Performance Comparison of Fuzzy and Choquet Fuzzy Integral Control for Line of Sight Stabilization Application

¹Ravindra Singh, ²M. Hanumandlu, ³Shahida Khatoon and ³Ibraheem

¹Lastec, Drdo Delhi, India ²Department of Electrical Engineering, IIT Delhi, India ³Department of Electrical Engg. Jamia Millia Islamia Delhi, India

Abstract: Stabilization and tracking systems maintain the orientation of optical sensor" payloads" so they are pointed in the scenario dependent directions and held steady in inertial space along the selected orientation. The stabilization-tracking systems are mechanical assemblies that precisely control the angular position of the sensor's line of sight (LOS), so that it is isolated from its base-foundation dynamics and is pointed towards its intended target. These system form part of modern fire control systems (FCSs). The performance of fire control system mounted on a mobile platform, decreases exponentially with increase in the disturbance on the line of sight (LOS). The conventional controller designed to stabilize the LOS are dependent on mathematical model of the plant. In this modeling process usually the higher order dynamics is ignored and plants are linearized around the operating point. Fuzzy-knowledge-based-controller (FKBC) design presents a good methodology to stabilize the line of sight against disturbances and nonlinearities present in the system, but tuning of input and output membership function parameters is quite a complex process. To overcome this, a choquet fuzzy integral based control algorithm is developed for this servo system with nonlinear property and some uncertainties. In this paper we present the design of FKBC and choquet fuzzy controller and their performance comparison.

Key words: Line of sight (LOS) • Bandwidth (B.W) • Fuzzy Controller • Choquet Fuzzy Integral • Q-measure
 • λ-measure • Electro-optical fire control system (EOFCS)

INTRODUCTION

The electro-optical fire control system on movable carrier causes the vibration in the azimuth and elevation direction which induces causes the image blur and leads the tracking performance to fail. Hence, the LOS stabilization technology must be used to isolate the LOS from carrier disturbance in order to make sure accurate aiming, tracking and firing for the target.

The LOS stabilized systems are basically motion control systems [1,2]. A few methods for the LOS stabilized control have been proposed during recent years [2]. However, a majority of these algorithms are complex and difficult to be realized.

Here, a choquet fuzzy integral based control strategy is proposed. In this approach, fuzzy integral is treated as the defuzzified output of the non-additive fuzzy rules [7].

These fuzzy rules involve input fuzzy sets having overlapping information [3,4]. This overlapping information between adjacent fuzzy sets is captured through λ -measure [5]. As a result the overlapping fuzzy sets are represented by the fuzzy measures that are used for computing the Choquet fuzzy integral. Choquet fuzzy control enables the system to have the quicker dynamic response and smaller overshoot. In this paper, firstly a brief description of λ -measure and q-measure followed by Choquet integral is given. After that the method of identification is described and finally the results obtained by choquet fuzzy controller are discussed.

Problem Statement

Simulation Results of Proposed Choquet Fuzzy Integral Controller: In this paper, the plants under consideration consist of a gimballed payload that is driven by

Corresponding Author: Ibraheem, Department of Electrical Engg. Jamia Millia Islamia Delhi, India. E-mail: ibraheem 2k@yahoo.com

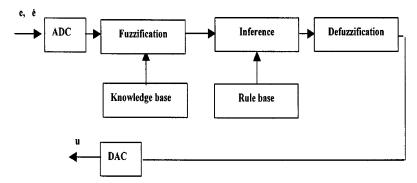


Fig. 1: Basic structure of fuzzy control scheme

a permanent magnet DC torque drives. A dual axis dynamically tuned gyro is used to sense the inertial angular rate of the gimbal in elevation and azimuth. The relevant parameters of gimbal system plant dynamics are as follows:

Gimbal inertia: 0.5 kg-sq mWeight of payload: 35 kg

Load pole: 1 Hz

Gimbal resonance: 140 HzTorque rating: 3.5nm (peak)

• Torque sensitivity(kt): 0.786 Nm/A

• Back emf constant(kb): 0.786 V/(rad/sec)

• Gyro scale factor: 5.73 V/rad.

