Micropolar Fluid Flow in a Channel with Shrinking Walls

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Abstract: A numerical study is presented for the two dimensional flow of a micropolar fluid in a channel with shrinking walls. The fluid motion is assumed to be steady, laminar and incompressible. The micropolar model due to Eringen is used to describe the working fluid. The governing equations of motion are reduced to a set of nonlinear coupled ordinary differential equations (ODEs) in dimensionless form by using similarity transformations. A numerical algorithm based on finite difference discretization is employed to solve these ODEs. The results obtained are further improved by Richardson’s extrapolation for higher order accuracy. The micropolar fluids reduce shear stress and increase couple stress at the walls as compared to the Newtonian fluids which may be beneficial in the flow control of polymeric processing.

Key words: Shrinking walls • Micropolar fluids • Couple stress • Microrotation

INTRODUCTION

Laminar flows through channels have applications in the fields of binary gas diffusion, ablation cooling, filtration, microfluidic devices, surface sublimation, grain regression (as in the case of combustion in rocket motors) and the modeling of air circulation in the respiratory system [1]. Sutton and Barto [2] described an exact solution of Navier-Stokes equations for motion of an incompressible viscous fluid in a channel with different pressure gradients. Their solutions are helpful in verifying and validating computational models of complex unsteady motions, to guide the design of fuel injectors and controlled experiments. Simulation of flow through microchannels with design roughness was presented numerically by Rawool et al. [3]. A numerical investigation is made by Robinson [4] for the problem of steady laminar incompressible flow in a porous channel with uniform suction at both walls. Taylor et al. [5] studied three dimensional flow of a viscous incompressible fluid driven along a channel by uniform suction through parallel porous walls. The investigations of Taylor [5] were further extended to a more general three dimensional stagnation point which can capture the phenomena in a single class of state by Hewitt et al. [6]. Two dimensional viscous incompressible fluid flow between two porous walls with uniform suction was analyzed by Cox [7]. Berman [8] proposed the two dimensional laminar steady flow through a porous channel which was driven by suction or injection. Similarly one/two dimensional laminar fluid flow in a porous channel with wall suction or injection was examined analytically by Laurent et al. [9].

Qi et al. [10] modeled and analyzed the problem of fully developed laminar pulse flow of an incompressible fluid through rectangular ducts using Green’s function. The problem of fluid flow in a channel with porous walls was solved by Karode [11]. Zheng et al. [12] investigated asymptotic solutions for steady laminar flow of an incompressible viscous fluid along a channel with accelerating rigid porous walls. The exact solution for two dimensional steady laminar flow through a porous channel was generalized by Terril [13], Shrestha and Terril [14, 15], Brady [16], Waston et al. [17] and Cox [18] under various conditions. Deng and Martinez [19] worked on two dimensional flow of a viscous fluid in a channel partially filled with porous medium with wall suction. Wang [20] worked on viscous flow due to stretching sheet with slip and suction and proved a closed form unique solution for two dimensional flows. For axisymmetric stretching both existence and uniqueness were shown.

In all the investigations cited above, the contributors are limited themselves to the classical Newtonian fluids only. The study of non-Newtonian fluids has attracted much attention of the researchers because of their practical applications. With the growing importance of non-Newtonian fluids in modern technology and
industries, investigations of such fluids are desirable. A number of industrially important fluids including molten plastics, polymers, pulps, food and fossil fuels, which may saturate in underground beds, display non-Newtonian behavior. Due to complexity of fluids, several non-Newtonian fluid models have been proposed. In the category of such fluids, micropolar fluid model is most prominent. Hoyt and Fabula [21] predicted experimentally that the fluid, which can not be characterized by Newtonian relationships, indicates significant reduction of shear stress near a rigid body and it can be well explained by the micropolar model introduced by Eringen [22]. The micropolar fluid theory describes the flow of a class of non-Newtonian fluids that are endowed with micro-inertia. This allows the fluid to withstand stress and body couples. Misra et al. [23] investigated the problem of steady incompressible viscoelastic electrically conducting fluid and heat transfer in a channel with stretching walls numerically. Their results show that back flow occurs near the central line of the channel due to the stretching walls and this reverse flow can be stopped by applying a strong magnetic field. Numerical simulation of two dimensional steady laminar incompressible flow of a micropolar fluid between two disks was considered by Kamal et al. [24]. The problem of laminar flow of a micropolar fluid in a rectangular microchannel was analyzed numerically by Shangjun et al. [25] using Chebyshev collection method. Two dimensional flow of micropolar fluids in a channel with suction or injection through the channel walls was investigated by Kelson et al. [26]. Si et al. [27] investigated analytical solution to the micropolar fluid flow through a semiporous channel with expanding or contracting walls by using the homotopy analysis method (HAM). Recently Ashraf et al. [28] presented numerical simulation of MHD stagnation point flow and heat transfer of a micropolar fluid over a shrinking sheet using an algorithm based on finite differences.

