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Clustering of Hyperspectral Image Data by Constructing a Diffuse Map

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Abstract: One of the problems of image processing and machine learning problems is the possibility of separating the classes. In our case, the proposed map is constructed in which the spectra of different surfaces areas associated with different materials, is clearly divided. In this approach, we rely on the method of "diffuse map". The basic idea is that the multidimensional data are projected into the mathematical variety of small dimension while preserving the mutual relations between the data. We describe a diffuse process that reveals hidden patterns that exist between the spectra of separating the different layers of the background. The model is based on a random walk on a graph. A walk on the graph is in the areas of condensation density. Random walk divides the region into separate clusters, which are caused by hidden relationships between elements of the set. As a result, we have used the technique of stochastic processes in the Markov chains for the separation of the clusters and identify relationships between data.

Key words: Diffuse map · Walk on the graph · Weight functions · Background · Clustering.

INTRODUCTION

The main objective of image processing is the detection of indeterminate zone area as belonging to a particular known class.

Recognition technology is represented by a set of basic algorithms: AdaBoost, SVM, Neural networks, Linear Discriminate Analysis. After analyzing these algorithms can be concluded that the existing methods of detection of indeterminate zones hyperspectral images are not as effective. Therefore, the solution of problems of recognition of indeterminate zones hyperspectral images remain relevant.

In our case, the solution of problems of recognition of indeterminate zones in hyperspectral imaging is proposed to construct a map in which the spectra of different surfaces areas associated with different materials, is clearly divided.

Methodology: In a considered approach we will refer to the method of «diffuse maps», described in [1] (Ronald R. Coifman, Stephane Lafon Diffusion maps, Appl. Comput. Harmon. Anal, Mathematics Department, Yale University, New Haven, CT 06520, USA). The method is widely considered in [2-5].

This method was applied for the first time to the modeling of the three-dimensional objects on the basis of the multiple representations of an object by the two-dimensional projections (photos). The essence of the method is that the multidimensional data are projected in the mathematical varifold of small dimension with the preservation of the mutual relations between data. Thus the topology of the varifold is modeling the distinction between the projections. That is, the variation of data is described by the varifold built by the diffuse map. In a case when the varifold is three-dimensional, it is described by the three-dimensional model of the projections.

Description of the Algorithm: In our case we must construct such a function at which the spectra of various surfaces of areas connected with various materials will be clearly divided. In this article we will describe the diffuse process which will reveal the hidden regularities existing between the spectra dividing various layers of background.

The model which we offer is based on the random walk on the graph. The basis of this algorithm are the model proposed by more than half a century ago in the works

Let's mark

$$I = \{L_{ij}\}\ j = 1.....r{:}\ I = 1.....mj$$

The set of the objects presented by multidimensional vectors. The first index marks the numbering of objects in a class, the second index numbers the classes themselves.

Each object is a multidimensional vector of the dimension n.

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$$I_{ij} \underline{\Delta} \left(p_{ij}^{1} ......p_{ij}^{n} \right) \in R^{n} .j = 1........r : i = 1......m_{i}$$

Let's construct a graph

$$G = (V.E). \tag{1}$$

where the set of points corresponds to the objects

$$\{L_{ij}\}\ j = 1.....r;\ I = 1.....mj$$

And a set of edges

$$E = \omega_{\varepsilon} \left(I_{i1.j1} . I_{i2.j2} \right)$$

is a measure of the local proximity between the vectors induced by the rate L_2. This can be illustrated with the following figure:

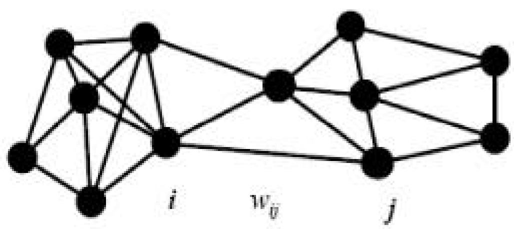

Figure 1. Graph (1). Points are the objects of the system, the edges are the existence of the local aboutness between the objects in the rate L_2

Then we initialize a random walk of a point on the graph (1). The walk on the graph occurs in the areas of density condensation, because the probabilities of the transition from one knot to another in dense sites is higher than the transition from one point of condensation to another [6].

Thus, the random walk divides all the area V into separate clusters which are caused by the hidden interrelations between the elements of the I-set.

Now we will give more formal axiomatic definition of the weight functions connected with the edges of the graph (1). The function ω_ε is defined as a correlation of the local aboutness between the points of the graph possessing of the following properties. For all x.y□I the weight function possesses the following properties:

- Symmetry. $\omega_\varepsilon\ (x.y) = \omega_\varepsilon\ (y.x)$
- Not negativity. $\omega_\varepsilon\ (x.y) \geq 0$
- Property of sparseness. Take as given the real positive number $\varepsilon > 0$. Then in case of $\|x\text{-}y\|$ we have $\omega_\varepsilon\ (x.y) \to 0$, otherwise, if $\|x\text{-}y\| \ll \varepsilon$, a property is carried out $\omega_\varepsilon\ (x.y) \to 1$.