• Gyro dynamics, singe pole: 100 Hz

• Data acquisition resolution:16 bits (max. input = $\pm 10 \text{ V}$)

• Dead band due to:10% stiction friction

• Digital-to-analog converter: 16 bits resolution,

The design considerations were as follows:

- Acceptable residual jitter on LOS <=100 micro radian
- Steady state error for step response <= 0.1%
- Overshoot <= 40%
- Rise time <=50 ms
- Typical disturbance frequencies 0.1 to 5 Hz
- Typical amplitude of disturbance input = 0.2 rad/sec

Fuzzy Controller: Figure 1 shows the basic structure of the fuzzy controller whose algorithm is briefly explained next. The input variables are error (e) and the rate of change of error (e). The output of the fuzzy controller gives the incremental control force (u). The membership functions were defined using the standard Gaussian function.

$$f(x, \sigma, c) = \exp[-(x-c)^2/2\sigma^2]$$

The proposed membership functions are continuous in the universe of discourse and therefore the inferred control output is smooth. Further, Gaussian function is not equal to zero anywhere and so provides a continuously fading over-lap with all the other membership functions. Figure 4 shows the membership functions for input and output variables. The scaling process is a trial- and- observation procedure. The selection of membership function parameters (c and ó) for different fuzzy sets of a variable is very important issue. The system performance is very sensitive to this selection. Since this selection depends heavily on the knowledge base of the designer, the experience of the designer is very vital. Table 1 indicates the definition of membership functions for the input variables (e and e) and the output variable (u). These membership functions were arrived at after a few iterations.

The rule base forms an important element to process the fuzzified inputs. The expert knowledge is usually in the form of "if-then" rules, which are easily implemented by fuzzy conditional statements in fuzzy logic. The collection of fuzzy control rules constitutes the rule base.

Table 1: Parameters of fuzzy membership functions.

Variables	e		e		e	
Finction Paraneters						
Fuzzy Sets	c	σ	c	σ	c	σ
nb	-1.0	0.35	-0.1	0.141	-0.1	0.141
nm	-0.25	0.1	-0.66	0.141	-0.57	0.142
ns	-0.1	0.04	-0.2	0.12	-0.15	0.1
z	0.0	0.013	0.0	0.05	0.0	0.007
ps	0.1	0.04	0.2	0.12	0.15	0.1
pm	0.25	0.1	0.66	0.141	0.57	0.142
pb	0.1	0.35	0.1	0.141	0.1	0.141

Tabl	le 2.	Rule	hase

e [·]							
e	nb	nm	ns	Z	ps	pm	pb
nb	nb	nb	nb	nb	nm	ns	z
nm	nb	nb	nb	nm	ns	z	ps
ns	nb	nb	nm	ns	Z	ps	pm
z	nb	nm	ns	z	ps	pm	pb
ps	nm	ns	z	ps	pm	pb	pb
pm	ns	z	ps	pm	pb	pb	pb
pb	Z	ps	pm	pb	pb	pb	pb

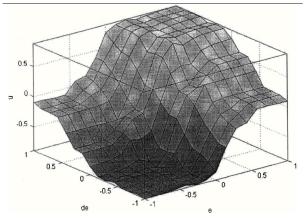


Fig. 2: Control surface of the fuzzy control law.

The aspect of rule base is discussed at length by Lee. The set of rules are given in the form of a matrix in Table 2. There are total 49 rules for all the possible combinations of input fuzzy sets.

A detailed discussion on various inference mechanisms is presented by Lee. In this paper

the most popular inference mechanism Mamdani was used. The output of the fuzzy inference block is a fuzzy output, which is to be defuzzified. This defuzzified number represents the incremental control voltage, to be fed to the actuator. The defuzzification has been done using centroid method. Figure 2 shows the control surface of the fuzzy control law. Figure 3 shows the block diagram for the simulation of control system using the fuzzy controller.

Since the output of the fuzzy controller is the incremental control output. The net control output is obtained by integrating the output of the fuzzy controller. The fuzzy control law computation is done in the digital domain. Therefore, analog inputs are quantized to simulate the digitization process.