The present study aims at studying numerically the flow characteristics of a viscous incompressible micropolar fluid in a channel with shrinking walls. The transformed ordinary differential equations are discritized by using the central finite differences.

**Basic Analysis:** Let us consider two dimensional steady laminar flow of an incompressible micropolar fluid in a parallel plate channel with shrinking walls. The body force and body couples are assumed to be absent. The two walls are located at \( y = -a \) and \( y = a \), where \( 2a \) is the channel width as shown in Fig. 1.

![Fig. 1: A sketch of the physical problem](image)

The general equations governing the motion of micropolar fluids as given by Eringen [22] may be expressed as

Continuity equation: \( \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \) \hspace{1cm} (1)

Momentum equation:

\[
(\lambda + 2\mu + k) \nabla (\nabla \cdot \mathbf{v}) - (\mu + k)\nabla \times \nabla \times \mathbf{v} + k \nabla \times \mathbf{v} - \nabla p + \rho f = \rho \ddot{\mathbf{v}}
\] \hspace{1cm} (2)

Angular momentum equation:

\[
(\alpha + \beta + \gamma) \nabla (\nabla \cdot \mathbf{u}) - \gamma (\nabla \times \nabla \times \mathbf{u})
\]

\[
+ k \nabla \times \mathbf{u} - 2K \mathbf{u} + \rho_j \mathbf{J} = \rho j \mathbf{\Phi}
\] \hspace{1cm} (3)

In these equations \( \mathbf{v} \) is the fluid velocity, \( \rho \) is the microrotation, \( P \) is the pressure, \( f \) and \( J \) are body force and body couple per unit mass respectively, \( j \) is the micro-inertia, \( \lambda, \mu, \alpha, \beta, \gamma \) and \( k \) are the micropolar material constants (or viscosity coefficients) and the dot signifies material derivative.

We may express the velocity \( \mathbf{v} \) and the microrotation \( \mathbf{u} \), respectively, in the following form

\[
\mathbf{v} = (u(x, y), v(x, y), 0), \quad \mathbf{u} = (0, 0, \phi(x, y))
\] \hspace{1cm} (4)

where \( \phi \) is the component of the microrotation normal to the \( xy \)-plane.

In view of Eq. (4) the equations of motion (1) - (3) for our problem take the form

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\] \hspace{1cm} (5)

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{(\mu + \kappa)}{\rho} \nabla^2 u + \frac{\kappa}{\rho} \frac{\partial \phi}{\partial y}
\] \hspace{1cm} (6)
\[
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{-1}{\rho} \frac{\partial p}{\partial y} + \frac{(\mu + \kappa)}{\rho} \nabla^2 v - \frac{\kappa}{\rho} \frac{\partial \phi}{\partial x} .
\]
\[
(7)
\]
\[
\rho \left[ \frac{\partial}{\partial x} (u \frac{\partial \phi}{\partial x}) + \frac{\partial}{\partial y} (v \frac{\partial \phi}{\partial y}) \right] = \gamma \nabla^2 \phi + \kappa \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) - 2\kappa \psi .
\]
\[
(8)
\]

