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Spectrum of $(\in,\in \vee q)$-Fuzzy Prime h-ideals of a Hemiring

[1]M. Shabir and [2]T. Mahmood

[1]Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan
[2]Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan

Abstract: In this paper we define $(\in,\in \vee q)$-fuzzy prime h-ideal, $(\in,\in \vee q)$-fuzzy semiprime h –ideal. We also construct spectrum of $(\in,\in \vee q)$-fuzzy prime h-ideals in two different ways. In the last section we also discuss imlpication-based $(\in,\in \vee q)$-fuzzy prime h-ideals.

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INTRODUCTION

Semirings, introduced by Vandiver [1] in 1934., appears in natural way for studying optimization theory, graph theory, theory of discrete event dynamical systems, matrices, determinants, generalized fuzzy computation, theory of automata, formal languages theory, coding theory, analysis of computer programmes. Hemirings, which are semirings with commutative addition and absorbing additive identity, are used in some applications to the theory of automata, the theory of formal languages and in computer sciences [2-5].

Ideals of semirings play an important role in the structure theory of semirings. However, in general, they do not coincide with the usual ring ideals. So many results in ring theory have no analogues in semirings using only ideals. In order to overcome this difficulty in [6] Henriksen defined a more restricted class of ideals in semirings, called k-ideals, with the property that if a semiring R is a ring, then a complex in R is a k-ideal if and only if it is a ring ideal. Another more restricted class of ideals in hemirings, called now h-ideals, has been given and investigated by Iizuka [7].

In 1965, Zadeh [8] introduced the concept of fuzzy set. Since then many efforts have been made to use this concept in algebraic structures. In this connection, Rosenfeld [9] introduced the notion of fuzzy subgroups. In [10] J. Ahsan et. al initiated the study of fuzzy semirings.

The notions of "belongingness ($\in$)" and "quasi-coincidence (q)" of fuzzy points and fuzzy sets proposed and discussed in [11, 12]. In [13], Zhan and Dudek introduced the concept of prime fuzzy h-ideals of a hemiring and established some interesting results. In [14], Kumbhojkar addressed the questions raised by Zhan and Dudek, redefined prime fuzzy h-ideals, defined semiprime fuzzy h-ideals of a hemiring and constructed spectrum of prime fuzzy h-ideals. Since prime spectrum on the set of prime ideals of a commutative hemiring with unity plays an important role in commutative algebra, algebraic geometry, lattice theory and semiprime ideals of hemirings determine the topology. So keeping in view all these facts in this paper we extend the ideas of Kumbhojkar and define $(\in,\in \vee q)$-fuzzy prime h-ideal, $(\in,\in \vee q)$-fuzzy semiprime h-ideal. We also construct spectrum of $(\in,\in \vee q)$-fuzzy prime h-ideals in two different ways. In the last section we also discuss implication-based $(\in,\in \vee q)$-fuzzy prime h-ideals.

PRELIMINARIES

For basic definitions we refer to [4, 14].

A fuzzy subset f of a universe X is a function $f: X\to[0,1]$. A fuzzy subset f of X of the form

$$f(y)=\begin{cases} t\in(0,1] & \text{if } y=x \\ 0 & \text{if } y\neq x \end{cases}$$

is called the fuzzy point with support x and value t and is denoted by x_t. Furthermore for $t\in(0,1]$, level subset of f is denoted and defined by $U(f,t)=\{x\in X: f(x)\geq t\}$.

Corresponding Author: T. Mahmood, Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan

In [12] Pu and Liu defined and discussed $x_t \alpha f$, where $\alpha \in \{\in, q, \in \vee q, \in \wedge q\}$. A fuzzy point x_t is said to belong to (resp. quasi-coincident with) f, written as $x_t \in f$ $(\text{resp. } x_t q f)$, if $f(x) \geq t$ $(\text{resp. } f(x) + t > 1)$. $x_t \in \vee qf$ $(\text{resp. } x_t \in \wedge qf)$ means that $x_t \in f$ or $x_t q f$ $(\text{resp. } x_t \in f \text{ and } x_t q f)$. To say that $x_t \overline{\alpha} f$ means that $x_t \alpha f$ does not hold.

