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He's Homotopy Method for Investigation of Flow and Heat Transfer in a Fluid Saturated Porous Medium

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Abstract: In this paper, thermally fully developed flow is investigated analytically in a porous medium inside a channel bounded by two isoflux parallel plates. The Darcy-Brinkman-Forchheimer equation is solved analytically using Homotopy Perturbation Method (HPM) and numerically by finite volume method. The effect of Darcy number, viscosity ratio and Forchheimer number on velocity profile, temperature distribution and Nusselt are studied. The results show an excellent agreement between analytical and numerical solutions, which indicates that HPM is a reliable method for solving such non-linear differential equations pending appropriate choice of linear operator.

Key words: Homotopy perturbation method • Porous media • Fully developed flow • Brinkman-Forchheimer model

INTRODUCTION

The study of forced convection in porous media has attracted considerable interests since it is important in many applications such as electronic cooling, solar collectors and ground water studies [1-8]. The presence of such a wide application has made the heat transfer in porous media as one of the mostly emphasized subjects in the literature. Forced convection in channels with various cross sections filled with fluid- saturated porous medium is one of the issues in this field. Many authors have examined the characteristic of flow and heat transfer in channels filled with porous media. For instance, Kaviany [9] numerically studied laminar flow in a porous channel bounded by isoflux parallel plates based on Brinkman extended Darcy model. Vafai and Kim [10] using this model, analytically considered forced convection in thermally fully developed flow between two flat plates. Amiri and Vafai [11] numerically investigated the effects of non-thermal equilibrium and dispersion on the fully developed flow and heat transfer characteristic in a channel filled with a porous medium with variable porosity. Lee and Vafai [12] studied the effect of local thermal non-equilibrium condition using a two-equation model involving transverse conduction contributions. They obtained exact solutions for both fluid and solid phase temperature fields. Nield *et al*. [13] considered forced convection in a channel filled with fluid-saturated porous medium with isothermal or isoflux boundaries using numerical method for a parallel plate channel and Rassoulinejad-Mousavi and Abbasbandy [14] solved the Brinkman-Forchheimer problem for a circular tube by use of spectral homotopy analysis method. Nield [15] considered, thermally developing forced convection in a porous medium inside a parallel plate channel with uniform wall temperature and axial conduction as well as viscous dissipation effects. Hooman *et al*. [16] investigated first and the second law characteristics of fully developed forced convection inside a porous-saturated duct with rectangular cross-section using Darcy-Brinkman flow model. In their work, the Nusselt and Bejan numbers and the dimensionless entropy generation rate were presented in terms of the system parameters. Hooman and Merrikh [17] analyzed thermally and hydrodynamically fully developed forced convection in a rectangular duct filled with a hyper-porous medium. In their work, the Darcy-Brinkman model was adopted and a Fourier series type solution was utilized to obtain the exact velocity and temperature distributions within the duct. Cheng [18] investigated the effects of the modified Darcy number, the buoyancy ratio and the inner radius-gap ratio on fully developed natural convection heat and mass transfer in a vertical annular porous medium with asymmetric wall temperatures. El-Din [19] analytically considered the effect of thermal and mass buoyancy forces on fully developed laminar forced convection in a vertical channel. Barletta *et al*. [20] investigated fully developed

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parallel flow in an annular region filled with a porous medium with a varying magnetic field. Hung *et al*. [21] conducted an analytical study on fully developed forced convection in a homogeneous porous medium and obtained a closed form solution for the temperature distribution in transverse direction. Kou [22] by use of non-Darcy flow model solved fully developed laminar mixed convection in a vertical channel filled with porous media.

Despite the success of numerical solutions, analytical solutions have received considerable attention in various problems because they can be used to test inverse techniques and validating numerical models. In addition, they provide insight into the physics of flow and heat transfer. Recently, some new analytical methods have been developed for solving nonlinear ordinary and partial differential equations (ODEs and PDEs). Using these methods, many nonlinear problems in different fields of engineering can be solved analytically. Homotopy Perturbation Method (HPM) is one of the applicable and novel methods that introduced by He [23, 24]. Recently it has been used by many researchers to find analytical solutions of non-linear ODEs and PDEs. For example, Seyf and Rassoulinejad-Mousavi [25] solved 2D Darcy-Brinkman equation in a channel with moving walls and subjected to uniform wall injection/suction. Their analytical results obtained from HPM showed very good agreement with CFD. Ganji and sadighi [26] used HPM and variational iteration methods to solve nonlinear heat transfer and porous media equations and found a very good agreement between these methods and exact solution. Biazar *et al*. [27] solved general form of porous medium equation by HPM and compared the results with Adomian decomposition method. Seyf and Layeghi [28] using this method investigated the effect of Reynolds number and ratio of evaporator to condenser length on pressure drop and vapor velocity profile in a flat plate heat pipe.

