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Hydromagnetic Boundary Layer Flow over Stretching Surface with Thermal Radiation

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Abstract: The quality of final product in many metallurgical and technological processes is found to be significantly influenced by the hydromagnetic flow and thermal radiation. Magnetic field exerts drag forces in electrically conducting fluid and thermal radiation influences the rate of heat transfer. In this paper, the magnetohydrodynamics (MHD) boundary layer flow in viscous fluid over a stretching sheet with radiation effects is studied. The stretching velocity and temperature of the sheet are assumed to follow the power law. The governing boundary layer equations of partial differential equations are transformed into a system of ordinary differential equations by similarity transformation. The system is solved numerically using Runge-Kutta-Fehlberg method with shooting technique in the Maple software environment. The effects of magnetic parameter M, temperature exponent n, Prandtl number Pr and thermal radiation parameter N on the surface skin friction coefficient, surface heat transfer coefficient and velocity and temperature boundary layers are analysed. The magnetic parameter decreases the skin friction coefficient thus reduces the momentum boundary layer thickness. The magnetic parameter, Prandtl number and thermal radiation parameter decrease the heat transfer coefficient and therefore, the thermal boundary layer thickness increases.

Key words: Boundary layer . magnetohydrodynamics . radiation . runge-kutta method . stretching surface

INTRODUCTION

In recent years, the study of hydromagnetic boundary layers on stretching surfaces has attracted considerable interest due to its wide applications especially in engineering and industrial processes. Numerous investigations have been conducted on the magnetohydrodynamic (MHD) flows and heat transfer. MHD was initially known in the field of astrophysics and geophysics and later becomes very important in engineering and industrial processes. For example, MHD can be found in MHD accelerators and generators, electric transformers, power generators, refrigeration coils, pumps, meters, bearing, petroleum production and metallurgical processes which involve cooling of continuous strips or filaments. In metallurgical processes, the rates of cooling and stretching of the strips can be controlled by drawing the strips in an electrically conducting fluid subject to a magnetic field, so that a final product of desired characteristics can be achieved [1-3].

Hassanpour *et al*. [4] investigetd numerically the MHD mixed convective flow in a lid-laden cavity filled with porous medium using Lattice Boltzmann method. They found that the fluid circulation within the cavity is reduced by increasing magnetic field strength and the heat transfer depends on the magnetic field strength and the Darcy number. Chamkha [5] and Abo-Eldahab [6] considered MHD problem in three-dimensional flow, while Ishak *et al*. [7] studied the effect of a uniform transverse magnetic field on the stagnation-point flow over a stretching vertical sheet. Different aspects of MHD flow and heat transfer due to a stretching surface by considering vertical sheet, stretching cylinder and moving extensible surface [8, 9] were examined.

Many processes in engineering occur at high temperatures and the full understanding of the effect of radiation on the rate of heat transfer is necessary in the design of an equipment. The effect of radiation on the boundary layer flow was studied by Elbashbeshy and Dimian [10], Hossain *et al*. [11], Bataller [12] and Cortell [13]. The radiation effect is considered by Bataller [12] in the study of boundary layer flow over a static flat plate (Blasius flow) and Cortell [13] in the study of boundary layer flow over a moving flat plate (Sakiadis flow) in a quiescent fluid. The problems of Bataller [12]and Cortell [13] have been extended by Ishak [14] and he found the existence of dual solutions

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when the plate and the fluid move in the opposite directions. Ferdows *et al*. [15] found that radiation increases the skin friction coefficient and the rate of heat transfer in natural convection flow over inclined porous surface.

In this paper, the hydromagnetic flow and heat transfer on a vertical surface of variable temperature that is stretched with a power-law velocity is studied. The study extends the work of Ishak *et al*. [7] by considering the radiation effects.

