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Boundary Layer Flow over a Stretching Sheet with a Convective Boundary Condition and Slip Effect

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Abstract: The steady laminar boundary layer flow over a stretching sheet with partial slip under a convective surface boundary condition is investigated. Using a similarity transformation, the governing partial differential equations are reduced into a set of nonlinear ordinary differential equations, before being solved numerically by a shooting method using Maple software. The effects of the governing parameters namely Prandtl number, convective parameter and slip parameter on the fluid flow and heat transfer characteristics are presented graphically and analyzed.

Key words: Boundary layer . stretching sheet . partial slip . convective boundary condition

INTRODUCTION

The study of flow over a stretching sheet has generated much interest in recent years due to its important contribution especially in many engineering processes and industries. The applications in industries involved such as the aerodynamic extrusion of plastic sheets, glass-fiber production, condensation process of metallic plate in a cooling bath and glass and also in polymer industries.

It seems that Crane [1] was the first who reported the analytical solution for the laminar boundary layer flow past a stretching sheet. After this pioneering work, the study of fluid flow over a stretching sheet has received wide attention among researchers. Gupta and Gupta [2] added new dimension to the study with suction and injection. Pop and Na [3] solved the boundary layer flow over a permeable stretching sheet in the presence of a magnetic field. A similar work was done by Kumaran *et al*. [4], who analyzed transition effect of boundary layer flow due to an imposed and withdrawal of magnetic field over a viscous flow past a stretching sheet. is seen that the first paper on non-Newtonian with stretching sheet in nanofluids was introduced by Hamad and Bashir [5] considering the problem of two-dimensional laminar mixed convection flow of a power-law non-Newtonian nanofluids over a vertical stretching sheet. Later, Khan and Pop [6] analyzed numerically the problem of laminar fluid flow past a stretching sheet in a nanofluid. The study of flow over a stretching sheet later continued by Arnold *et al*. [7], who considered the viscoelastic fluid flow and heat transfer characteristics with the effects of viscous dissipation and internal heat generation or absorption. Besides that,

A collection of papers mentioned above which studied the stretching flow problems apply the no-slip condition. On the other hand, in certain circumstance, partial slip may occur between the fluid and the moving surface. Emulsions, suspensions, foams and polymer solutions are the examp les when the fluid is particulate which partial slip effect may take place (Yoshimura and Prudhomme [8]). Wang [9] has investigated the flow due to a stretching surface with partial slip. A few years later, Wang [10] continued the study on viscous flow due to a stretching sheet with suction and injection. Besides that, the magnetohydrodynamic (MHD) flow over a stretching sheet with partial slip was analyzed analytically by Fang *et al*. [11]. Most recently, Siddiqui *et al*. [12] has solved analytically the two boundary value problems for MHD flow of an Oldroyd-B fluid between two infinite parallel plates with the effect of partial slip.

In recent years, investigations on the boundary layer flow problem with a convective surface boundary condition have gained much interest among researchers, since first introduced by Aziz [13], who considered the thermal boundary layer flow over a flat plate in a uniform free stream with a convective surface boundary condition. This problem was then extended by Bataller [14] by considering the Blasius and Sakiadis flows, both under a convective surface boundary condition and

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in the presence of thermal radiation. Ishak [15] obtained the similarity solutions for the steady laminar boundary layer flow over a permeable plate with a convective boundary condition. Very recently, Makinde and Aziz [16] investigated numerically the effect of a convective boundary condition on the two dimensional boundary layer flows past a stretching sheet in a nanofluid.

The aim of the present study is to investigate the effect of a convective surface boundary condition on the laminar thermal boundary layer flow past a stretching sheet with partial slip. The boundary layer equations governed by the partial differential equations are first transformed into a system of nonlinear ordinary differential equations, before being solved numerically using a shooting method. The effects of the Prandtl number, convective parameter and the slip parameter on the flow field and heat transfer characteristics are thoroughly examined and discussed. To our best of knowledge, the investigations of the proposed problem are new and the results have not been published before.

PROBLEM FORMULATION

Consider a two dimensional laminar boundary layer flow past a stretching sheet of uniform surface temperature T_w immersed in a quiescent viscous fluid. It is assumed that the sheet is stretched with linear velocity $u_w = ax$, where a is a positive constant. The continuity, momentum and energy equations which describing the flow problem can be written as [11, 13, 15].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

with the boundary conditions for the velocity field

$$\begin{aligned} &u = u_w + L\frac{\partial u}{\partial y}, \quad v = 0, \quad \text{at} \quad y = 0 \\ &u \to 0 \qquad \text{as} \quad y \to \infty \end{aligned} \qquad (4)$$

where u and v are the velocity components in the x and y directions, respectively, T is the temperature, ν is the kinematic viscosity, α is the thermal diffusivity and L is the proportional constant of the velocity slip.