For any two fuzzy subsets f and g of X, $f \leq g$ means that, for all $x \in X$, $f(x) \leq g(x)$. For two fuzzy subsets f and g of X, $f \wedge g$ and $f \vee g$ will mean the following fuzzy subsets of X

$$(f \wedge g)(x) = f(x) \wedge g(x)$$

$$(f \vee g)(x) = f(x) \vee g(x)$$

for all $x \in X$.

$(\in, \in \vee q)$-FUZZY PRIME AND $(\in, \in \vee q)$-FUZZY SEMIPRIME H-IDEALS

From now to onward throughout R will denote a commutative hemiring with 1.

Definition: A fuzzy h-ideal f of R is said to be an $(\in, \in \vee q)$-fuzzy prime h-ideal of R if it is non constant and for all $x,y \in R$ and $t \in (0,1]$

(1a) $(xy)_t \in f \Rightarrow x_t \in \vee qf$ or $y_t \in \vee qf$

Theorem: For a fuzzy subset f of R the following are equivallent:

(1a) $(xy)_t \in f \Rightarrow x_t \in \vee qf$ or $y_t \in \vee qf$

(1b) $f(x) \vee f(y) \geq f(xy) \wedge 0.5$

for all $x,y \in R$ and $t \in (0,1]$.

Corollary: A fuzzy h-ideal f of R is an $(\in, \in \vee q)$-fuzzy prime h-ideal of R if and only if it satisfies (1b).

Lemma: A fuzzy h-ideal f of R is an $(\in, \in \vee q)$-fuzzy prime h-ideal of R if and only if for all $t \in (0,0.5]$, the non-empty level subset $U(f,t) = \{x : f(x) \geq t\}$ is either prime h-ideal of R or is R itself.

Lemma: A non-empty subset P of R is prime h-ideal of R if and only if C_P is an $(\in, \in \vee q)$-fuzzy prime h-ideal of R.

Definition: A fuzzy h-ideal f of R is said to be an $(\in, \in \vee q)$-fuzzy semiprime h-ideal of R if it is non constant and for all $x \in R$ and $t \in (0,1]$.

(1′a) $(x^2)_t \in f \Rightarrow x_t \in \vee qf$

Theorem: For a fuzzy subset f of R the following are equivallent:

(1′a) $(x^2)_t \in f \Rightarrow x_t \in \vee qf$

(1′b) $f(x) \geq f(x^2) \wedge 0.5$

for all $x \in R$ and $t \in (0,1]$.

Corollary: A fuzzy h-ideal f of R is an $(\in, \in \vee q)$-fuzzy semiprime h-ideal of R if and only if it satisfies (1′b).

Lemma: A fuzzy h-ideal f of R is an $(\in, \in \vee q)$-fuzzy semiprime h-ideal of R if and only if for all $t \in (0,0.5]$, the non-empty level subset $U(f,t)$ is either semiprime h-ideal of R or is R itself.

Lemma: A non-empty subset P of R is semiprime h-ideal of R if and only if C_P is an $(\in, \in \vee q)$-fuzzy semiprime h-ideal of R.

Remark: Every $(\in, \in \vee q)$-fuzzy prime h-ideal of R is $(\in, \in \vee q)$-fuzzy semiprime h-ideal of R, however converse is not true in general.

Example: Let $N_0 = \{0\} \cup N$ and $p_1, p_2, p_3, \ldots$ be the distinct prime numbers in N_0. If $J^0 = N_0$ and $J^l = p_1 p_2 p_3 \ldots p_l N_0$, $l = 1,2,3,\ldots$, then $J^0 \supset J^1 \supset J^2 \supset \ldots J^n \supset J^{n+1} \supset \ldots$ As every non-zero element of N_0 has unique prime factorization, J^l is a semiprime h-ideal for $l = 2,3,\ldots$ but not a prime h-ideal. Then for such values of l, by Lemma 3.10, C_{J^l} is an $(\in, \in \vee q)$-fuzzy semiprime h-ideal of R, but by Lemma 3.5, C_{J^l} is not an $(\in, \in \vee q)$-fuzzy prime h-ideal of R.