The step response of the fuzzy control law is shown in Fig. 5. Figure 6 illustrates the corresponding control output. Figure 7 shows the residual jitter on LOS for random disturbance signal. Figures 8 and 9 illustrate the controller output corresponding to a random commanded input signal and the corresponding error in command following.

A Brief Description on Fuzzy Integral: In this section, we briefly describe basic fuzzy measures and fuzzy integrals. Several properties of the Choquet fuzzy integrals are then discussed.

Let $X=\{x_1, x_2, \dots, x_n\}$ be a finite set and let P(X) denote the power set of X or set of all subsets of X. A fuzzy measure over a set X is a function.

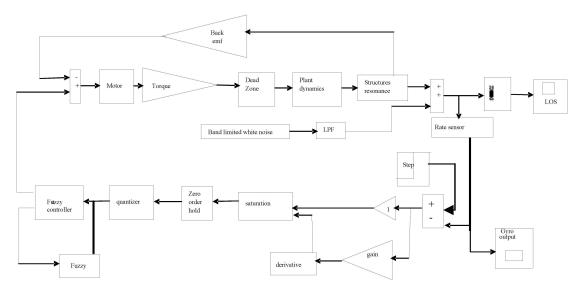


Fig. 3: Block diagram of the stabilization loop using fuzzy controller.

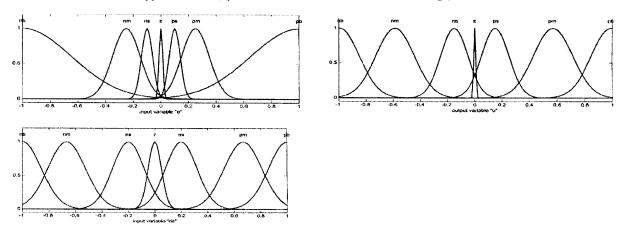


Fig. 4: Membership Functions of error(e), rate of change of error (e) and incremental control output(u)

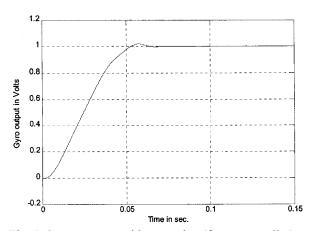
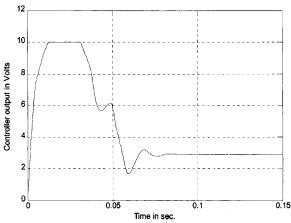
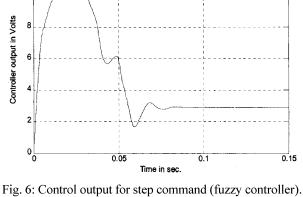


Fig. 5: Step response with saturation (fuzzy controller).





 $g(\Phi)=0, g(X)=1;$

g: $P(X) \rightarrow [0,1]$ such that:

If A, B \subset P(X) and A \subset B, then g(A) = g(B).

Sugeno introduced the so called λ fuzzy measure satisfying the following additional property: for all

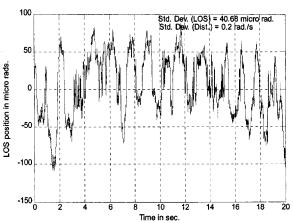


Fig. 7: Residual jitter on LOS for random signal.

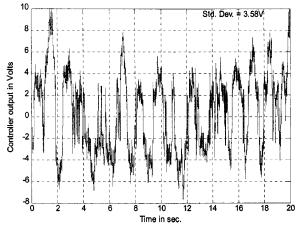


Fig. 8: Control output for disturbance attenuation.

$$A,\,B\subset X \text{ with } A\cap B=\Phi$$
 g (AUB)=g(A)+g(B)+ $\lambda g(A)g(B),$ for some fixed $\lambda \geq$ -1

The value of λ can be found from g(X) = 1, which is equivalent to solving

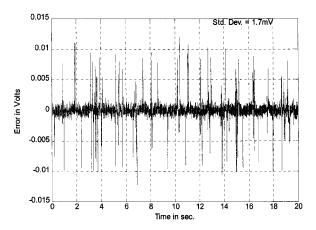


Fig. 9: Error in following random commanded input.