The above review shows that all of the closed forms analytical solutions presented for fully developed flow in channels are mainly for Darcy and Darcy-Brinkman models. Finding an analytical exact solution for Darcy-Brinkman equation is easy in the light of [9] but when it comes to consider the effects of form drag by adding a non-linear term, the problem becomes more complicated and has not closed form solution. Therefore, in the present study HPM is applied to find an approximate solution for Darcy-Brinkman-Forchheimer equation in a porous saturated channel bounded by two isoflux parallel plates.

Fundamentals of Homotopy Perturbation Method: To explain the basic idea of homotopy perturbation method, lets us consider following differential equations:

$$A(r) = f(r) \qquad r \in \Omega \tag{1}$$

With the following boundary conditions:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \qquad r \in \Gamma \tag{2}$$

Where $f(r)$, is a known analytical function, A is a general differential operator, B is a boundary operator, $\partial/\partial n$ denotes differentiation along the normal vector drawn outward from Ω and Γ is boundary of the domain Ω.

The operator A can be decomposed into two parts, linear *(L)* and nonlinear *(N)* parts, Hence, Eq. (1) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

The Homotopy perturbation structure is shown in the following form:

$$D(h,s) = (1-s)[L(h) - L(u_0)] + s[A(h) - f(r)] = 0, s \in [0,1] \tag{4}$$

Where

$$h(r,s) = \Omega \times [0,1] \to R \tag{5}$$

In Eq. (4), $s \in [0,1]$ is an embedding parameter, which increases from 0 to 1 and u_0 is the initial approximation of Eq. (1) that satisfies boundary conditions. By Eq. (4), it easily follows that:

$$D(h,0) = L(h) - L(u_0) = 0 \tag{6}$$

$$D(h,1) = A(h) - f(r) = 0 \tag{7}$$

Changing values of s form zero to unity lead to changing $h(r,s)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(h) - L(u_0)$ and $A(h) - f(r)$ are called homotopic. According to HPM, we can first use the embedding parameter s as a “small parameter”, so by applying the perturbation technique, we assume that the solution of Eq. (4) can be written as a power series in s, as following:

$$h = h_0 + h_1 s + h_2 s^2 + h_3 s^3 + \ldots \tag{8}$$

The best approximation for Eq. (1) can be readily obtained by setting $s = 1$ as follows:

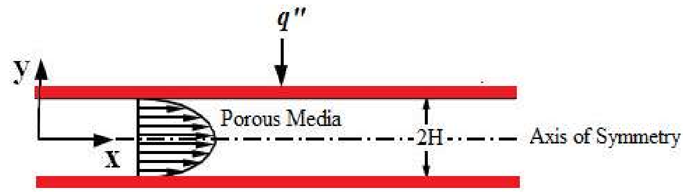

Fig. 1: Definition Sketch

$$u = \lim_{s \to 1} h = h_0 + h_1 + h_2 + h_3 + ... \quad (9)$$

The series (9) is convergent for most cases. However, the convergent rate depends on the nonlinear operator $A(h)$ and suitable choice of linear term. It is worth mentioning that J.H. He [23, 29] has proved the convergence of this method.

Problem Definition and Governing Equations: Figure 1 illustrates the schematic of steady laminar Newtonian fluid flow into a straight channel uniformly filled with an isotropic porous medium. The fluid is driven by a constant pressure gradient and is assumed to flow from left to the right. Flow is hydrodynamically and thermally fully developed which velocity and temperature fields do not depend on axial direction of the channel. The governing equation of a viscous and incompressible fluid in fully developed flow and heat transfer conditions is as follows:

$$\mu_{eff} \frac{d^2u(y)}{dy^2} - \frac{\mu u(y)}{K} - \frac{\rho C_F u(y)^2}{\sqrt{K}} + G = 0 \quad (10)$$

Where μ and ρ are viscosity and density of fluid, respectively. μ_{eff} is the effective viscosity of the fluid in the porous medium. K, C_F, u and p are permeability, form drag coefficient, velocity and pressure, respectively. Also $G = -\frac{dp}{dx}$ is the negative constant pressure gradient along the channel.

