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Aesthetics Vase Design with Transition Curves

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Abstract: Transition curves with monotone curvature profile are widely applied in fair curve design. Advantage of using transition curve is that it can avoid the undesirable curvature extrema. Demonstration of transition curve in designing surface of revolution, particular of vase design can be often seen in literature. For these demonstrations, authors mainly focused on the curvature control of the vase profile. To improve the quality of design, others beauty judgment should take into account. Referred to Birkhoff, in order to correlates the aesthetic measure with quality of elegance it is necessary to consider the symmetries and proportion. In the recent paper, we discussed the low energy transition curve. This paper demonstrates its application in aesthetic vase design. Specific location of curvature extremas is first evaluated base on the desirable symmetries and proportion. Whereby they are evaluate through considering the meet points of horizontal and vertical tangents line at the curvature extrema. Low energy transition curve satisfying the curvature distribution is then computed to form the vase profile. The symmetries and proportion of resulting vase profile is then compared against the idea vase given by Birkhoff.

Key words: Aesthetic . vase design . transition curve

INTRODUCTION

Transition curve are useful in fair curve design. They are commonly applied for smooth blending between circular arcs and straight line. Clotohid segment is the good example of transition curve. It is especially well known by its application in highway and railway design. (Meek, 1992; Walton, 2009) described the usage of clothoid as transition curve. Clothoid have excellent curvature properties but it can only form finite spectrum of curve. Thus for application in which linear curvature profile is not straightly required, others transition curve which is more flexibility would have an advantage. In an addition, clothoid admit only an approximate representation in CAD system, polynomial curve which is fully compatible with existing CAD system is preferred.

Lots of studies regarding to polynomial transition curve can be found in literature. (Walton 1996a, 1996b) introduced the cubic Bezier spiral and Phtyagorean hodograph (PH) quintic spiral. (Farouki, 1997) improved the PH spiral by adding more flexibility in controlling the arc length. (Walton, 1998) extended the studies of Bezier spiral, emphasized in control of the curvature and inflection points. (Walton, 2002) described single PH quintic spiral in C shape and S shape transition curve between circles. (Walton, 2003) discussed cubic Bezier spiral which allow users specify the points under certain condition. And (Habib, 2009) described transition curve with local shape control parameter allow user to control the curve shape under the stable manner.

Authors in the literature commonly demonstrated the resulting transition curve for designing curve profile for surface of revolution (Walton, 1996b; 1998; 2002; 2010; Habib, 2009). Transition curve is suitable for this application because it can avoid undesirable curvature extrema. In these designs, concentrations have been focusing on the curvature distribution of the curve profile. Hence some others aspect in design might not able to take into consideration. This drawback is causes by limited control and lack of flexibility of the transition curve. User could not concerned about both end point constrains and the smooth curvature variation simultaneously. Thus computation of a more flexible and easy of control transition curve is needed.

In the recent study (Chan, 2011) we described the low energy transition curve. This transition curve allows user to specify the end points position, tangent and curvature. Here, we extend the study to the single piece of C shape transition between circles. The resulting curve is easy to compute and of high flexibility. To demonstrate the flexibility of this transition curve, we applied this curve in the aesthetic vase design. The aesthetic vase design requires more then smooth curvature variation. According to Birkhoff, to correlates the aesthetic measure with quality of elegance, it is necessary to consider the symmetric and

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proportion. Thus the high flexible transition curve is needed such that resulting curve profile can satisfy the aesthetic portions and exhibit smooth curvature variation. The resulting vase profile is computed according to the portion of Birkhoff's ideal aesthetic vase.

QUINTIC HERMITE CURVE

In this study, we adopted the quintic Hermite curve for computation of transition curve. The Quintic Hermite curve interpoalte end point position, first and second derivatives. Thus it guarantee second order parametric continuity. Given two end points, P_0, P_1, beginning and ending tangent T_0, T_1, beginnig and ending curvature k_0 and k_1 respectively. The quintic Hermtie curve is given by:

$$\bar{C}(t) = P_0H_0 + P_0^{'}H_1 + P_0^{''}H_2 + P_1^{''}H_3 + P_1^{'}H_4 + P_1H_5, t = [0,1]$$

where $P_0^{'}$, $P_1^{'}$ are the end point first derivatives and $P_0^{''}$, $P_1^{''}$ are the end point second derivatives. While H_i, $i \in \{1,2,...,5\}$ are the Hermite blending function discribed in (Farin, 2002). The first and second derivatives is definded as:

$$P_j^{'} = m_j \cdot \hat{t}_j$$

$$P_j^{'} = m_j \cdot \hat{n}_j \cdot k_j, j \in \{0,1\}$$

where $\hat{t}_j$ and $\hat{n}_j$ represent the ending unit tangent and unit normal respectively. While m_j are the free parameter controlling magnitude of first and second derivatives. These well estimation derivatives guarantee the second order geometric continuity. m_j play important role in controlling the curve shape and variation of curvature. They must be constrained to be positive such that it would not change the tangent direction. In this sutdy, we compute the C shape transition curve by assign suitable value to m_0 and m_1.

C SHAPE TRANSITION CURVE

C shape transition curve in most of the litreature is computed using two pieces of curves (Walton, 1996a; 1996b; 2010). However, the use of single curve has the benefit that designer have fewer entities to be concerned with (Habib, 2009). Compotation of C shape transition curve using single curve are discribed in (Walton, 2002; Habib, 2009). In this study, we would discribed the computation of C shape transition curve using single piece of curve. The resultign transition curve have at most one curvature extrema. The computation method of this C shape transition curve require minor calculation step, thus is said to be simple. Refer (Levein, 2009) user always prefer computation method which is simple and easy..