$$\lambda + 1 = \prod_{i=1}^{n} (1 + \lambda g_i)$$
 (1)

Let $A = \{x_i, x_{i+1}, ..., x_n\}$. When g is a λ fuzzy measure, the values of g (A_i) can be computed recursively as

$$g(A_n)=g(\{x_n\})=g_n$$
 (2)

$$g(A_i)=g_i+g(A_{i+1})+\lambda g_i g(A_{i+1}), \text{ for } 1=i=n.$$
 (3)

When a fuzzy measure is available on a finite set X, one can express the fuzzy integral as a computational scheme to integrate all values from the individual subset nonlinearly. In other words, the fuzzy integral relies on the concept of a fuzzy measure, which provides the degree to which some subsets of elements satisfy some characteristic.

We recall a general definition here. Given a class of functions $F \subseteq \{h : X \neg R \}$ and a class of fuzzy measures $m \subseteq M$ a functional

$$I:F\times m\rightarrow R$$
, $h,g\rightarrow I(h,g)$ is a fuzzy integral.

Clearly, there are a number of interesting families of fuzzy integrals in terms of the underlying fuzzy measures. One particular interest that we consider in this work is the Choquet fuzzy integral.

Description of Choquet Fuzzy Integral: Choquet Fuzzy Integral: The Choquet fuzzy integral is a fuzzy integral based on any fuzzy measure that provides alternative computational scheme for aggregating information. Assume $h(x_1),h(x_2),\ldots,h(x_n)$ are the evidence provided by the input sources x_1,x_2,\ldots,x_n , respectively and g is a λ ? fuzzy measure, then we can construct a Choquet Fuzzy Integral as

$$\int_{X} h(.) \circ g(.) \tag{4}$$

For a finite set of X, the Choquet Fuzzy Integral can be computed as follows:

$$E_{g}(h) = \sum_{i=1}^{n} [h(x_{i}) - h(x_{i-1})] g(A_{i})$$
 (5)

Where $h(x_1) = h(x_2) = ... = h(x_n)$ and $h(x_0) = 0$.

Another computation formula for the finite set case can also be represented by

$$E_{g}(h) = \sum_{i=1}^{n} (x_{i})[g(A_{i})-g(A_{i+1})]$$
 (6)

 R^k : If $x_1(j)$ is A_1^k and $x_2(j)$ is A_2^k and $x_3(j)$ is , ... and xn(j) is A_n^k , such that

$$A_i \cap A_{i+1} = \Phi$$
, then $y_k(j) = h_k(X(j))G_k$ (7)

In (4) the input information for the kth rule is aggregated as

$$h_{k}(X(j)) = \mu_{0k} + \mu_{1kx1}(j) + \dots + \mu_{nkxn}(j)$$

$$h_{k}(X(j)) = \mu_{0k} + \sum_{i=1}^{n} \mu_{ikxi}(j)$$
(8)

Where $\mu 0K=1$, k varies from 1 to r, r being the number of rules, n is the total number of inputs, j is the jth training sample, μ ik is the fuzzified value of ith input and for the kth rule, defined as

$$\mu ik(X) = \exp(-|aki(xi - cki)| |ki|)$$
 (9)

Where indices i, k indicate ith and kth rule, respectively. cki is the central value of the fuzzy set for the ith premise variable xi corresponding to kth rule, 1/aki represents the width of the fuzzy set.

Now, hk(X(j)) is the combined input which is the s-norm of the weighted input for the kth fuzzy rule and fuzzy measure Gk is defined corresponding to hk(X(j)). The final output is calculated using Choquet integral, which is a nonlinear addition of the new inputs hk(X(j)). The final output of the model given by Choquet integral in (6), aggregates yk from (7) as

$$y(j) = \sum_{k=1}^{r} h_k(X(j))G_k = \sum_{k=1}^{r} h_k(X(j))(g_{hk} - g_{h(k+1)})$$
 (10)

as the input information is combined in hk(X(j)), Gk is chosen as in (7) such that fuzzy measures are related by the λ -measure as follows:

$$g_{hk} = gk + g_{h(k+1)} + \lambda g_k g_{h(k+1)} = f$$
 (11)

(3), it is obvious that the calculation of the Choquet Fuzzy Integral with respect to? fuzzy measure requires the knowledge of the fuzzy density g and the input value h.

