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Intuitionistic Fuzzy Linear Programming Problems

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Abstract: In this paper, a new definition of intuitionistic fuzzy linear programming problem (IFLPP) is given in which the technological co-efficients are intuitionistic fuzzy numbers with linear membership and non-membership functions. Intuitionistic fuzzification of objective function and constraints has also been done.The determination of a crisp maximizing decision is used for defuzzification. Stepwise algorithm is given for solving an IFLPP and it is checked with a numerical example using fuzzy decisive set method.

Key words: Intuitionistic fuzzy set, intuitionistic fuzzy linear programming problem, intuitionistic fuzzy optimization, membership and non-membership functions, non-hesitancy degree, fuzzy decisive set method.

INTRODUCTION

The subject of decision making is the study of how decisions are actually made and how they can be made better or more successful. In this case, the decision making problem becomes an optimization problem of maximizing / minimizing the expected utility. In fuzzy decision making problems, the concept of maximizing decision was first proposed by Bellman and Zadeh [1]. The first formulation of Fuzzy Linear Programming Problem (FLPP) was proposed by Zimmermann [2]. FLPP with fuzzy coefficients was formulated by Negoita [3] and called robust programming.

Shaocheng [4] considered the FLPP with fuzzy constraints and defuzzified it by first determining an upper bound for the objective function. Further he solved the so obtained crisp problem by using the fuzzy decisive set method introduced by Sakawa and Yana [5]. Gasimov and Yenilmez [6] solved FLPP with linear membership functions.

Fuzzy set theory has been extensively used to capture uncertainty and vagueness in decision making problems. However, there is no way to model hesitancy in fuzzy set theory. Intuitionistic Fuzzy (IF) set theory introduced by Atanassov (1986) [7] and developed by Angelov, Yager and many others, addresses this issue of uncertainty. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than or equal to one.

The concept of IFS, can be viewed as an alternative approach to define a fuzzy set, in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Thus, it is expected that IFS can be used to simulate human decision-making process and activities requiring human expertise and knowledge that are inevitably imprecise or totally reliable.

Burillo et al [8] proposed the definition of Intuitionistic Fuzzy Number (IFN) and studied perturbations of IFN. B.S.Mahapatra and G.S.Mahapatra [9] have presented intuitionistic fuzzy fault tree analysis using trapezoidal intuitionistic fuzzy number. Angelov Plamen [10] introduced Intuitionistic Fuzzy Optimization (IFO) for LPP in which the non-membership function is considered as the complement of membership function. Lisy Cherian and Sunny Kuriakose [11] proposed an IFLPP with a non-membership function. Lei Yang et al [12] proposed a normalization technique for ascertaining non-membership functions of IFS.

This paper introduces LPP in an IF environment by constructing membership and non-membership functions for objective functions and constraints of IFLPP. It also explains a solution procedure for IFLPP in which the technological coefficients are intuitionistic fuzzy numbers.
The paper is organized as follows: Section 2 briefly describes the basic definitions and notations of IFS, IFN and FLPP. Formulation of IFLPP with intuitionistic fuzzy technological coefficients and the conversion of

IFLPP into classical optimization problem have been done in Section 3. In Section 4, the algorithm of the intuitionistic fuzzy decisive set method is presented and the application of this method is illustrated. The paper is concluded in Section 5.

PRELIMINARIES

Definition 1. [2] *An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form A* $= \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ *where the functions* $\mu_A : X \to [0,1]$ *and* $\nu_A : X \to [0,1]$ *define the degree of membership and the degree of non-membership of the element* $x \in X$ *respectively, and for every* $x \in X$ *in A,* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ *holds.*
Note: Here after, in this paper, μ *represents membership values and* ν *represents non-membership values.*

Definition 2. [2] *For every common fuzzy subset A on X, Intuitionistic Fuzzy Index of x in A is defined as* $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ *. It is also known as degree of hesitancy or degree of uncertainty of the element* x *in A. Obviously, for every* $x \in X$ *,* $0 \leq \pi_A(x) \leq 1$.