The boundary conditions for the velocity and microrotation fields are
\[
u = bx, \quad v = 0, \quad \phi = 0 \quad \text{at} \quad y = \pm a
\]
\[
(9)
\]

Here \( b < 0 \) for shrinking of the channel walls. In order to obtain the velocity and the microrotation fields for the problem under consideration, we have to solve Eqs. (5) - (8) subject to the appropriate boundary conditions (9). For this purpose, we define the following similarity variables to convert the governing partial differential Eqs. (5) - (8) into ordinary differential equations
\[
\eta = \frac{y}{a}, \quad u = bx \eta' (\eta),
\]
\[
v = -abf (\eta) , \phi = -\frac{b}{a} xg (\eta).
\]
\[
(10)
\]

The above proposed velocity field is compatible with the continuity Eq. (5) and, therefore, represents the possible fluid motion. Since the pressure term is eliminated from the governing equations, as we shall see later, therefore the number of unknowns is reduced by one and we do not need to consider the continuity equation any more.

Using Eq. (10) in Eq. (6) and (7), the following equations have been found
\[
- \frac{1}{\rho} \frac{\partial p}{\partial x} = b^2 x f^* f'' - b^2 x f f'' - \frac{(\mu + k)}{\rho} \frac{bx f''}{a} - \frac{b x g' k}{\rho a^2}.
\]
\[
(11)
\]
\[
- \frac{1}{\rho} \frac{\partial p}{\partial y} = ab^2 f f'' + \frac{(\mu + k)}{\rho} \frac{bf''}{a} - \frac{b g k}{\rho a}.
\]
\[
(12)
\]

In order to eliminate the pressure term, differentiating Eqs. (11) and (12) w. r. t \( x \) and \( y \) respectively, we get
\[
\frac{1}{\rho} \frac{\partial^2 p}{\partial x^2} = b^2 x f f'' - b^2 x f f'' - \frac{(\mu + k)}{\rho} \frac{bx f''}{a} + \frac{b x g' k}{\rho a^2},
\]
\[
(13)
\]
\[
\frac{1}{\rho} \frac{\partial^2 p}{\partial y^2} = 0 .
\]
\[
(14)
\]

Using Eq. (14) in Eq. (13) and after simplification we get,
\[
(1 + C_1) f'''' - C_1 g'' = R \left( f'' - f' \right). \]
\[
(15)
\]

Integration of Eq. (15) with respect to \( \eta \) yields
\[
(1 + C_1) f''' - C_1 g' = R \left( f' - f \right) + B .
\]
\[
(16)
\]

Using Eq. (10) in Eq. (8) and after simplification we get,
\[
C_3 g'' + C_1 C_2 (f' - 2g) = f' g - fg'
\]
\[
(17)
\]

Here \( C_1 = \frac{\kappa}{\mu} \) is the vortex viscosity parameter,
\[
C_2 = \frac{\mu}{\rho b}
\]
\[
(18)
\]
\[
is the microinertia density parameter,
\[
C_3 = \frac{\gamma}{\rho a^2 b}
\]
\[
is the spin gradient viscosity parameter, \( R = \frac{\rho a^2 b}{\mu} < 0 \) is the shrinking Reynolds number and \( B \) is a constant of integration.

Boundary conditions (9) in view of Eq. (10) in dimensionless form are as under
\[
f (\pm 1) = 0, f' (\pm 1) = l, g (\pm 1) = 0
\]
\[
(19)
\]

**Numerical Solution:** We have to solve Eqs. (16) and (17) subject to the boundary conditions (18). We use a finite difference based numerical algorithm to solve these nonlinear ordinary differential equations. Following the method of reduction of order of differential equations as used by [24,33,34], it is better to reduce the order of the Eq. (16) by one with the help of the substitution \( f' = q \), so that the boundary value problem consisting of Eqs. (16) and (17) and boundary condition (18) takes the following form.