The parameter ε sets the local structure of a neighborhood. The function ω_ε sets the local geometry of the similarity of pair of objects in a radius neighborhood ε.

As a rule « the Gaussian kernel » is chosen as a function ω_ε [7].

$$\omega_{\varepsilon}\left(x.y\right) = \exp(-\frac{\left\|x - y\right\|^{2}}{2\varepsilon} \tag{2}$$

Now let's describe the SP of walk on the graph (1). Let's determine the weight of each point of the graph

$$d\left(x\right) = \sum_{j=1}^{m} \omega_{\varepsilon}\left(x.y\right)$$

We normalize the weight functions ω_ε as lines of a stochastic Markov matrix. More formally, we will consider a matrix

$$P = \{P(x.y)\}_{i,j=1....m}$$

determined as

$$p(x.y) = \frac{\omega_{\varepsilon}\left(x.y\right)}{d\left(x\right)}$$

The size P (x.y) can be interpreted as the probability of the transition from a point x to a point y in one step. Let's define now the probability of the transition from X to Y for the time t $p_t(x.y)$. [8] The matrix made of the elements $p_t(x.y)$ sets the splitting of the graph into clusters at the convergence of the parameter t to infinity.

Indeed if the point was in a position x, then with the greatest probability in a large number of steps the random walk will lead towards those points which form one cluster with x. Thus, all points y for which $p_t(x.y)$ is close to 1 form one cluster with x. Thus the points y relating to another cluster will be separated from x by the property of proximity to 0 of the function $p_t(x.y)$ [9].

The following figure illustrates the efficiency of the division of the mixed known clusters. If to generate data as two rings linked with each other (designated by red and green color), there is no known linear methods that can obviously separate them. Nevertheless the random walk on the graph presented by these rings, is logically capable to separate them, as at the random walk the probability to remain in the same ring is higher than the probability to transit from one ring into another.

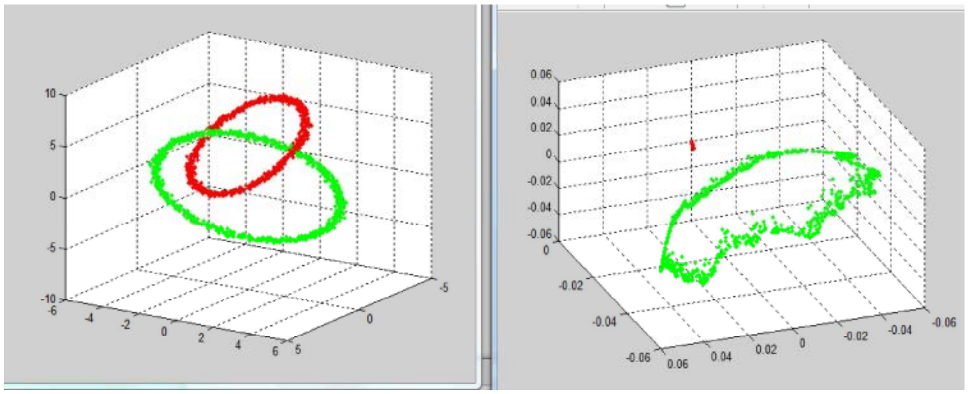

Figure 2. Demonstration of efficiency of division of classes by means of diffuse maps. There are the basic data on the left and the representation of diffusive maps use on the right.

For example, for the vectors presented in Figure 3, it is obvious that the groups on the right and on the left belong to different clusters.

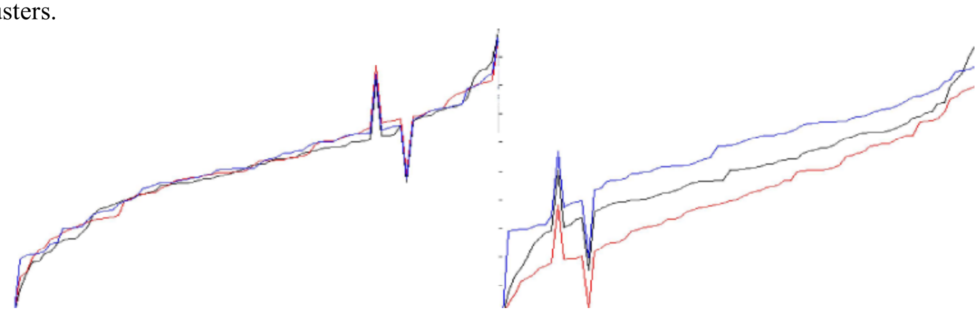

Figure 3. Two groups of spectra presented by graphs on the right and on the left, respectively.