Definition 3. [9] *An Intuitionistic Fuzzy Number (IFN)* $\tilde{A}^I$ *is*
i) an intuitionistic fuzzy subset of the real line,
ii) normal, that is, there is some $x_0 \in R$ *such that* $\mu_{\tilde{A}^I}(x_0) = 1, \quad \nu_{\tilde{A}^I}(x_0) = 0$,
iii) convex for the membership function $\mu_{\tilde{A}^I}(x)$, *that is,*
$\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$,
for every $x_1, x_2 \in R, \quad \lambda \in [0,1]$,
iv) concave for the non-membership function $\nu_A(x)$, *that is,*
$\nu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \leq max(\nu_{\tilde{A}^I}(x_1), \nu_{\tilde{A}^I}(x_2))$
for every $x_1, x_2 \in R, \quad \lambda \in [0,1]$.

Definition 4. [5] *A linear programming problem with fuzzy technological coefficients (FLPP) is defined as*

$$Maximize\ z = \sum_{j=1}^{n} c_j x_j$$

$$subject\ to \quad \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_i,\ 1 \leq i \leq m$$

$$x_j \geq 0,\ 1 \leq j \leq n$$

where at least one $x_j > 0$ *and* $\widetilde{a}_{ij}$ *is a fuzzy number with the following linear membership function*

$$\mu_{\widetilde{a}_{ij}}(x) = \begin{cases} 1 & if \;\; x < a_{ij} \\ \frac{a_{ij}+d_{ij}-x}{d_{ij}} & if \;\; a_{ij} \leq x < a_{ij}+d_{ij} \\ 0 & if \;\; x \geq a_{ij}+d_{ij} \end{cases}$$

where $x \in R$ *and* $d_{ij} > 0$ *for all i=1,2,...,m ; j=1,2,...,n.*

INTUITIONISTIC FUZZY LINEAR PROGRAMMING PROBLEMS

Necessity of IFLPP

For the linear programming problems in the crisp environment, the aim is to maximize or minimize a linear objective function under linear constraints. But in many practical situations, the decision maker may not be in a position to specify the objective function and/or constraints functions precisely but rather can specify them in a fuzzy sense.

It is possible to represent deeply existing nuances in problem formulation defining objectives and constraints by IF sets. The main advantages of the Intuitionistic Fuzzy Optimization(IFO) problems are that they give the richest apparatus for formulation of optimization problems and the solution of IFO problems satisfy the objective(s) with higher degree of determinacy than the fuzzy and crisp cases.

Formulation of IFLPP

In the framework of IFLPP, there are no additional assumptions about the nature of objective functions and constraints. According to different hypotheses being considered, distinct IFLP problems are obtained.

If the decision maker does not know exactly the values of the coefficients taking part in the problem(in this paper $a_{ij}^{'s}$ because the costs $C_j^{'s}$, $b_i^{'s}$ and $x_j^{'s}$ are supposed to be fixed throughout) and moreover, the vagueness is not of probabilistic kind, the inexact values can be determined by means of models using IF numbers. Here the general form of such an IFLPP is considered and the solution procedure is illustrated.

Definition 5. *An IFLPP with intuitionistic fuzzy technological coefficients is defined as*

$$\text{Maximize } z = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to} \sum_{j=1}^{n} \tilde{a}^I_{ij} x_j \leq b_i,\ 1 \leq i \leq m \qquad (1)$$

$$x_j \geq 0,\ 1 \leq j \leq n$$

where at least one $x_j > 0$ *and* $\widetilde{a}^I_{ij}$ *is an IFN.*

Definition 6. *Any vector* $x \in R^n$ *which satisfies the constraints and non negative restrictions of (1) is said to be an Intuitionistic Fuzzy Feasible Solution.*

Definition 7. *Let S be the set of all intuitionistic fuzzy feasible solutions of (1). Any vector* $x_0 \in S$ *is said to be an Intuitionistic Fuzzy Optimum Solution to(1) if* $Cx_0 \geq Cx$ *for all* $x \in S$ *where C =* $(c_1, c_2, ..., c_n)$ *and* $Cx = c_1x_1 + c_2x_2 + ... + c_nx_n$.