Solve
\[
f' = q
\]
\[
(19)
\]
\[
(1 + C_1) q'' - C_1 g' = R (q'' - f'') + B
\]
\[
(1 + C_1) q''' \approx R (q' - f') + B
\]
\[
(20)
\]
\[
C_3 g'' + C_1 C_2 (q' - 2g) = q' g - fg'
\]
\[
(21)
\]
subject to the associated boundary conditions
\[
f (\pm 1) = 0, q (\pm 1) = 1, g (\pm 1) = 0
\]
\[
(22)
\]

For the numerical solution of the present problem we first discretize the domain \([-1,1]\) uniformly with step \( h \). Eq. (19) is integrated using Simpson’s rule Gerald [35] with the formula given in Milne [36]. Eqs. (20) and (21) are discretized at a typical grid point \( \eta = \eta_i \) of the interval
by employing central difference approximations for the derivatives and then are solved iteratively by Successive over relaxation (SOR) method, Hildebrand [37], subject to the appropriate boundary conditions given in Eq. (22). In order to accelerate the iterative procedure and improve the accuracy of the solution, we use the solution procedure which is mainly based on the algorithm described by Syed et al. [38].

An iterative procedure is started with some initial guess for the values of the constant of integration $B$ and the solution vectors $q, g, f$. The iterative procedure is stopped if the following criteria is satisfied for four consecutive iterations

$$\max \left( \left\| q^{(i+1)} - q^{(i)} \right\|_2, \left\| g^{(i+1)} - g^{(i)} \right\|_2, \left\| f^{(i+1)} - f^{(i)} \right\|_2 \right) < \text{TOL}_{\text{iter}}$$

Here $\text{TOL}_{\text{iter}}$ is the prescribed error tolerance and we have taken at least $10^{-12}$ for our calculations during execution of a computer program in FORTRAN 90.

**RESULTS AND DISCUSSION**

In this section we present our findings in tabular and graphical form together with the discussion and their interpretations. As our objective is to develop a better understanding of the effects of micropolar structure of fluids, we choose to present shear and couple stresses at the two walls and the velocity and microrotation fields across the channel for a range of values of the shrinking parameter $R$ and a few cases of material properties of micropolar fluids represented by the micropolar parameters $C_1, C_2, C_3$. All the cases of the micropolar parameters in the present work are shown in Table 1. These values of $C_1, C_2, C_3$ are taken arbitrarily to investigate their influence on the flow as chosen customarily in the literature by [24, 29-32]. In order to establish the validity of our numerical computations and to improve the order of accuracy of the solutions, numerical results are computed for three grid sizes 0.02, 0.01 & 0.005 and then extrapolated using Richardson’s extrapolation. The comparison of numerical values of normal velocity for the three grid sizes and its extrapolated values is given in Table 2. Excellent comparison validates our numerical computation and the use of extrapolation.

First of all we present the effect of the shrinking Reynolds number $R$ on the shear and couple stresses at the channel walls. The numerical values of $f(-1)$ & $f(1)$ and $g(-1)$ & $g(1)$ which are proportional to the shear and couple stresses at the lower and upper walls respectively are given in Table 3 for typical values of the micropolar parameters $C_1, C_2, C_3$. We observe that the shear stresses exerted by the lower wall and upper wall are equal in magnitude but opposite in direction. The magnitudes of the shear and couple stresses decrease at the both walls by increasing the magnitude of $R$. The values of constant of integration $B$ for calculations at three grid sizes denoted by $B_1, B_2, B_3$ for the effect of $R$ are given in Table 4.

<table>
<thead>
<tr>
<th>Case No</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>1.2</td>
<td>1.4</td>
</tr>
</tbody>
</table>