Defining the dimensionless variables as:

$$Y = \frac{y}{H}, X = \frac{x}{PeH}, U = \frac{\mu u}{GH^2} \quad (11a\text{-}c)$$

In dimensionless form, the momentum equation takes the form:

$$M \frac{d^2U}{dY^2} - \frac{U}{Da} - \frac{FMU^2}{\sqrt{Da}} + 1 = 0 \quad (12)$$

We have defined the Darcy, viscosity ratio, Forchheimer and Peclet numbers by:

$$F = \frac{C_F \rho G H^3}{\mu_{eff} \mu}, Da = \frac{K}{H^2}, M = \frac{\mu_{eff}}{\mu}, Pe = \frac{\rho C_P H u^*}{K} \quad (13a\text{-}c)$$

Eq. (12) has the following boundary conditions:

$$U\big|_{Y=1} = 0 \text{ No-slip boundary condition at top wall} \quad (14)$$

$$\left.\frac{dU}{dY}\right|_{Y=0} = 0 \text{ Symmetry conditions at centerline} \quad (15)$$

The bulk mean temperature (T_m) and mean velocity (U), can be defined as:

$$T_m = \frac{1}{Hu^*} \int_0^H uT\,dy,\ u^* = \frac{1}{H} \int_0^H u\,dy \quad (16a\text{-}b)$$

Further dimensionless variables are introduced as:

$$\overline{U} = \frac{U}{u^*},\ \theta = \frac{T - T_w}{T_m - T_w} \quad (17a,b)$$

The Nusselt number is defined as:

$$Nu = \frac{2Hq''}{k(T_w - T_m)} \quad (18)$$

Where q'' is wall heat flux.

Homogeneity and local thermal equilibrium is assumed so the steady-state energy equation is:

$$\rho C_P u \frac{\partial T}{\partial x} = k \frac{\partial}{\partial y}(\frac{\partial T}{\partial y}) \quad (19)$$

Where k, C_p are thermal conductivity and specific heat, respectively.

From the first law of thermodynamics, we have:

$$\frac{\partial T}{\partial x} = \frac{2q''}{\rho C_P H u^*} \quad (20)$$

Substituting Eq (20) in to Eq (19) gives:

$$\frac{2q'' u}{Hu^*} = k \frac{\partial}{\partial y}(\frac{\partial T}{\partial y}) \quad (21)$$

The dimensionless form of Eq (21) becomes:

$$2\frac{d^2\theta}{dY^2}+\overline{U}Nu=0 \tag{22}$$

Where the boundary conditions are as follows:

$$\left.\frac{d\theta}{dY}\right|_{Y=0}=0 \quad \theta|_{Y=1}=0 \tag{23a-b}$$

After solving the above equation and finding the velocity and temperature profiles the Nusselt number can be calculated using compatibility condition.

$$\int_0^1 \overline{U}.\theta=1 \tag{24}$$

Solution Procedure: The momentum equation (Eq. 12) with its associated boundary conditions (Eqs. 14, 15) is solved analytically to obtain velocity profile in the channel and then using the obtained velocity distribution, the temperature distribution can be found using Eq. 22 and its associated boundary conditions i.e., Eq. (23a-b) and afterwards Nusselt number can be obtained using Eq. 24. The Darcy-Forchhimer-Brinkman is a nonlinear ODE and to the knowledge of authors, it has not an exact closed form solution. Hence, the main objective of this paper is to present an approximate analytical solution for this equation using a novel method of solving nonlinear ODEs, called Homotopy Perturbation Method.

To solve Eq. (12), we define the linear operator which is the linear part of the Eq. (12), as follows:

$$L(\psi)=M\frac{d^2\psi}{dY^2}-\frac{\psi}{Da} \tag{25}$$

Where ψ is an auxiliary function.

The second step is to guess an arbitrary initial approximation like:

$$U_{ini}=(Y)=0 \tag{26}$$

Where subscript *ini* refers to an initial approximation of Eq. (12).