We assume the bottom surface of the plate is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient, h_f. Based on this assumption, the boundary condition for the thermal field can be written as [13, 15]:

$$\begin{aligned} &-k\frac{\partial T}{\partial y} = h_f(T_f - T_w) \quad \text{at} \quad y = 0 \\ &T \to T_\infty \qquad \text{as} \quad y \to \infty \end{aligned} \qquad (5)$$

Here, k is the thermal conductivity and T_w is the uniform temperature over the top surface of the plate.

We look for similarity solutions for Eqs. (1)-(3) with the boundary conditions (4)-(5) by introducing the following transformation [13-15]:

$$\eta = \left(\frac{a}{\nu}\right)^{1/2} y, \quad \psi = (\nu a)^{1/2} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \qquad (6)$$

where f is the dimensionless stream function, θ is the dimensionless temperature and the stream function $\psi(x,y)$ is define as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ which identically satisfies Eq. (1).

Substituting (6) into equations Eqs (2) and (3) subject to the boundary conditions (4) and (5), we obtain the following nonlinear ordinary differential equations:

$$f''' + ff'' - f'^2 = 0 \qquad (7)$$

$$f''' + ff'' - f'^2 = 0 \qquad (7)$$

$$\frac{1}{Pr}\theta'' + f\theta' = 0 \qquad (8)$$

In order to get similarity solutions of Eqs. (1)-(3), we shall take [13-15]:

$$h_f = cx^{-1/2} \qquad (9)$$

where c is a constant. Without this assumption, the generated solutions are local similarity solutions.

The transformed boundary conditions can be written as

$$f'(0) = 1 + Kf''(0), \; f(0) = 0, \; \theta'(0) = -\gamma[1 - \theta(0)]$$

$$f'(\eta) \to 0, \; \theta(\eta) \to 0 \text{ as } \eta \to \infty \qquad (10)$$

where primes denote differentiation with respect to η, $K = L(a/\nu)^{1/2}$ is the slip parameter and $\gamma = \frac{c}{k}\sqrt{\nu / U_\infty}$ is the convective parameter.

The quantities of physical interest are the local skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2/2}, Nu_x = \frac{q_w x}{k(T_f - T_\infty)} \tag{11}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}, q_w = -k\frac{\partial T}{\partial y}\bigg|_{y=0} \tag{12}$$

with μ and k being the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables (6), Eq. (11) becomes

$$\frac{1}{2}C_f \, Re_x^{\frac{1}{2}} = f''(0), Nu_x \, Re_x^{-\frac{1}{2}} = -\theta'(0) \tag{13}$$

where $Re_x = u_w x/\nu$ is the local Reynolds number.

RESULTS AND DISCUSSION

The nonlinear ordinary differential equations (7) and (8) subjected to the boundary conditions (10) were solved numerically using a shooting method. The numerical computations have been carried out for various values of the parameters involved, namely Prandtl number Pr, slip parameter K and convective parameter γ.

Figure 1-5 show the velocity and temperature profiles which satisfy the far field boundary conditions (10) asymptotically and thus supporting the numerical results obtained. We note that the convective parameter γ and the Prandtl number Pr give no effect to the flow field, which is clear from Eqs. (7)-(10). Thus, the velocity profile shown in Fig. 1 is independent of γ and Pr.

The effect of the convective parameter γ on the thermal field when Pr = 1 and K = 1 is shown in Fig. 2. It is seen from this figure that increasing γ is to increase both the surface temperature $\theta(0)$ and the temperature gradient at the surface $\theta'(0)$ (in absolute sense). Thus, the local Nusselt number, which represents the heat transfer rate at the surface, increases as γ increases. The convective parameter γ at any location x is directly proportional to the heat transfer coefficient associated with the hot fluid h_f. The thermal resistance on the hot fluid side is inversely proportional to h_f. Thus, as γ increases, the hot plate side convection resistance decreases and consequently, the surface temperature $\theta(0)$ increases [13].

The effects of the slip parameter K on the velocity and temperature profiles are illustrated in Fig. 3 and 4, respectively, both for Pr = 1 and γ= 0.5. It is seen from Fig. 3 that the velocity gradient at the surface $f''(0)$ decreases (in absolute sense) as K increases. Thus, the skin friction coefficient decreases in the presence of slip at the boundary and is inversely proportional to the magnitude of the slip. This results in decreasing manner of the magnitude of the temperature gradient at the surface $\theta'(0)$ as can be seen from Fig. 4. This is due to the fact that in the presence of slip at the boundary, the friction at the fluid-solid interface decreases and in consequence reduces the heat produced by the friction. Thus, both the skin friction coefficient $f''(0)$ and the heat transfer rate at the surface $-\theta'(0)$ decrease as the slip effect increases.

