the new data bank are computed using data collected on October 12<sup>th</sup>, 2001 (After turning SA off). The test point was processed at the same previous three dates of Figure 12. However, the first two dates (1998 and 1999) were processed using the correction data bank of 1999, where the third date (2000) was processed using the new correction data bank (constructed using 2001 data). Results are given in Figure 13.

By Figure 13, it can be stated that, when updating the data bank after turning the SA off, the RNS technique succeeded in getting a reliable estimation for the test point in SPP mode with positional accuracy of 0.36m. This was also examined on a dataset of the test point after October 12<sup>th</sup>, 2001 and for assurance, possessed the same result. Note here that the consistent results obtained in Figure 13 are conditioned by using the appropriate data bank for each date based on the state of SA.

## CONCLUSIONS

Based on the performed trials, analysis and obtained results, many important conclusions can be extracted concerning the accuracy of SPP using both classical approaches and the proposed RNS technique. Such conclusions can be summarized as follows:

- Considering the broadcast orbital data, the maximum achievable positional accuracy using SPP can reach the order of about 10m by using a session of 8 hours.

- Considering the precise orbital data, SPP positional accuracy can reach the order of 4m using a session of 6 hours.
- Turning the SA off increases the positional accuracy of SPP to about 3m using a session of 3.5 hours. This result is obtained using precise orbital data.
- The developed RNS technique can improve SPP positional accuracy to the few decimeters level. This technique is based on the estimation of the coordinate corrections at the considered point, using the corresponding discrepancies at some surrounding reference points, using a certain prediction model.
- In the considered area (which is about 120 km x 92 km), the weighted mean prediction model (with raising the distances to the 7<sup>th</sup> power) is the superior one in the RNS technique.
- For the considered area, the implementation of 9 reference points can yield a positional accuracy for SPP in the order of 27 cm. This result was obtained using 11 hours of synchronized GPS data at both the reference points and the test point.
- In the developed RNS technique, a positional SPP accuracy of 31 cm can be obtained using 3 minutes of synchronized GPS data at both the reference points and the test point.
- Reducing the session length below 3 minutes degrades the efficiency of the RNS technique significantly.
- The application of the RNS technique is based on the construction of a data bank for the considered reference network. Such data bank should contain the coordinate discrepancies between the adjusted coordinates and SPP coordinates of the reference points, obtained every 3 minutes, within one complete day.
- In the RNS technique, coordinate discrepancies at the considered point cannot be interpolated from the corresponding discrepancies, at the reference points, computed at different time (hour) during the day.
- Interpolating the coordinate discrepancies at the considered point using discrepancies at the surrounding reference points computed at the same time within the day but in a different date yields an accuracy of 37 cm for SPP.
- The appropriate application of the developed RNS technique is based on the implementation of the suitable data base. Such data base should be constructed in the same state of SA (on or off, which is now mainly off) as the considered test point.

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**APPENDIX**

**Mean**

$$X_{\text{corr}} = \frac{X_1 + X_2 + \dots + X_n}{n} \tag{5}$$

Where  $X_{\text{corr}}$  is the Correction in X-Coordinate, calculated as a function in number of network points. Note that similar equations can be written for Y and Z Coordinates for all used prediction models.

**Weighted Mean**

$$X_{\text{corr}} = \frac{P_1 X_1 + P_2 X_2 + \dots + P_n X_n}{n} \tag{6}$$

where P is the weight of each Coordinate correction and can be calculated as follows:  $P = \frac{1}{L^m}$ , where L is the distance from each network point to the required test point, while m is the power of this distance starting from 2, 3, ..., 8, 9.

**Nearest Neighbor**

$$X_{\text{corr}} = X_{\text{nb}} \tag{7}$$

Where  $X_{\text{nb}}$  is the X-Correction at the nearest network point to the test point.

**Search Radius:** Taking different values for circles radii, where the test point is the center of this circle and thus calculating a weighted mean for its correction using equation (6) (for all network points within this circle) with a certain power. Here in this power is taken as the 7<sup>th</sup> power as agreed upon in page 6.

**Triangulation:**

$$X_{\text{corr}} = \frac{(X_2 - X_0)(X_3 - X_2) - (X_2 - X_0)(X_3 - X_2)}{(X_2 - X_1)(X_3 - X_2) - (X_2 - X_1)(X_3 - X_2)}(X_1 - X_0) + \frac{(X_3 - X_1)(X_1 - X_3) - (X_3 - X_0)(X_1 - X_3)}{(X_3 - X_2)(X_1 - X_3) - (X_3 - X_2)(X_1 - X_3)}(X_2 - X_0) + \frac{(X_1 - X_0)(X_2 - X_1) - (X_1 - X_0)(X_2 - X_1)}{(X_1 - X_3)(X_2 - X_1) - (X_1 - X_3)(X_2 - X_1)}(X_2 - X_0) \tag{8}$$

Where  $X_1, Y_1, X_2, Y_2, X_3, Y_3$  are the X and Y coordinates of the three points forming a triangle surrounding the test point, while  $X_0$  is the X-Coordinate of the test point. Note that if more than one triangle is available, then the best solution regarding the test point accuracy is considered.

**Polynomial Model:**

$$X_{\text{corr}} = \begin{matrix} aX_0 + b & 1^{\text{st}} \text{ Order} \\ aX_0^2 + bX_0 + c & 2^{\text{nd}} \text{ Order} \\ aX_0^2 + bX_0^2 + cX_0 & 3^{\text{rd}} \text{ Order} \end{matrix} \tag{9}$$

Where in each case of 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> order Polynomials, the values of a, b, c and d are determined through least square adjustment using all available network points. Note that,  $X_0$  is the X-Coordinate of the test point substituted by in the previous equation, in order to obtain the correction at this point.

**Logarithmic Trend:**

$$X_{\text{corr}} = a \ln X_0^b \tag{10}$$

**Power Trend:**

$$X_{\text{corr}} = aX_0^b \quad (11)$$

**Exponential Trend:**

$$X_{\text{corr}} = a^{e^{bx_0}} \quad (12)$$

Where in the previous three models, the values of a and b are again determined through least square adjustment using all available network points.