In our next discussion Ω denotes the set of all $(\in, \in \vee q)$-fuzzy prime h-ideals of R, also we assume for every $f \in \Omega$, $f(0) = 1$.

SPECTRUM OF $(\in, \in \vee q)$-FUZZY PRIME H-IDEALS

We denote and define $\Omega = \{f : f \text{ is } (\in, \in \vee q)\text{-fuzzy prime h-ideal of R}\}$. If $\lambda \in \Omega$, then $V(\lambda) = \{f \in \Omega : \lambda \leq f\}$. If $\lambda = C_{\{a\}}$, where $a \in R$, then

$$V(\lambda) = V(a) = \{f \in \Omega : C_{\{a\}} \leq f\} = \{f \in \Omega : f(a) = 1\}$$

Further $r_h(\lambda) = \wedge\{f \in \Omega : \lambda \leq f\}$ is called $(\in, \in \vee q)$-fuzzy h-prime radical of λ. Further

$$V'(\lambda) = \Omega - V(\lambda) = \{f \in \Omega : \lambda > f\}$$

Lemma: $r_h(\lambda)$ is $(\in, \in \vee q)$-fuzzy semiprime h-ideal containing λ.

Proof: Proof is straightforward, hence omitted.

Lemma: If λ, μ are fuzzy subsets of R. Then

(i) $\lambda \le \mu$ implies $V(\mu) \subseteq V(\lambda)$

(ii) $V(\lambda) \cup V(\mu) \subseteq V(\lambda \wedge \mu)$

(iii) If A and B are h-ideals of R, then $V(C_A) \cup V(C_B) = V(C_{A \cap B})$

(iv) If η is $(\in, \in \vee q)$-fuzzy h-ideal of R generated by λ and $r_h(\eta)$ is $(\in, \in \vee q)$-fuzzy h-prime radical of η. Then $V(\lambda) = V(\eta) = V(r_h(\eta))$.

(v) If $\{\lambda_i : i \in \Lambda\}$ is a family of fuzzy subsets of R, then $V(\vee\{\lambda_i : i \in \Lambda\}) = \cap\{\vee(\lambda_i) : i \in \Lambda\}$

(vi) If $A \subseteq R$, then $V(C_A) = \cap\{V(a) : a \in A\}$

(vii) $V(a) \cup V(b) = V(ab)$, for all $a, b \in R$.

Proof

(i) Let $f \in V(\mu)$. Then $\mu \le f \Rightarrow \lambda \le \mu \le f \Rightarrow f \in V(\lambda)$.

(ii) As $\lambda \wedge \mu \le \lambda$ and $\lambda \wedge \mu \le \mu$, by (i) $V(\lambda) \subseteq V(\lambda \wedge \mu)$ and $V(\mu) \subseteq V(\lambda \wedge \mu) \Rightarrow V(\lambda) \cup V(\mu) \subseteq V(\lambda \wedge \mu)$.

(iii) By using Lemma 2.5 and (ii) $V(C_A) \cup V(C_B) \subseteq V(C_{A \cap B})$. Now let $f \in V(C_{A \cap B})$. Which implies $C_{A \cap B} \le f \Rightarrow f(x) = 1$ for all $x \in A \cap B$. Then $f(x) = 1$ for all $x \in A$ or $f(x) = 1$ for all $x \in B \Rightarrow C_A \le f$ or $C_B \le f \Rightarrow f \in V(C_A)$ or $f \in V(C_B) \Rightarrow f \in V(C_A) \cup V(C_B)$.
Hence $V(C_A) \cup V(C_B) = V(C_{A \cap B})$.

(iv) First we prove $V(\lambda) = V(\eta)$. Let Γ be the class of all $(\in, \in \vee q)$-fuzzy h-ideals of R containing λ. Then $\eta = \cap\Gamma$. Then by the fact $\lambda \le f \Leftrightarrow \eta \le f$, result follows immediately. Now we prove $V(\eta) = V(r_h(\eta))$. Let Γ be the class of all $(\in, \in \vee q)$-fuzzy prime h-ideals of R containing η. Then $r_h(\eta) = \cap\Gamma$. Then by the fact $r_h(\eta) \le f \Leftrightarrow \eta \le f$, result follows immediately.