Furthermore, Fig. 5 shows the temperature profiles for various values of Pr for fixed K = 1 and γ= 0.5. It is evident from this figure that the temperature gradient at the surface increases as Pr increases, which implies an increase in the heat transfer rate at the surface. This is

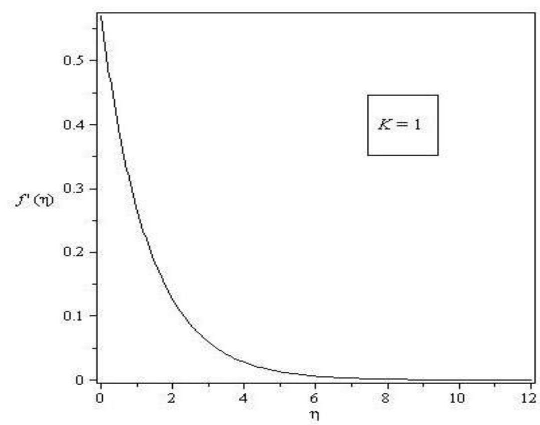

Fig. 1: Velocity profiles for K = 1

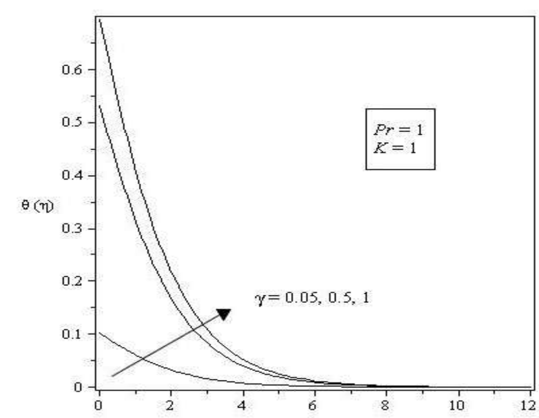

Fig. 2: Temperature profiles for different values of γ with fixed Pr = 1 and K = 1

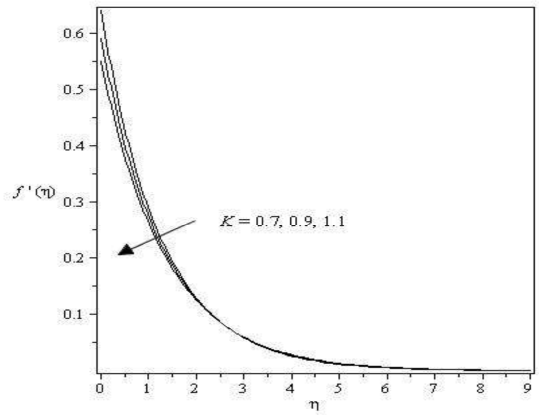

Fig. 3: Velocity profiles for different values of K

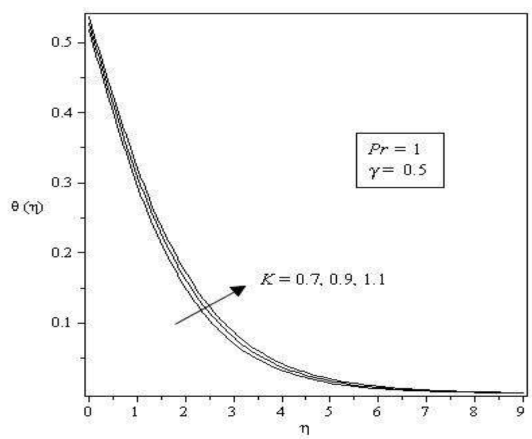

Fig. 4: Temperature profiles for different values of K with fixed Pr = 1 and $\gamma = 0.5$

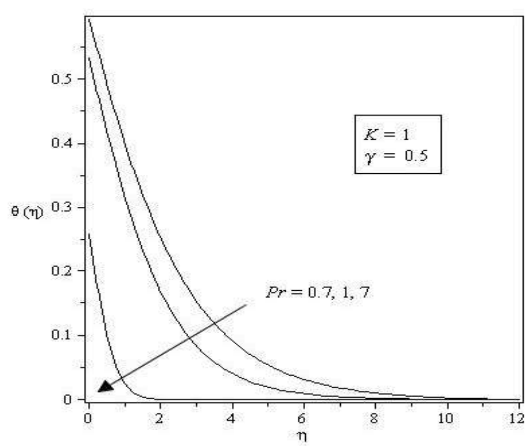

Fig. 5: Temperature profiles for different values of Pr with fixed K = 1 and $\gamma = 0.5$

because a higher Prandtl number fluid has a relatively low thermal conductivity, which reduces conduction and thereby the thermal boundary layer thickness and as a consequence increases the heat transfer rate at the surface [15].

CONCLUSION

The steady laminar boundary layer flow over a stretching sheet with partial slip under a convective surface boundary condition is studied. The similarity transformations are used to reduce the governing partial differential equations into ordinary differential equations. The effect of Prandtl number, convective parameter and slip parameter on the fluid flow were shown graphically and discussed. A numerical study was performed to solve the problem of boundary layer flow over a stretching sheet with a convective surface boundary condition and slip effect using a shooting method programmed in Maple. It was found that both the skin friction coefficient and the heat transfer rate at the surface decrease as the slip effect at the boundary increases. Moreover, the heat transfer rate at the surface increases as the convective parameter increases.

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