According to homotopy perturbation method, we separate the linear and non-linear parts of the main equation (Eq. 12) and construct the following homotopy equation:

$$\begin{aligned}D(h,s)=(1-s)\left[M\frac{d^2h(Y)}{dY^2}-\frac{h(Y)}{Da}-0\right]+\\ s\left[1+M\frac{d^2h(Y)}{dY^2}-\frac{h(Y)}{Da}-\frac{FM\,h(Y)^2}{\sqrt{Da}}\right]=0\end{aligned} \tag{27}$$

Substituting Eq. (8) into Eq. (27) and equating the coefficient with identical power of s we obtain a system of equations with n+1 differential equations to be solved simultaneously; where n is the order of s in Eq. (8). For n= 3 the system of linear ODEs is as follows:

Zeroth-order: $\frac{d^2}{dY^2}h_0(Y)-\frac{h_0(Y)}{MDa}=0$

First order: $-\frac{h_1(Y)}{MDa}+\frac{d^2}{dY^2}h_1(Y)+\frac{1}{M}-\frac{F\,h_0(Y)^2}{\sqrt{Da}}=0$

Second order: $-\frac{2F\,h_0(Y)\,h_1(Y)}{\sqrt{Da}}-\frac{h_2(Y)}{MDa}+\frac{d^2}{dY^2}h_2(Y)=0$

(28a-d)

Third order: $-\frac{F\left(2h_0(Y)h_2(Y)+h_1(Y)^2\right)}{\sqrt{Da}}-\frac{h_3(Y)}{MDa}+\frac{d^2}{dY^2}h_3(Y)=0$

And the boundary conditions are:

$$h_i(1)=0, h_i'(0)=0 \quad i\geq 0 \tag{29}$$

Solving Eqs. (28a-d) with corresponding boundary conditions, the following functions can be obtained successively.

$h_0(Y)=h_2(Y)=0$

$$h_1(\overline{y})=\frac{Da\left(-e^{\frac{Y}{\sqrt{m}\sqrt{Da}}}-e^{-\frac{Y}{\sqrt{m}\sqrt{Da}}}+e^{\frac{1}{\sqrt{m}\sqrt{Da}}}+e^{-\frac{1}{\sqrt{m}\sqrt{Da}}}\right)}{e^{\frac{1}{\sqrt{m}\sqrt{Da}}}+e^{-\frac{1}{\sqrt{m}\sqrt{Da}}}}.$$

(30a-c)

$$h_3(Y)=\frac{1}{3}\Bigg(\Bigg(\Bigg(3e^{-\frac{Y}{\sqrt{m}\sqrt{Da}}}\sqrt{m}\,Y+2e^{\frac{Y+4}{\sqrt{Da}\sqrt{m}}}\sqrt{Da}\,m+6e^{-\frac{Y-2}{\sqrt{Da}\sqrt{m}}}\sqrt{m}\,Y+12e^{-\frac{Y-2}{\sqrt{Da}\sqrt{m}}}m\sqrt{Da}-3e^{\frac{Y}{\sqrt{m}\sqrt{Da}}}\sqrt{m}\,Y$$

$$+e^{\frac{2Y+3}{\sqrt{Da}\sqrt{m}}}\sqrt{Da}\,m+2e^{\frac{Y}{\sqrt{m}\sqrt{Da}}}m\sqrt{Da}+e^{-\frac{2Y-3}{\sqrt{Da}\sqrt{m}}}\sqrt{Da}\,m+3e^{\frac{Y+4}{\sqrt{Da}\sqrt{m}}}\sqrt{m}-3e^{\frac{Y}{\sqrt{m}\sqrt{Da}}}\sqrt{m}+3e^{-\frac{Y-4}{\sqrt{Da}\sqrt{m}}}\sqrt{m}\,Y$$

$$-3e^{-\frac{1}{\sqrt{m}\sqrt{Da}}}\sqrt{Da}\,m-6e^{\frac{2+Y}{\sqrt{m}\sqrt{Da}}}\sqrt{m}\,Y+2e^{-\frac{Y-4}{\sqrt{Da}\sqrt{m}}}\sqrt{Da}m-3e^{\frac{Y+4}{\sqrt{Da}\sqrt{m}}}\sqrt{m}Y-3e^{\frac{5}{\sqrt{m}\sqrt{Da}}}\sqrt{Da}\,m+2e^{-\frac{Y}{\sqrt{m}\sqrt{Da}}}\sqrt{Da}\,m$$