(v) Let $f \in \cap\{\vee(\lambda_i) : i \in \Lambda\}$. Then $f \in \vee(\lambda_i)$ for all $i \in \Lambda \Rightarrow \lambda_i \le f$ for all
$i \in \Lambda \Rightarrow \vee\{\lambda_i : i \in \Lambda\} \le f \Rightarrow f \in V(\vee\{\lambda_i : i \in \Lambda\})$
$\Rightarrow \cap\{\vee(\lambda_i) : i \in \Lambda\} \subseteq V(\vee\{\lambda_i : i \in \Lambda\})$.
Working backward we get the desired result.

(vi) Follows directly from (v).

(vii) Straightforward.

Theorem: The set $\Im(\Omega) = \{V(\lambda) : \lambda \in \Omega\}$ forms a topology on the set Ω.

Proof

(i) As $V'(o) = \phi$ and $V'(R) = \Omega$, where $R \in \Omega$, defined by $R(x) = 1$, for all $x \in R$. So $\phi, \Omega \in \Im(\Omega)$.

(ii) Let $\{V'(\lambda_i) : i \in I\}$ be an arbitrary collection of elements of $\Im(\Omega)$. To prove $\vee_{i \in I} V'(\lambda_i)$ is in $\Im(\Omega)$.

$$\vee_{i \in I} V'(\lambda_i) = \vee_{i \in I} \{f \in \Omega : \lambda_i > f\}$$
$$= \{f \in \Omega : \exists\ k \in I \text{ such that } \lambda_k > f\}$$

Note that

$$\left(\sum_{i \in I} \lambda_i\right)(x) = \bigvee_{x + a_1 + a_2 + \ldots = b_1 + b_2 + \ldots} \begin{Bmatrix} \lambda_1(a_1) \wedge \lambda_2(a_2) \wedge \ldots. \\ \lambda_1(b_1) \wedge \lambda_2(b_2) \wedge \ldots \end{Bmatrix}$$

where $a_i, b_i \in R$ for all $i \in I$ and also only a finite number of the a_i's and b_i's are not zero. Since $\lambda_i(0) = 1$, therefore we are considering the infimum of a finite number of terms because 1′s are effectively not being considered. Now, if for some $k \in I$, $\lambda_k > f$, then there exists $x \in R$ such that $\lambda_k(x) > f(x)$. Consider the particular expression for which $a_k = x$, $b_k = 0$ and $a_i = b_i = 0$ for all $i \ne k$. We see that $\lambda_k(x)$ is an element of the set whose supremum is defined to be $\left(\sum_{i \in I} \lambda_i\right)(x)$.

Thus $\left(\sum_{i \in I} \lambda_i\right)(x) \ge \lambda_k(x) > f(x)$. This implies $\left(\sum_{i \in I} \lambda_i\right)(x) > f(x)$ that is $\sum_{i \in I} \lambda_i > f$.

Hence $\lambda_k > f$ for some $k \in I$ implies $\sum_{i \in I} \lambda_i > f$.

Conversely, suppose that $\sum_{i \in I} \lambda_i > f$ then there exists $x \in R$ such that $\left(\sum_{i \in I} \lambda_i\right)(x) > f(x)$.

$$\Rightarrow \bigvee_{x + a_1 + a_2 + \ldots + z = b_1 + b_2 + \ldots + z} \begin{Bmatrix} \lambda_1(a_1) \wedge \lambda_2(a_2) \wedge \ldots. \\ \lambda_1(b_1) \wedge \lambda_2(b_2) \wedge \ldots \end{Bmatrix} > f(x)$$

$$\Rightarrow \lambda_1(a_1') \wedge \lambda_2(a_2') \wedge \ldots\ldots \lambda_1(b_1') \wedge \lambda_2(b_2') \wedge \ldots\ldots > f(x) \quad (*)$$

($x + a_1' + a_2' + \ldots = b_1' + b_2' + \ldots$ being the corresponding breakup of x, where only a finite number of a_i's and b_i's are not zero.)