MATHEMATICAL FORMULATIONS

Consider a steady two-dimensional flow of viscous, incompressible and electrically-conducting fluid over a vertical stretching surface in the presence of magnetic field and thermal radiation. The x-axis is coincident with the vertical surface and the y-axis is perpendicular to the surface. u and v are defined as the velocity components along the x-and y-axes, respectively. The stretching sheet velocity is assumed to be in the form of $u = ax^m$ where a is a positive constant. The velocity at a short distance from the surface allows a thin boundary layer to develop near the surface. The surface temperature, T_w is assumed to follow the power law $T_w = T_\infty + bx^n$ where b is a constant and T_∞ is the ambient temperature. It is also assumed that the magnetic Reynolds number is small in such a way that the induced magnetic field is negligible. Both viscous dissipation and Ohmic heating terms are neglected because their values are generally small. Under these assumptions along with Boussinesq and boundary layer approximations, the governing equations of partial differential equations for the conservation of mass, momentum and energy are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B(x)}{\rho} \pm g\beta\left(T - T_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

subject to

$$u = ax^m, \quad v = 0, \quad T = T_w(x) \ \text{ at } y = 0 \tag{4}$$

$$u \to 0, \quad T \to T_\infty \ \text{as } y \to \infty$$

where T is the fluid temperature in the boundary layer, B(x) is the variable magnetic field strength, ν is the kinematic viscosity, ρ is the fluid density, $\alpha = \frac{k}{\rho C_p}$ is the thermal diffusivity, σ is the electric conductivity, k is the thermal conductivity, β is the coefficient of thermal expansion, g is the acceleration due to gravity, "+" and "-" signs correspond to the buoyancy assisting and the buoyancy opposing flow regions, respectively, C_p is the specific heat at constant pressure, q_r is the radioactive heat flux and T_w is the surface temperature of the stretching surface.

The radiative heat flux q_r [16-18] is:

$$q_r = -\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* and K are the Stefan-Bolzman constant and Rosseland mean absorption coefficient, respectively. The assumption that the temperature differences within the flow are sufficiently small allows T^4 to be expressed as a linear function of temperature by Taylor series expansion,

$$T^4 \approx T_\infty^4 + (T - T_\infty)4T_\infty^3 = 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

The substitution of (5) and (6) in (3) yields

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(1 + N\right)\frac{\partial^2 T}{\partial y^2} \tag{7}$$

where

$$N = 16\sigma^* T_\infty^3/(3kK)$$

is the radiation parameter [19].

When the variable magnetic field $B(x)=B_o x^{(m-1)/2}$, the system (1)-(4) admits similarity solutions. The momentum and energy equations along with the boundary conditions can be transformed into a system of coupled ordinary differential equations by the following transformation:

$$\eta = \left(\frac{ax^{m-1}}{\nu}\right)^{1/2} y \tag{8}$$

$$\psi = \left(a\nu x^{m+1}\right)^{1/2} f\left(\eta\right) \tag{9}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

where η is the similarity variable and ψ is the stream function defined by $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ which satisfy the continuity equation (1). The momentum equation (2) and the energy equation (7) are reduced to

$$f''' + \frac{m+1}{2} ff'' - mf'^2 - M^2 f' + \lambda\theta = 0 \qquad (11)$$

$$\frac{1}{Pr}(1+N)\theta'' + \frac{m+1}{2} f\theta' - nf'\theta = 0 \qquad (12)$$

respectively, where prime indicates differentiation with respect to η and $Pr = \nu/\alpha$ is the Prandtl number.
The transformed boundary equations are:

$$\begin{aligned} &f(0)=0, \quad f'(0)=1, \quad \theta(0)=1, \\ &f'(\eta)\to 0, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty \end{aligned} \qquad (13)$$

In the absence of the thermal radiation N = 0, the system (11)-(13) is reduced to the system considered by Ishak *et al*. [7] and the analytical solutions for the special case of n = 2m-1 and N = 0 are given by Grubkha and Bobba [20]. The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as:

$$C_f = \frac{2\tau_w}{\rho U_w^{\,2}}, Nu_x = \frac{x\, q_w}{k(T_w - T_\infty)} \qquad (14)$$

respectively, where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu\left(\frac{a^{3/2} x^{\frac{3m-1}{2}} f''(0)}{\nu^{1/2}}\right)$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k\left(bx^n\theta'(0)\left(\frac{ax^{m-1}}{\nu}\right)^{1/2}\right) \qquad (15)$$

with μ is the dynamic viscosity. Using the non-dimensional variables in (8)-(10), we obtain

$$\frac{1}{2}C_f\, Re_x^{1/2} = f''(0), Nu_x\, Re_x^{-1/2} = -\theta'(0) \qquad (16)$$

where $Re_x = U_w x/\nu$ is the local Reynolds number.