For this example the matrix $p_t(x.y)$ where t=10 will be presented as shown in the Figure 4.

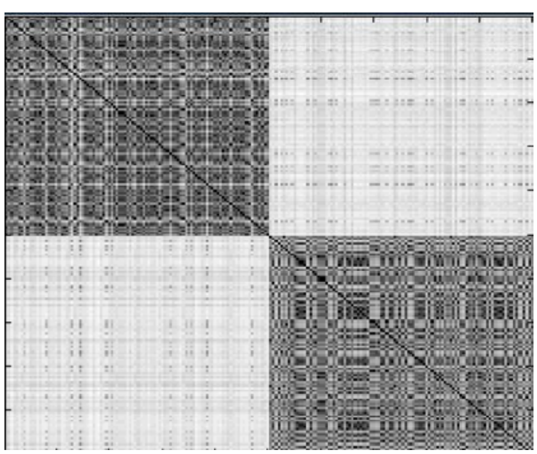

Figure 4. Matrix $p_t(x.y)$ where t=10.

As it has already been shown in [10], the diffuse distance $p_t(x.y)$ can be calculated by the following formula

$$p_t^2(x.y) = \sum \lambda_j^{2t}\left(\psi_j(x) - \psi_j(y)\right)$$

where λ1, λ2,..λm - are the eingenvalues of the matrix P, where

$$|\lambda_1| \geq |\lambda_2| \geq |\lambda_m|$$

and ψ1, ψ2,… ψm - are the corresponding proper vectors. Omitting terms at eingenvalues close to 0 and leaving unaltered the first w of the most significant summands, we will have

$$p_t(x.y) = \sum_{k=1}^{\omega}\left(\lambda_j^{2t}\left(?_j(x) - ?_j(y)\right)\right)$$

Let's define the mapping

$$?_t(x) = \left(\lambda_1^t\left(?_1(x)....\lambda_\omega^t\left(?_\omega(x)\right)\right.\right.$$

- It is easy to note that it has the following properties:
- The mapping occurs in the space of the dimension w.
- The mapping is not linear.
- The distance between the images of the points is equal to the diffuse distance, that is, to the probability to get from a point x to a point y at the random walk on the graph (1) for the time t. Let us call such mapping «The diffuse map».

RESULTS

The results of the application are shown in Figures 5 and 6. Shows the division of clusters after the application of diffuse maps. One color signifies spectra of one and the same background site. It is evident that as the spectra are differently directed and noisy the clusters are completely mixed and the classification of them is impossible. The method provides the mapping of the spectra in the virtual space of the relations and after the dimension decrease the space of spectra is projected in the three-dimensional space where the images of the spectra relating to one material are well divided.

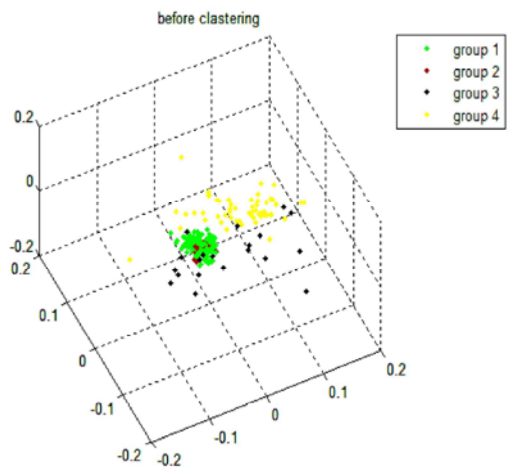

Fig. 5: The visualization of the three random lengths of waves of the hyper spectral image

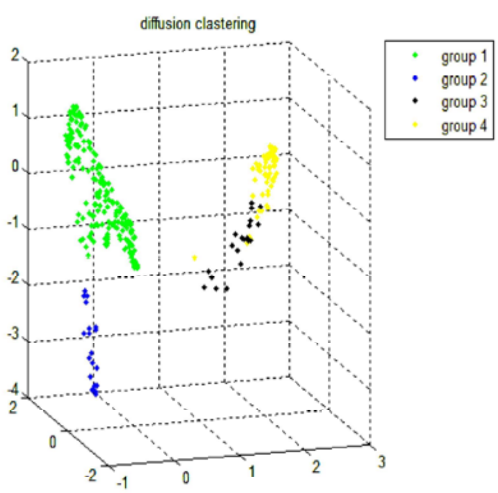

Fig. 6: The visualization of spectra after the use of the diffuse maps.

CONCLUSION

We have constructed a map in which the spectra of the different materials are separated clearly. The method of the stochastic processes in Markov chains is applied to the division of clusters and identification of the interrelations between the data and also the representations of clusters in the space of a small dimension.

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