Definition 8. *The IF set of optimal values G which is a subset of* R^n*, is defined as*

$$\mu_G(z) = \begin{cases} 0 & \text{if } z < z_l \\ \frac{z - z_l}{z_u - z_l} & \text{if } z_l \leq z < z_u \\ 1 & \text{if } z \geq z_u \end{cases}$$

$$\text{and} \qquad (2)$$

$$\nu_G(z) = \begin{cases} 1 & \text{if } z < z_l \\ 1 - c - \mu_G(z) & \text{if } z_l \leq z < z_u \\ 0 & \text{if } z \geq z_u \end{cases}$$

Here 'c' is called the intuitionistic fuzzy index and the value of 'c' is chosen such that $0 < c < \frac{z_u - z}{z_u - z_l} < 1$.
As 'z' approaches its maximum value, the value of 'c' approaches 'zero'.

Definition 9. *The intuitionistic fuzzy set of the* i^{th} *constraint,* c_i*, which is a subset of* R^m *, is defined as*

$$\mu_{c_i}(x) = \begin{cases} 0 \\ \text{if } b_i < \sum_{j=1}^{n} a_{ij}x_j \\ \frac{b_i - \sum_{j=1}^{n} a_{ij}x_j}{\sum_{j=1}^{n} d_{ij}x_j} \\ \text{if } \sum_{j=1}^{n} a_{ij}x_j \leq b_i < \sum_{j=1}^{n} (a_{ij} + d_{ij})x_j \\ 1 \\ \text{if } b_i \geq \sum_{j=1}^{n} (a_{ij} + d_{ij})x_j \end{cases}$$

$$\text{and} \qquad (3)$$

$$\nu_{c_i}(x) = \begin{cases} 1 \\ \text{if } b_i < \sum_{j=1}^{n} a_{ij}x_j \\ 1 - c - \mu_{c_i}(x) \\ \text{if } \sum_{j=1}^{n} a_{ij}x_j \leq b_i < \sum_{j=1}^{n} (a_{ij} + d_{ij})x_j \\ 0 \\ \text{if } b_i \geq \sum_{j=1}^{n} (a_{ij} + d_{ij})x_j \end{cases}$$

Definition 10. *The linear membership and non-membership functions of the IFN* $\widetilde{a}^I_{ij}$ *are defined as*

$$\mu_{\widetilde{a}^I_{ij}}(x) = \begin{cases} 1 & \text{if } x < a_{ij} \\ \frac{a_{ij} + d_{ij} - x}{d_{ij}} & \text{if } a_{ij} \leq x < a_{ij} + d_{ij} \\ 0 & \text{if } x \geq a_{ij} + d_{ij} \end{cases}$$

$$\text{and} \qquad (4)$$

$$\nu_{\widetilde{a}^I_{ij}}(x) = \begin{cases} 0 & \text{if } x < a_{ij} \\ 1 - c - \mu_{\widetilde{a}^I_{ij}}(x) & \text{if } a_{ij} \leq x < a_{ij} + d_{ij} \\ 1 & \text{if } x \geq a_{ij} + d_{ij} \end{cases}$$

where $x \in R$ *and* $d_{ij} > 0$ *for all* $i = 1, 2, ..., m;\quad j = 1, 2, ..., n$ *and also* $0 < c < \frac{z_u - z}{z_u - z_l} < 1$.