Now on the contrary if we suppose that (*) does not hold, that is if all the elements of the set (whose supremum we are taking) are individually less than are equal to $f(x)$, then

$$\left(\sum_{i\in I}\lambda_i\right)(x)=\bigvee_{x+a_1+a_2+\ldots+z=b_1+b_2+\ldots+z}\left\{\begin{matrix}\lambda_1(a_1)\wedge\lambda_2(a_2)\wedge\ldots.\\ \lambda_1(b_1)\wedge\lambda_2(b_2)\wedge\ldots\end{matrix}\right\}\leq f(x)$$

which is a contradiction. Thus (*) holds.
Thus,

$$\lambda_1(a_1')\wedge\lambda_2(a_2')\wedge\ldots\ldots\lambda_1(b_1')\wedge\lambda_2(b_2')\wedge\ldots\ldots$$
$$>f(x)\geq f(a_1')\wedge f(a_2')\wedge\ldots\ldots\wedge f(b_1')\wedge f(b_2')\wedge\ldots\ldots$$

Let

$$\lambda_1(a_1')\wedge\lambda_2(a_2')\wedge\ldots\lambda_1(b_1')\wedge\lambda_2(b_2')\wedge\ldots=\lambda_1(x_1')$$

and

$$f(a_1')\wedge f(a_2')\wedge\ldots\ldots f(b_1')\wedge f(b_2')\wedge\ldots\ldots=f(x_1')$$

where $1\in I$

So, $\lambda_1(x_1')>f(x_1')$ it follows that $\lambda_1>f$ for some $l\in I$.

Hence $\sum_{i\in I}\lambda_i>f$ implies that $\lambda_1>f$ for some $l\in I$.

Hence the two statements $\sum_{i\in I}\lambda_i>f$ and $\lambda_1>f$ for some $l\in I$ are equivalent.
Hence

$$\vee_{i\in I}V'(\lambda_i)=\cup_{i\in I}\{f\in\Omega:\lambda_i>f\}$$
$$=\cup_{i\in I}\left\{f\in\Omega:\sum_{i\in I}\lambda_i>f\right\}=V'\left(\sum_{i\in I}\lambda_i\right)$$

As $\sum_{i\in I}\lambda_i\in\Omega$ thus, $\vee_{i\in I}V'(\lambda_i)\in\Im(\Omega)$.

(iii) Let $V'(\lambda_1),V'(\lambda_2)\in\Im(\Omega)$.

Then

$$V'(\lambda_1)\wedge V'(\lambda_2)=\{f\in\Omega:\lambda_1>f\text{ and }\lambda_2>f\}$$
$$=V'(\lambda_1\wedge\lambda_2)$$

Hence it follows that $\Im(\Omega)$ forms a topology on the set Ω.

Theorem: If $a\in R$ and $V'(a)=\Omega-V(a)$. Then $\mathbf{B}=\{V'(a):a\in R\}$ is a base for a topology T on Ω. Then open sets of this topology are $V'(C_A),A\subseteq R$. Further this topology is completely determined by $(\in,\in\vee q)$-fuzzy semiprime h-ideals of R.

Proof: As for $a\in R,V(a)=\{f\in\Omega:f(a)=1\}$, so $V'(a)=\{f\in\Omega:f(a)\neq1\}$. Now as $V'(1)=\Omega$, so $\cup\{V'(a):a\in R\}=\Omega$. Also by using (vii) of Lemma 4.2, $V'(ab)=V'(a)\cap V'(b)$ for all $a,b\in R$ Thus B is base for some topology T on Ω. Now let $U\in T$,-then for some $A\subseteq R$,

$$\mathbf{U}=\cup\{V'(a):a\in A\}=\cup\{\Omega-V(a):a\in A\}$$
$$=\Omega-\cap\{V(a):a\in A\}$$
$$=\Omega-V(C_A)\text{ (by (vi) of Lemma 4.2)}=V'(C_A)$$

Further as $V(C_A)=V(C_{\langle A\rangle})=V(r_h(C_{\langle A\rangle}))$. Hence T is completely determined by $(\in,\in\vee q)$-fuzzy semiprime h-ideals of R.