Fuzzy Modeling: In the fuzzy modeling, the consequent part of the rule makes use of the Choquet integral and q-measure, so as to combine the input information non-linearly. The use of q-measure makes both λ and fuzzy density independent. A rule of the following form explains the model.

The fuzzy measures are computed using the new rule inputs as they correspond to rules. Initially we combine two single element sets $h_1(X(i))$ and $h_2(X(i))$. Then we obtain a two element set. Next we combine this with a single element set to yield a three element set. This is continued until all rule inputs are covered. When we deal with single element sets we require their fuzzy densities. If the set is of more than two elements we need to consider their fuzzy measure. f is the scaling function and λ -measure brings in the interaction effect. First g_k 's which are fuzzy densities are randomly initialized and ghk is calculated, i.e. $g_{hr} = g_r$. Then $g_{h(k+1)}$ is recursively calculated by (11), by varying k from r-1 to 1. $h_k(X(j))$ gives the overall grade of the premise part of the kth rule. Using qmeasure in (8), the fuzzy densities and measure are modified as

$$g_k = g_k/f \ and \ g_{h(k+1)} = g_{h(k+1)} / f$$
 hence new g_{hk} is calculated according to (8).

Given a set of density generator values [f', f'', ..., f], the q-measure is given in (9):

q:
$$\Omega \rightarrow [0,1]$$
 on W by $q(h) = f(h)/f(X)$, $\forall h \subseteq X$ (12)

Using the above normalization, a fuzzy measure can be constructed for any choice of the variable $\lambda \varepsilon$ [-1, ∞). The Sugeno λ -measure is a special case that arises when λ is selected subject to the constraint, f(X) = 1. As in [13] the solution to λ is simplified with the use of q-measure discussed here.

The value of λ is learned by taking the derivative of f in [11] with respect to λ (as the scaling function should be converging one and hence λ should converge). However, in the literature, initially λ calculations were

tedious time consuming [2]. By using q-measure, this problem can be eliminated. As λ is learned with respect to scaling function the problem of its choices does not arise. So, λ is updated as

$$\lambda (j+1) = \lambda(j) - \partial f / \partial \lambda \tag{13}$$

Where

$$\partial f/\partial \lambda = g_k g_{h(k+1)}$$

The proposed fuzzy rule is similar to that of T-S model as far as premise part is concerned but is different in the consequent part. The formation of $h_k(x_i)$ which is the input to the Choquet Integral is entirely unique thus giving better output. This input information is obtained by fuzzifying the input data and then sorting the fuzzified data to form the source of information to the Choquet integral system.

Learning of the Model: Fuzzy curve method [18] is employed along with a heuristic approach for the determination of the number of rules. We then specify the membership function and initialize the parameters of the membership functions of the model. The count of the maxima and minima of the fuzzy curve gives the minimum number of rules needed to approximate each fuzzy curve. This is a heuristic devised on the concept that fuzzy model interpolates between maxima and minima. If the maxima and minima are far apart, or the curve is not smooth, a rule may be added. For n we will have nfuzzy curves that will yield n different numbers m₁, m₂, m_n. The minimum number of rules needed to construct the model is $m_{min} = max (m_1, m_2, \dots, m_n)$ and the maximum number of rules is $m_{max} = (m_1 \times m_2 \times ... \times m_n)$. The approaches for determining the minimum number of rules are discussed in [2].

Fine tuning of rules can be achieved by minimizing the objective function J. It is a function of normalized mean square error with respect to the parameters a_{kl} , c_{kl} and g_k . The following form is assumed for J.

$$J = \frac{1}{2My_r} \sum_{j=1}^{M} e^2(j)$$
 (14)

Where, $e(j) = y_d(j) - y(j)$ and $y_r = [\max \{y_d\} - \min \{y_d\}]^2$. Here, yd is the desired output and y is the actual output. The reason for using normalized mean square error is that it provides a universal platform for model evaluation irrespective of application and target value specification while selecting an input to the model. Choquet fuzzy integral is the output of the model that we are seeking to build.