Intuitionistic Fuzzy Optimization Model

Generally, an optimization problem includes objectives and constraints. Intuitionistic fuzzy optimization, a method of uncertainty optimization, is derived from [2]. In [1], maximization of the degree of acceptance and minimization of the degree of rejection of the IF objective function and constraints are defined as follows:

$$\begin{aligned} &\max \quad \mu(x), \quad \min \quad \nu(x) \\ &subject \;\; to \\ &\mu(x) \quad \geq \quad \nu(x) \\ &0 \quad \leq \quad \mu(x) \quad + \quad \nu(x) \quad \leq \quad 1 \\ &\mu(x), \quad \nu(x) \quad \geq \quad 0, \quad x \quad \geq \quad 0 \end{aligned}$$

where $\mu(x)$ and $\nu(x)$ denote the degrees of acceptance and rejection of x respectively. It is an extension of fuzzy optimization in which the degrees of rejection of objectives and constraints are considered together with the degrees of satisfaction. The above problem is equivalent to the following:

$$\begin{aligned} &\max \quad \alpha, \quad \min \beta \\ &subject \;\; to \\ &\alpha \leq \mu(x) \\ &\beta \geq \nu(x) \\ &\alpha \geq \beta \quad and \;\; 0 \quad \leq \quad \alpha+\beta \leq 1; \quad \alpha, \beta \geq 0; \quad x \geq 0 \end{aligned}$$

where α denotes the minimal acceptable degree and β denotes the maximal degree of rejection. The IFO model can be changed into the following certainty(non-fuzzy) optimization model [1] as follows:

$$\begin{aligned} &\max(\alpha-\beta) \\ &subject \;\; to \\ &\alpha \leq \mu(x) \\ &\beta \geq \nu(x) \\ &\mu(x) \geq \nu(x) \\ &\alpha \geq \beta; \quad and \;\; 0 \quad \leq \quad \alpha+\beta \leq 1; \quad \alpha, \beta \geq 0; \quad x \geq 0 \end{aligned} \tag{5}$$

Here α, β are treated as variables in the objective function and so the linear programming problem is converted into non-linear programming problem which cannot be solved by using usual simplex methods. So, the following method, which is being first of its kind, is used to solve LPP in an IF environment.

THE INTUITIONISTIC FUZZY DECISIVE SET METHOD

In this method, a combination of the bisection method and phase one of the simplex method of linear programming problem is used to obtain a feasible solution.

For fixed values of α and β, the problem (5) is a linear programming problem. Obtaining the optimal solution (α^*,β^*) to (5) is equivalent to determining the maximum value of α and the minimum value of β so that the feasible set is nonempty. The algorithm of this method for (5) is presented below.

Algorithm for solving an IFLPP

Step 1 : Set α, β in the interval (0,1) such that $\beta = 1 - c - \alpha$ where $c \in (0, 1)$ and the difference between α and β should not approach the value zero and test whether a feasible set satisfying the constraints of the problem (5) exists or not using phase one of the simplex method.

If a feasible set exists,

set $\alpha^L = \alpha$; $\beta^L = \beta$ and $\alpha^R = \beta$; $\beta^R = \alpha$

Otherwise, set $\alpha^L = \beta$; $\beta^L = \alpha$ and $\alpha^R = \alpha$; $\beta^R = \beta$

and go to the next step.

Step 2 :

For the value of $\alpha = (\alpha^L + \alpha^R)/2$ and $\beta = (\beta^L + \beta^R)/2$, update the values of α^L,α^R,β^L and β^R,using the bisection method as follows :

(i) If feasible set is nonempty for α and β, set $\alpha^L = \alpha$ and α^R as it's value in the preceding step. $\beta^L = \beta$ and β^R as it's value in the preceding step.

(ii) If feasible set is empty for α and β, set $\alpha^R = \alpha$ and α^L as it's value in the preceding step. $\beta^R = \beta$ and β^L as it's value in the preceding step.