IMPLICATION-BASED $(\in,\in\vee q)$-FUZZY PRIME H-IDEALS

Fuzzy propositional calculus is an extension of the Aristotelian propositional calculus. In fuzzy propositional calculus the truth set is taken [0, 1] instead of {0, 1}, which is the truth set in Aristotelian propositional calculus. In fuzzy logic some of the operators like $\wedge,\vee,\neg,\rightarrow$ can be defined by using truth tables. One can also use the extension principle to obtain the definitions of these operators.

In fuzzy logic the truth value of a fuzzy proposition λ is denoted by $[\lambda]$. In the following we give fuzzy logic and its corresponding set theoretical notations, which we will use in the paper hereafter.

$[x\in\lambda]=\lambda(x)$, $[x\notin\lambda]=1-\lambda(x)$
$[P\wedge Q]=\min\{[P],[Q]\}$, $[P\vee Q]=\max\{[P],[Q]\}$
$[P\rightarrow Q]=\min\{1,1-[P]+[Q]\}$
$[\forall xP(x)]=\inf[P(x)]$
$\neg P$ if and only if $[P]=1$.

Operator name	Definition of the operator
Early Zadeh	$I_m(x,y)=\max\{1-x,\min\{x,y\}\}$
Lukasiewicz	$I_a(x,y)=\min\{1-x,1-x+y\}$
Standard star (Godel)	$I_g(x,y)=\begin{cases}1 & \text{if } x\leq y\\ y & \text{if } y<x\end{cases}$
Contraposition of Godel	$I_{cg}(x,y)=\begin{cases}1 & \text{if } x\leq y\\ 1-x & \text{if } y<x\end{cases}$
Gains-Rescher	$I_{gr}(x,y)=\begin{cases}1 & \text{if } x\leq y\\ 0 & \text{if } y<x\end{cases}$
Kleen-Dienes	$I_b(x,y)=\max\{1-x,y\}$
Goguen	$I_{gg}(x,y)=\begin{cases}1 & \text{if } x\leq y\\ \frac{y}{x} & \text{if } y<x\end{cases}$

A function $I:[0,1]\times[0,1]\to[0,1]$ is called fuzzy implication if its monotonic with respect to both variables separately and fulfills the binary implication truth table

$$I(1,0)=0, I(0,0)=I(0,1)=I(1,1)=1$$

By monotonicity of I

$$I(0,x)=I(x,1)=1 \text{ for all } x\in[0,1].$$

There have been defined many implication operators in [15]. We consider in the following some important Implication Operators:

In the following definition, we consider the implication operators in Lukasiewicz system of continuous-valued logic.

Definition: A fuzzy set λ of R is called a fuzzified left (right) prime h-ideal if it satisfies:

(i) For any $x,y\in R, \neg[x\in\lambda]\wedge[y\in\lambda]\Rightarrow[x+y\in\lambda]$,

(ii) For any $x,y\in R, \neg[y\in\lambda]\Rightarrow[xy\in\lambda]$(resp. $\neg[x\in\lambda]\Rightarrow[xy\in\lambda]$),

(iii) For any $x,y\in R, \neg[xy\in\lambda]\Rightarrow[x\in\lambda]\vee[y\in\lambda]$,

(iv) For any $a,b,x,z\in R$ with $x+a+z=b+z, \neg[a\in\lambda]\wedge[b\in\lambda]\Rightarrow[x\in\lambda]$. A fuzzy set λ of a hemiring R is called a fuzzified prime h-ideal if and only if it is both fuzzified left and right prime h-ideal of R.

In [23] the concept of t-tautology is given, that is $\neg_t P$ if and only if $[P]\geq t$.

Now we define the implication based fuzzy h-ideal:

Definition: A fuzzy set λ of R is called a t-implication-based, $t\in(0,1]$ is a fixed number, left (right) prime h-ideal if it satisfies:

(i) For any $x,y\in R, \neg_t[x\in\lambda]\wedge[y\in\lambda]\Rightarrow[x+y\in\lambda]$,

(ii) For any $x,y\in R, \neg_t[y\in\lambda]\Rightarrow[xy\in\lambda]$(resp. $\neg[x\in\lambda]\Rightarrow[xy\in\lambda]$),

(iii) For any $x,y\in R, \neg_t[xy\in\lambda]\Rightarrow[x\in\lambda]\vee[y\in\lambda]$,

(iv) For any $a,b,x,z\in R$ with $x+a+z=b+z, \neg_t[a\in\lambda]\wedge[b\in\lambda]\Rightarrow[x\in\lambda]$. A fuzzy set λ of a hemiring R is called a t-implication-based, $t\in(0,1]$ is a fixed number, prime h-ideal if and only if it is both t-implication-based left and t-implication-based right prime h-ideal of R.