RESULTS AND DISCUSSION

The system of boundary layer equations (11)-(13) is solved numerically using the Runge-Kutta-Fehlberg method with shooting technique for various parameter

Table 1:The values of $f''(0)$ and $-\theta'(0)$ for various values of M when $Pr = \lambda = m = 1$ and N = 0

	Ishak *et al.* [7]		Present results	
M	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0	-0.5607	1.0873	-0.56075	1.08727
0.1	-0.5658	1.0863	-0.56585	1.08626
0.2	-0.5810	1.0833	-0.58103	1.08326
0.5	-0.6830	1.0630	-0.68303	1.06301
1	-1.0000	1.0000	-1.00000	1.00000
2	-1.8968	0.8311	-1.89683	0.83113
5	-4.9155	0.4702	-4.91553	0.47027

Table 2:The values of $f''(0)$ and $-\theta'(0)$ for various value of M when $Pr = \lambda = m = n = N = 1$

M	$f''(0)$	$-\theta'(0)$
0	-0.4578	0.7477
0.1	-0.4631	0.7468
0.2	-0.4792	0.7442
0.5	-0.5873	0.7267
1	-0.9236	0.6721
2	-1.8594	0.5342
5	-4.9108	0.2975

values. The effects of temperature exponent parameter n, Prandtl number Pr, magnetic parameter M and radiation parameter N are assessed while the velocity exponent parameter m and buoyancy parameter λ are held fixed. Only the case of buoyancy assisting flow λ=1 is considered. The values for the magnetic parameter that have been tested are 0, 0.2, 0.5, 1 and 2, the values for the radiation parameter are 0, 1, 2 and 3 and the values for Prandtl number Pr are 1, 3, 5 and 10. To verify the validity and accuracy of the present analysis, numerical results for the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ are compared with results of Ishak *et al*. [7] for the case of no radiation. Ishak *et al*. [7] used the implicit finite-difference of Keller-Box method.

Table 1 and 2 show the excellent agreement between the numerical results of $f''(0)$ and $-\theta'(0)$ by Runge-Kutta-Fehlberg shooting technique and the results via Keller-Box method of Ishak *et al*. [7]. Table 1 and 2 also show the effect of the magnetic parameter M on $f''(0)$ and $-\theta'(0)$ when Pr = 1, λ = 1, m = 1, n = 1 in the absence of thermal radiation, N = 0 and in the presence of thermal radiation, N = 1, respectively. In Table 1, 2 and 4, it can be observed that when M increases, the values of skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$ decrease regardless of the absence or presence of thermal radiation. Similar

Table 3: Values of $-\theta'(0)$ for various values of Pr and n when m = 1 and M = λ = N = 0

	Ishak *et al.* [7]			Present results		
n	Pr = 1	Pr = 3	Pr = 10	Pr = 1	Pr = 3	Pr = 10
-2	-1.0000	-3.0000	-10.0000	-1.0000	-3.0000	-10.0000
-1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.5820	1.1652	2.3080	0.5820	1.1652	2.3080
1	1.0000	1.9237	3.7207	1.0000	1.9237	3.7207
2	1.3333	2.5097	4.7969	1.3333	2.5097	4.7969

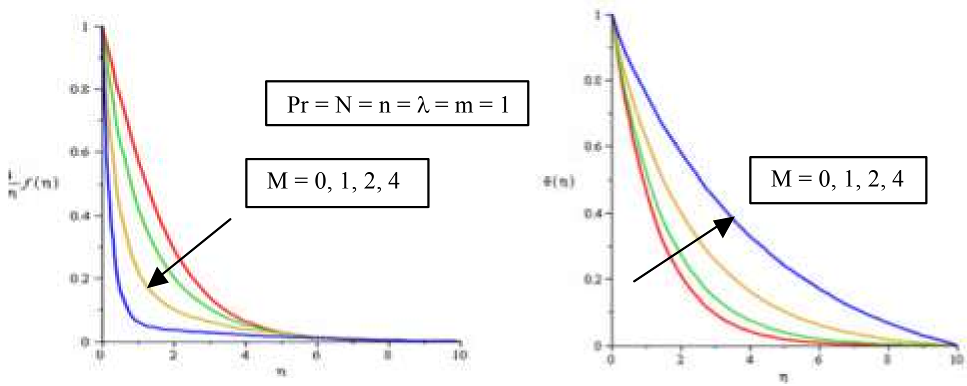

Fig. 1: Influence of the magnetic parameter M on the dimensionless velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$