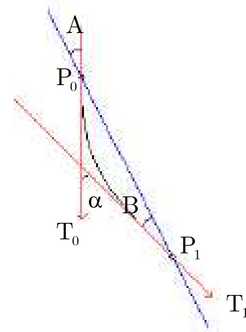

Fig. 1: C shape transition curve

Consider two specific points (P_0, P_1) with given tangent direction (T_0, T_1) and curvature (k_0, k_1) as showed in Fig. 1.

The tansition curve can be computed by input the desire end points position, tangent and curvature. With this information provided, the calculation method then proceed with finding of the relevent angle as show in Fig. 1. Refer to Fig. 1, A is the different of angle between T_0 with straight line joining P_0 and P_1. Whilst B is the diffferent of angle between T_1 with the line P_0P_1. And α is the winding angle between ending tangent. A C shape transition curve with at most one curvature extrema can then be obtain by setting the magnitide of first and second derivative is set as follow:

$$m_0 = \left(\frac{A}{\sin A} + \frac{B}{\sin B}\right) \cdot \|P_0P_1\| \cdot \frac{1}{2} + (\theta_0 + \alpha) \cdot k_0$$

$$m_1 = \left(\frac{A}{\sin A} + \frac{B}{\sin B}\right) \cdot \|P_0P_1\| \cdot \frac{1}{2} + (\theta_1 + \alpha) \cdot k_1$$

where θ_0 and θ_1 is the angle of ending tangent and $\|P_0P_1\|$ is the distance between end points. If the resulting curvature profile is not satisfied, it can be improve by slightly modifiying the m_0 and m_1. This metohd is restricted for cases where winding angle is less than $\pi/2$.

AESTHETIC VASE DESIGN

This section illustrates the application of resulting transition curve in aesthetic vase design. Position, tangent and curvature for data points satisfying the aesthetic vase proportion is first computer according to desirable symmetries and proportion. The symmetries

Table 1: End point data of curve profle

	C1	C2	C3	C4
P_0	(1,4)	(0.5,3)	(1,2)	(2,0)
P_1	(0.5,3)	(1,2)	(2,0)	(1,-4)
$\hat{t}_0$	$\begin{bmatrix}-0.707106781\\-0.707106781\end{bmatrix}$	$\begin{bmatrix}0\\-1\end{bmatrix}$	$\begin{bmatrix}0.707106781\\-0.707106781\end{bmatrix}$	$\begin{bmatrix}-0.052221931\\-0.998635504\end{bmatrix}$
$\hat{t}_1$	$\begin{bmatrix}0\\-1\end{bmatrix}$	$\begin{bmatrix}0.707106781\\-0.707106781\end{bmatrix}$	$\begin{bmatrix}-0.052221931\\-0.998635504\end{bmatrix}$	$\begin{bmatrix}-0.382683432\\-0.923879532\end{bmatrix}$
k_0	1/20	1/10	1/20	1/80
k_1	1/10	1/20	1/80	0
m_0	1.227038405	1.227038405	2.38302581	4.14992817
m_1	1.227038405	1.227038405	2.335241226	4.141444325

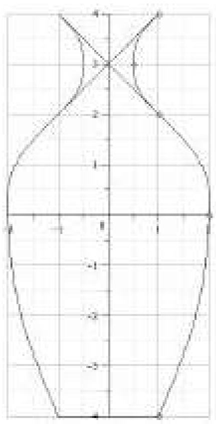

Fig. 2: Vase profile and its proportion distribution

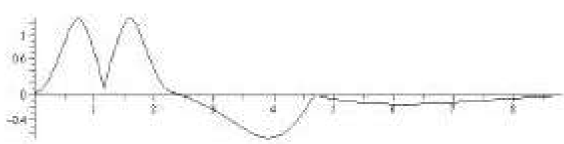

Fig. 3: Curvature profile of transition curve

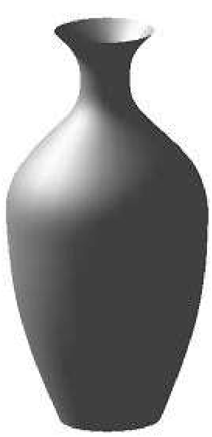

Fig. 4: Shaded rendition of aesthetic vase

and proportion can be determined by taking the meet point of Horizontal and vertical tangent line of the curvature extrema into account. Transition curve used to interpolate the data is then computed by setting the magnitudes of end points derivatives as discussed in section three. Figure 2 and 3 showed the resulting aesthetic vase profile and curvature profile respectively. The aesthetic vase is computed using four pieces of transition curves. The square boxes in Fig. 2 denoted the end points and the dark line represented the transition curves. The shaded rendition of aesthetic vase is illustrated in Fig. 4. And the data for computation of aesthetic curve showed in Fig. 4 is tabulated in Table 1

CONCLUSION

The C shape transition curve discribed in this study involve only simple computation method. In addition it allow user to spicify the ending point position, tangent direction and curvature. But user have no control on the curve shape. Anologus to others C shape transition curve in litreature, the resulting curve in this study have at most one curvature extrema. Thus for time being, we cannot work out the ideal transition curve. However, with the ultimate benefit of finding in this study together with those in litreature, we are a step nearer to the goal of computating for the ideal transition curve.

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