Consequently, for each α and β, test whether a feasible set of (5) exists or not, using phase one of the simplex method and determine the maximum value α^* and the minimum value β^* satisfying the constraints of (5).

When the intuitionistic fuzzy decisive set method is applied to a given IFLPP, the values of α and β in (0,1) remain constant for a particular value of 'c' in (0,1) which is the optimum solution.

Conversely when the solution is optimum, the value of 'c' remains constant in (0,1).

A Numerical Example

A Company produces two types of products P_1 and P_2. These products are processed on two different machines M_1 and M_2. It is noted that the time required to manufacture each product can vary from day to day due to breakdown of machines, overtime work, etc. At the same time, the time required to manufacture one unit of product is **some what close to 1 hour** for P_1 and 2 hours for P_2 on machine M_1 and 4 hours for P_1 and 3 hours for P_2 on machine M_2, **with hesitancy degree 0.2 each.** The capacity of the machine M_1 is 6 hours/day and that of M_2 is 12 hours/day and the company wants to keep the profit of Rs.7 for P_1 and Rs.5 for P_2. The company wants to determine the number of units of products P_1 and P_2 to be produced per day to maximize it's profit. It is assumed that all the amounts produced are consumed in the market.

Since the time required to manufacture each product is uncertain with hesitancy degree 0.2, the given problem can be Modeled as an IFLPP with intuitionistic fuzzy technological coefficients. So, the problem is formulated as follows:

$$\text{Maximize } 7x_1 + 5x_2$$

subject to

$$\widetilde{1}^I x_1 + \widetilde{2}^I x_2 \leq 6 \quad (6)$$

$$\widetilde{4}^I x_1 + \widetilde{3}^I x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

which take intuitionistic fuzzy parameters as $\widetilde{1}^I = L(1,1)$, $\widetilde{2}^I = L(2,4)$, $\widetilde{4}^I = L(4,2)$ and $\widetilde{3}^I = L(3,4)$ as used in [5] and [12].

By using the IF optimization model, problem (6) can be reduced to the following equivalent non-linear programming problem:

$$\text{maximize } (\alpha - \beta)$$

subject to

$$7x_1 + 5x_2 \geq 14 + 7\alpha \; ; \; 7x_1 + 5x_2 \geq 19.6 + 7\beta$$

$$(1+\alpha)x_1 + (2+4\alpha)x_2 \leq 6 \; ; \; (1.8-\beta)x_1 + (5.2-4\beta)x_2 \leq 6 \quad (7)$$

$$(4+2\alpha)x_1 + (3+4\alpha)x_2 \leq 12; (5.6-2\beta)x_1 + (6.2-4\beta)x_2 \leq 12$$

$$14x_1 + 10x_2 \geq 33.6$$
$$2.8x_1 + 7.2x_2 \leq 12$$
$$9.6x_1 + 9.2x_2 \leq 24$$
$$0 \leq \alpha, \beta \leq 1$$
$$x_1, x_2 \geq 0.$$

By using the Intuitionistic Fuzzy Decisive Set Method, the optimal solution to the problem (7) is obtained as α^* = 0.44949, β^* = 0.35051, x_1^* = 2.4495 and x_2^* = 0.

This means that the vector (x_1^*, x_2^*) is a solution to the problem (6) which has the best membership grade α^* and the least non-membership grade β^* with intuitionistic index c = 0.2. The Intuitionistic Fuzzy Decisive Set Method

CONCLUSION

The new concept of LPP in an IF environment is introduced in this paper. This concept allows to define a degree of rejection which may not simply a complement of degree of acceptance. A special type of membership and non-membership functions have been proposed to solve the IFLPP. Intuitionistic fuzzification of objective function and constraints has also been done in this paper. Also, an IFLPP is solved using intuitionistic fuzzy decisive set method. The most important application of this IFLPP is to fit data using IF linear regression. The authors work on IF regression models to fit the data.

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