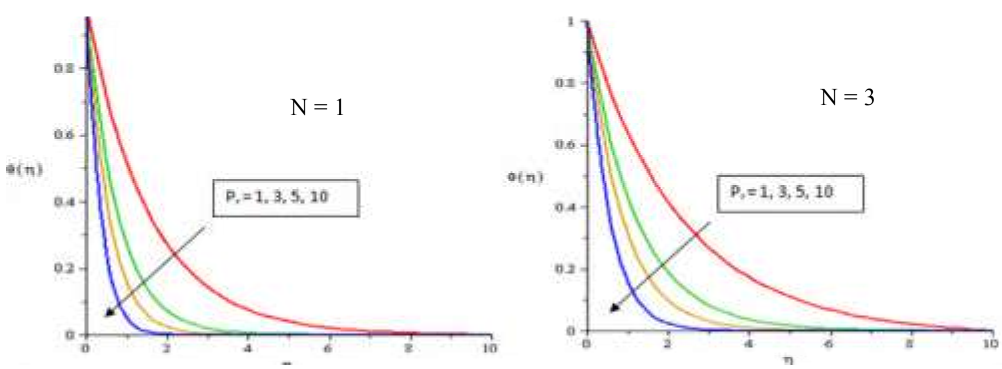

Fig. 2: Temperature profiles $\theta(\eta)$ when N = 1 and N = 3 for different values of Pr and m = M = n = λ =1

Table 4:Values of $-\theta'(0)$ for various values of M and N when m = Pr = n = λ = 1

M	N = 0	N = 0.5	N = 1	N = 2	N = 3
0	1.0873	0.8728	0.7477	0.6022	0.5173
0.2	1.0833	0.8691	0.7442	0.5991	0.5145
0.5	1.0630	0.8504	0.7267	0.5834	0.5001
1	1.0000	0.7921	0.6721	0.5345	0.4554
2	0.8314	0.6410	0.5342	0.4156	0.3410

effect can be seen as illustrated in Table 4 and 5, when the thermal radiation N increases, the value of $-\theta'(0)$

Table 5:Values of $-\theta'(0)$ for various values of Pr and N when m = M = n = λ = 1

Pr	N = 0	N = 0.5	N = 1	N = 2	N = 3
1	1.0000	0.7921	0.6721	0.5345	0.4554
3	1.8756	1.4894	1.2629	1.0000	0.8475
5	2.4963	1.9904	1.6915	1.3417	1.1371
10	3.6474	2.9253	2.496	1.9904	1.6915

decreases. However, when the temperature exponent n or Prandtl number Pr increases as shown in Table 3 and 5, the absolute value of the heat transfer coefficient $-\theta'(0)$ also increases.

Figure 1 shows the effects of the magnetic parameter M on the velocity and temperature functions when Pr = N = n = λ = m = 1. The presence of magnetic field exerts viscous drag forces on the flow field which results in the deceleration of momentum and thus the velocity boundary layer thickness decreases. Slower movement of fluid lead to the decreasing of the rate of heat transfer and thus the thermal boundary layer thickness increases with the increase of magnetic parameter. Figure 2 depicts the temperature profiles when the Prandtl number Pr increases for m = M = n = λ = 1 with thermal radiation parameter N = 1 and N = 3. The thermal boundary layer thickness for both cases of N = 1 and N = 3 decrease as Pr increases with thermal boundary layer for N = 3 is thicker than the thermal boundary layer for N = 1. Thermal radiation heats the fluid and thus decreases the rate of heat transfer from the wall to the fluid due to lesser temperature difference between the surface and the fluid. The Prandtl number which measures the strength of momentum diffusivity versus the thermal diffusivity shows that when the momentum diffusivity is greater than thermal diffusivity, the thermal boundary layer thickness decreases.

CONCLUSIONS

The magnetohydrodynamic (MHD) for a steady two-dimensional boundary layer flow along a vertical power law stretching surface with radiation effects has been investigated. The system of coupled boundary layer equations was solved by using the Runge-Kutta-Fehlberg method with shooting technique. The magnetic parameter decreases the momentum boundary layer thickness. The magnetic field and the thermal radiation increase the thermal boundary layers but the Prandtl number decreases the thermal boundary layer thickness.

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