**Table 1:** Six cases of values of $C_1, C_2, C_3$

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$f(\eta)$ at $h = 0.02$</th>
<th>$f(\eta)$ at $h = 0.001$</th>
<th>$f(\eta)$ at $h = 0.005$</th>
<th>Extrapolated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.8</td>
<td>-0.07285</td>
<td>-0.07286</td>
<td>-0.07286</td>
<td>-0.07286</td>
</tr>
<tr>
<td>-0.6</td>
<td>-0.09803</td>
<td>-0.09803</td>
<td>-0.09803</td>
<td>-0.09803</td>
</tr>
<tr>
<td>-0.4</td>
<td>-0.08632</td>
<td>-0.08633</td>
<td>-0.08633</td>
<td>-0.08633</td>
</tr>
<tr>
<td>-0.2</td>
<td>-0.0495</td>
<td>-0.04951</td>
<td>-0.04952</td>
<td>-0.04952</td>
</tr>
<tr>
<td>0</td>
<td>0.000044</td>
<td>0.000028</td>
<td>0.000021</td>
<td>0.000018</td>
</tr>
<tr>
<td>0.2</td>
<td>0.049586</td>
<td>0.049565</td>
<td>0.049556</td>
<td>0.049552</td>
</tr>
<tr>
<td>0.4</td>
<td>0.086408</td>
<td>0.086383</td>
<td>0.086372</td>
<td>0.086367</td>
</tr>
<tr>
<td>0.6</td>
<td>0.098113</td>
<td>0.098084</td>
<td>0.098071</td>
<td>0.098066</td>
</tr>
<tr>
<td>0.8</td>
<td>0.072943</td>
<td>0.072912</td>
<td>0.072897</td>
<td>0.072892</td>
</tr>
<tr>
<td>1</td>
<td>0.000089</td>
<td>0.000057</td>
<td>0.000042</td>
<td>0.000036</td>
</tr>
</tbody>
</table>

**Table 2:** Dimensionless normal velocity $f(\eta)$ on three grid sizes and extrapolated values for $R = -5, C_1 = 6, C_2 = 0.4, C_3 = 0.6$
The influence of the micropolar parameters $C_1, C_2$ & $C_3$ on the shear and couple stresses is given in Table 5. It is concluded that the shear stresses decrease where as the couple stresses increase at the walls by increasing the values of micropolar parameters $C_1, C_2$ & $C_3$. The values of constant of integration $B$ for various cases of the micropolar parameters are given in Table 6.

Comparison of Newtonian results with the Case 1, results of micropolar fluids, shows that for smaller values of $C_1, C_2$ & $C_3$ there is no significant effect of micropolar structure on the shear stresses. However as $C_1, C_2$ & $C_3$ are increased, the wall shear stresses decrease significantly indicating that microrotation decreases the viscous forces and increases the couple stresses at the walls. This fact has also been observed by [30] for the flow and heat transfer of micropolar fluids between two porous disks. Now in order to investigate the effect of the micropolar parameters $C_1, C_2$ & $C_3$ and the shrinking Reynolds number $R$ on the primary flow fields, we give graphical presentation of the streamwise & normal velocity profiles and the microrotation across the channel. The influence of the parameter $R$ on the streamwise velocity is presented in Fig. 2 for fixed values of the parameters $C_1, C_2$ & $C_3$. Streamwise velocity profiles are symmetric and parabolic for all values of $R$. The magnitude of the streamwise velocity of the fluid decreases near the surface of the walls and increases in the centre of the channel to obey the law of conservation of mass. The magnitude of maximum velocity increases by increasing the magnitude of the shrinking Reynolds number $R$.

Fig. 3 presents the profiles of normal velocity component across the channel to reflect the influence of $R$ on its behavior for typical values of micropolar parameters $C_1, C_2$ & $C_3$. The normal velocity takes its dimensionless value -1 at the lower wall and increases to 1 at the upper wall with a point of inflection at the centre of the channel where it changes its concavity. As the magnitude of $R$ is increased, the profiles reflect significant increase in the normal velocity within the channel away from the walls.