The parameter update equation is

$$w(j+1) = w(j) + \Delta w \tag{15}$$

B. Parameter Update Formula: We apply well-known gradient descent learning law [20] to update the parameters, aki, cki and gk by finding the respective increment as follows:

$$\Delta aki = -2\eta \partial J/\partial aki$$
, $\Delta cki = -2\eta \partial J/\partial cki$, $\Delta gk = -2\eta \partial J/\partial gk$

We will use the chain rule to compute the derivative of J with respect to aki given by

$$\partial J/\partial aki = \partial J/\partial e$$
. $\partial e/\partial v$. $\partial v/\partial aki$

Where is the learning rate and $\eta > 0$. Similarly, we can derive derivatives of J with respect to cki and gk.

By using the chain rule in above given equations, we obtain

$$\partial J/\partial aki = e/M yr(-1) \times (ghk - gh(k+1))_{\mu}ik (1-lki)| aki(xi-cki)| lki-1 (xi-cki)$$

$$\partial J/\partial cki = e/M yr(-1) \times (ghk - gh(k+1))_{\mu}ik (lki-1)| aki(xi-cki)| lki-1 aki$$

$$\partial J/\partial gk = e/M \text{ yr}(-1) \text{ xi}\mu\text{ik} (1+\lambda gh(k+1))$$

Fuzzy Partitioning of Premise and Consequent Variables: For the proposed architecture of the model, the number of fuzzy partitions of all the variables is equal to the number of rules. As per the Lin's method [11], initially the middle points of the first and the last fuzzy partitions are at the beginning and at the end of range of each variable and the middle points of the rest of the fuzzy partitions are located at the interval of $\{range/(m-1)\}$. We take an adaptive membership function for each input partition. The width is taken as $\{\alpha^* \text{ interval}\}$; where $\alpha \in [0.5,2]$.

Initialization of Parameters: Following are the steps to initialize the membership functions for premise variable:

• Divide the domain of each fuzzy curve into m intervals, i.e, a_{ki} , c_{ki} and l_{ki} (i = 1 to $n, k = 1, \dots, r$)

- such that the width of each interval is equal to the range of x_i / r-1, except the widths of first and last intervals both of which are half of the range of x_i / r-1.
- For any curve, label the centers of intervals x_{ik}(k=2,....k 1), for k=1 the centre value is the beginning of the range and for k=r the centre value is at the end of the range. Order x_{ik} by the corresponding value of the fuzzy curve, then x_{ir} corresponds to the interval containing the largest value.
- The length of an interval over which a rule is applied in the domain of the fuzzy curve is denoted by $?x_{i.}$ Initial fuzzy membership function of x_{i} for rule k is defined as $\mu_{i}^{k}(x_{i}) = \exp\{-|(x_{i}-x_{ki})/(\alpha\Delta x_{i})|_{ki}^{k}\}$, where α is typically in the range of [0.5, 2]. The initial parameters are $a_{ki} = 1/\alpha\Delta x_{i}$ and $c_{ki} = x_{ki}$.
- Set $l_{ki} = 1$ or 2.

Performance Comparison and Discussion: Table 3 shows the comparison of the choquet fuzzy controller and fuzzy controller based LOS stabilization loops. In this application, overall performance can be divided in two main categories, namely, disturbance attenuation characteristics and dynamic time response.

It can be verified easily that for this design, the choquet fuzzy controller scores over the fuzzy controller.