Corollary: Let I be an implication operator, $t\in(0,1]$ is a fixed number and λ be a fuzzy set of R. Then λ is a t-implication based fuzzy prime h-ideal of R if and only if the following conditions hold:

(i) For any $x,y\in R, I(\lambda(x)\wedge\lambda(y),\lambda(x+y))\geq t$,

(ii) For any $x,y\in R, I(\lambda(x),\lambda(xy))\geq t$(resp. $I(\lambda(y),\lambda(xy))\geq t$),

(iii) For any $x,y\in R, I(\lambda(xy),\lambda(x)\vee\lambda(y))\geq t$,

(iv) For any $a,b,x,z\in R$ with $x+a+z=b+z, I(\lambda(a)\wedge\lambda(b),\lambda(x))\geq t$.

Theorem

(1) Let $I=I_{gr}$. Then λ is a 0.5-implication-based fuzzy prime h-ideal of R if and only if λ is a fuzzy prime h-ideal of R.

(2) Let $I=I_g$. Then λ is a 0.5-implication-based fuzzy prime h-ideal of R if and only if λ is an $(\in,\in\vee q)$-fuzzy prime h-ideal of R.

Proof: (1) Let us assume λ be a 0.5-implication-based fuzzy prime h-ideal of R. Then

(i) For any $x,y\in R, I_{gr}(\lambda(x)\wedge\lambda(y),\lambda(x+y))\geq 0.5$,

(ii) For any $x,y\in R, I_{gr}(\lambda(x),\lambda(xy))\geq 0.5$(resp. $I(\lambda(y),\lambda(xy))\geq 0.5$),

(iii) For any $x,y\in R, I_{gr}(\lambda(xy),\lambda(x)\vee\lambda(y))\geq 0.5$,

(iv) For any $a,b,x,z\in R$ with $x+a+z=b+z, I_{gr}(\lambda(a)\wedge\lambda(b),\lambda(x))\geq 0.5$.

Now from (i) the only case which is possible is that $\lambda(x+y)\geq\lambda(x)\wedge\lambda(y)$. Similarly other conditions can be proved for λ to be prime fuzzy h-ideal of R. Converse of the Theorem is straightforward.

(2) Let us assume λ be a 0.5-implication-based fuzzy prime h-ideal of R. Then

(i) For any $x,y\in R, I_g(\lambda(x)\wedge\lambda(y),\lambda(x+y))\geq 0.5$,

(ii) For any $x,y\in R, I_g(\lambda(x),\lambda(xy))\geq 0.5$(resp. $I(\lambda(y),\lambda(xy))\geq 0.5$),

(iii) For any $x,y\in R, I_g(\lambda(xy),\lambda(x)\vee\lambda(y))\geq 0.5$,

(iv) For any $a,b,x,z\in R$ with $x+a+z=b+z, I_g(\lambda(a)\wedge\lambda(b),\lambda(x))\geq 0.5$.

Now (i) implies $\lambda(x+y)\geq\lambda(x)\wedge\lambda(y)$ or $\lambda(x)\wedge\lambda(y)>\lambda(x+y)\geq 0.5$. Then $\lambda(x+y)\geq\min\{\lambda(x)\wedge\lambda(y)\wedge 0.5\}$. (ii) and (iv) can be proved similarly.

Next (iii) implies $\lambda(x) \vee \lambda(y) \geq \lambda(xy)$ or $\lambda(xy) > \lambda(x) \vee \lambda(y) \geq 0.5$. Then $\lambda(x) \vee \lambda(y) \geq \lambda(xy) \wedge 0.5$.

Hence λ is an $(\in, \in \vee q)$-fuzzy prime h-ideal of R. Converse is straightforward.

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