### Table 3: Effect of the shrinking Reynolds number $R$ on the shear and couple stresses for $C_1 = 6, C_2 = 0.4, C_3 = 0.6$

<table>
<thead>
<tr>
<th>$R$</th>
<th>$f(-1)$</th>
<th>$f'(1)$</th>
<th>$g'(-1)$</th>
<th>$g'(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5.7057</td>
<td>5.7057</td>
<td>-3.5406</td>
<td>-3.5406</td>
</tr>
<tr>
<td>-100</td>
<td>-5.1027</td>
<td>5.1027</td>
<td>-3.5254</td>
<td>-3.5254</td>
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<tr>
<td>-200</td>
<td>-4.5237</td>
<td>4.5237</td>
<td>-3.5100</td>
<td>-3.5100</td>
</tr>
<tr>
<td>-300</td>
<td>-3.9786</td>
<td>3.9886</td>
<td>-3.4947</td>
<td>-3.4947</td>
</tr>
</tbody>
</table>

### Table 4: Values of constant of integration $B$ at the three grid sizes for various values of $R$ given in Table 3.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>79.87905</td>
<td>79.88000</td>
<td>79.88070</td>
</tr>
<tr>
<td>-100</td>
<td>116.07311</td>
<td>116.07894</td>
<td>116.08040</td>
</tr>
<tr>
<td>-200</td>
<td>158.87878</td>
<td>158.89860</td>
<td>158.90357</td>
</tr>
<tr>
<td>-300</td>
<td>208.80099</td>
<td>208.84177</td>
<td>208.85197</td>
</tr>
</tbody>
</table>

### Table 5: Effect of micropolar parameters $C_1, C_2$ & $C_3$ on the shear and couple stresses for $R = -5$.  

<table>
<thead>
<tr>
<th>Case No</th>
<th>$f(-1)$</th>
<th>$f'(1)$</th>
<th>$g'(-1)$</th>
<th>$g'(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.2150</td>
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<tr>
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<td>5.7365</td>
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</table>

### Table 6: Values of constant of integration $B$ at the three grid sizes for the cases of $C_1, C_2$ & $C_3$ given in Table 5.

<table>
<thead>
<tr>
<th>Case No</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
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<td>-10.4772</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>-138.4503</td>
</tr>
<tr>
<td>4</td>
<td>-193.4447</td>
<td>-193.4195</td>
<td>-193.4120</td>
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<tr>
<td>5</td>
<td>-244.9610</td>
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<td>6</td>
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</tr>
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</table>
The magnitude of the normal velocity increases by increasing magnitude of shrinking Reynolds number $R$. The behavior of the microrotation across the channel for fixed values of the micropolar parameters $C_1, C_2$ & $C_3$ and for different values of the shrinking Reynolds number $R$ is shown graphically in Fig. 4. The shear stresses at the two walls tend to rotate the fluid in opposite directions because of which the microrotation has opposite signs near the walls. When $R = 0$, the effect of shear stresses propagates at equal rate from the walls across the channel resulting into zero microrotation at the mid point of the channel.

Fig. 5-7 Show streamwise & normal velocity and microrotation profiles for six sets of values of $C_1, C_2$ & $C_3$ (given in Table 1) when $R$ is fixed. The values of the micropolar parameters $C_1, C_2$ & $C_3$ have significant effect on microrotation as compared to their effect on streamwise and normal velocity profiles.
Fig. 6: Variation of dimensionless normal velocity $f(\eta)$

Fig. 7: Variation of dimensionless microrotation $g(\eta)$

The maximum values of streamwise and normal velocities and microrotation increase as we increase the values of the $C_1, C_2 \& C_3$.

CONCLUSIONS

In the present work we have considered the numerical solution of the problem of two dimensional steady, laminar and incompressible flow of a micropolar fluid in a channel with parallel shrinking walls. The objective of the present study is to investigate the effects of the shrinking parameter $R$ and the micropolar parameters $C_1, C_2 \& C_3$ on the flow variables. The results indicate that the parameters $R$ and $C_1, C_2 \& C_3$ have a strong influence on the velocity and microrotation profiles and shear and couple stresses at the walls. The micropolar parameters $C_1, C_2 \& C_3$ have profound effect on microrotation as compared to their effect on streamwise and normal velocity profiles. For the fluids with larger values of these parameters, the effect of micropolar structure cannot be ignored. The micropolar fluids reduce magnitude of shear stress and increase that of couple stress at the walls as compared to the Newtonian fluids.

REFERENCES