Table 3: Results of choquet fuzzy controller and fuzzy controller LOS stabilization loop

	Choquet-fuzzy integral				
Characteristics	based controller	Fuzzy Controller			
Rise Time(ms)	30.4	37			
Settling Time(ms)	44	70			
% Overshoot	1	2			
Steady State Error(%)	0.01	0.05			
Residual jitter for 0.2 rad/	S				
disturbance rate(µrad)	34.25	44			

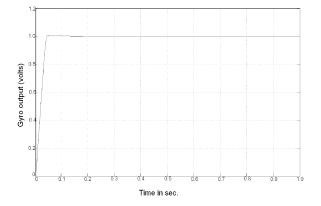


Fig. 10: Step Response Choquet Fuzzy Integral Controlled Stabilization Loop

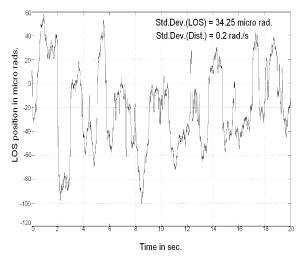


Fig. 11: Residual Jitter on LOS for random signal (Choquet Fuzzy Controller)

The disturbance attenuation of the system is much better in the choquet fuzzy control system for both deterministic and random disturbance inputs. It can be observed that choquet fuzzy controller performs well in the presence of nonlinearities also.

Here, Table 3. has the summary of the results of proposed controller and meet the desired specifications. The Fig. 3 shows the block diagram of choquet Fuzzy Controlled stabilization loop. Fig. 10 shows the step response of choquet fuzzy controller of stabilization loop of the proposed plant and Fig.11 depicts the LOS Jitter characteristics of Choquet Fuzzy Controller

CONCLUSION

Control laws using fuzzy control technique and choquet fuzzy integral technique were designed for a LOS stabilization problem. Very stringent requirements of disturbance attenuation and command following were met through both the approaches. Simulated results for both the designs were presented incorporating different nonlinearities such as dead band, saturation, quantization, etc. The fuzzy control laws, which were chosen for this study, gave results that were not as good as the choquet fuzzy integral controller, particularly in the presence of system nonlinearities. However, this fact cannot be generalized and results may vary from system to system. Till now, choquet integral has been widely used in the areas of pattern recognition and image processing. In this paper, we have tried to use it for dynamical plants to control the line of sight. Very stringent requirements of

fast response and very less steady state error are met through this approach of choquet fuzzy integral control. Simulated results are presented incorporating different nonlinearities such as dead band, saturation, quantization etc. It has been seen that in terms of dynamic time response characteristics, choquet fuzzy controller is performing as per desired specifications.

ACKNOWLEDGMENT

The authors extend heartfelt thanks to the Director, Laser Science and Technology Centre for his constant encouragement and initiative in pursuing the above-stated work. The authors are also thankful to all other colleagues, whose cooperation was sought.

REFERENCES

- Profeta, J.A., III, W.G. Vogt and M.H. Mickle, 1990. "Torque disturbance rejection in high accuracy systems", IEEE Trans. Aerospace Electronic System. 26(2): 232-237.
- Profeta, J.A., III, W.G. Vogt and M.H. Mickle, 1990. Disturbance estimation and compensation in linear system, IEEE Trans. Aerospace Electronic System. 26(2): 225-231.
- 3. Chiang, J.H., 1999. Choquet fuzzy integral-based hierarchical networks for decision analysis, IEEE Trans. Fuzzy Syst., 7(1): 63-71.
- 4. Lee, C.C., 1990. Fuzzy logic in control systems: fuzzy logic controller-part I, IEEE Trans. Electronic System Man Cyben. 20(2): 404-418.
- Lee, C.C., 1990. Fuzzy logic in control system: fuzzy logic controller-part II, IEEE Trans. Electronic System Man Cyben. 20(2): 419-435.
- Srivastava, S., M. Singh and A.N. Jha, 2002. Control and identification of a non linear systems affected by noise using wavelet network, in: Proceedings of International Workshop on Intelligent Systems Design and Application (ISDA-2002), Atlanta, USA, August,
- Mohammed, M.A., 2003. Q-measures: an efficientextension of the Sugeno λ-measure, IEEE Trans. Fuzzy Syst., 11(3): 419-426.
- 8. Zadeh, L.A., 1994. Aggregation operators and fuzzy systems modeling, Fuzzy Sets Syst., 67: 129-146.
- 9. Lang, L.C. and J.S. Kwon, 2000. On the representation of Choquet integrals of set valued functions and null sets, Fuzzy Sets. Syst., 112: 233-239.