$$-15\sqrt{Da}\,m\,e^{\frac{1}{\sqrt{m}\sqrt{Da}}}-15\sqrt{Da}\,m\,e^{\frac{3}{\sqrt{m}\sqrt{Da}}}+12e^{\frac{2+Y}{\sqrt{m}\sqrt{Da}}}\sqrt{Da}\,m+e^{\frac{-2Y-1}{\sqrt{m}\sqrt{Da}}}\sqrt{Da}\,m+3e^{-\frac{Y-4}{\sqrt{Da}\sqrt{m}}}\sqrt{m}-3e^{-\frac{Y}{\sqrt{m}\sqrt{Da}}}\sqrt{m}$$

$$+e^{\frac{2Y+1}{\sqrt{m}\sqrt{Da}}}\sqrt{Da}\,m\Bigg)f\,Da^2\Bigg)\Bigg/\Bigg(\Bigg(e^{\frac{4}{\sqrt{m}\sqrt{Da}}}+2e^{\frac{2}{\sqrt{m}\sqrt{Da}}}+1\Bigg)\Bigg(e^{\frac{1}{\sqrt{m}\sqrt{Da}}}+e^{-\frac{1}{\sqrt{m}\sqrt{Da}}}\Bigg)\Bigg)\Bigg).$$

Finally, by summing the results up and $s \to 1$ we write the velocity profile as

$$U(Y)=\sum_{i=0}^{3}h_i(Y),\qquad h_0(Y)=h_2(Y)=0. \tag{31}$$

Equation (31) is the analytical solution of problem by using HPM.

As a cross-check for the analysis, a finite volume method (FVM) has been selected for the solution of momentum and energy equations with their associated boundary conditions. The computational domain is descritized with a non-uniform grid. The discretized form of momentum and energy equations are as follows:

$$A\phi_{j-1}+B\phi_j+C\phi_{j-1}=D \tag{32}$$

Where ϕ could be velocity or temperature and A, B, C and D are constant depending on Da,M and F and grid size. The resulting tridiagonal systems of algebraic equations are solved using well established Thomas Algorithm [30].

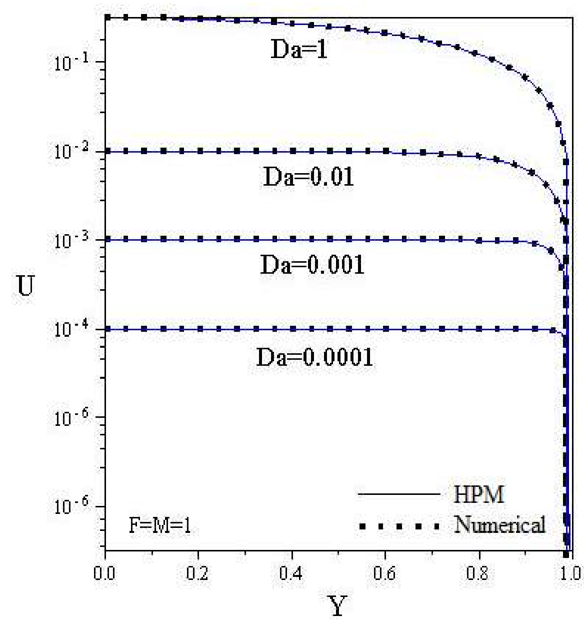

Fig. 2: Effect of *Da* on Dimensionless velocity profile for F=M=1

RESULTS AND DISCUSSION

The results are divided in two hydrodynamic and heat transfer parts. First, the effect of different values of key parameters in the governing equations such as Darcy number (Da), viscosity ratio (M) and Forchheimer number (F) are studied on hydrodynamic aspect of fluid flow in channel and then the influence of mentioned parameters are investigated on heat transfer characteristics of the problem. It is important to note that in order to check the validity of the presented analytical solution the HPM results are compared in plots with numerical ones in all of the presented figures of this section. The point that is clear in all figures is that the HPM results are in complete agreement with numerical ones.

The dimensionless velocity profile is demonstrated in Figure 2 for various values of Darcy numbers along the channel height for Da<1 which is the realistic range for industrial applications. It can be seen that with increasing Darcy number the velocity profile tends to the plane Poiseuille flow and with decreasing this parameter the velocity profile become more flat and tends to that of the slug flow. This is expected because low Darcy values are associated with higher fluid resistance (lower permeability) and consequently higher shear stress between fluid and porous structure.

Effect of viscosity ratio (M) is shown in Figure 3 by fixing the value of Darcy at 0.001 when F=1. It can be seen that increasing the value of M leads to increasing

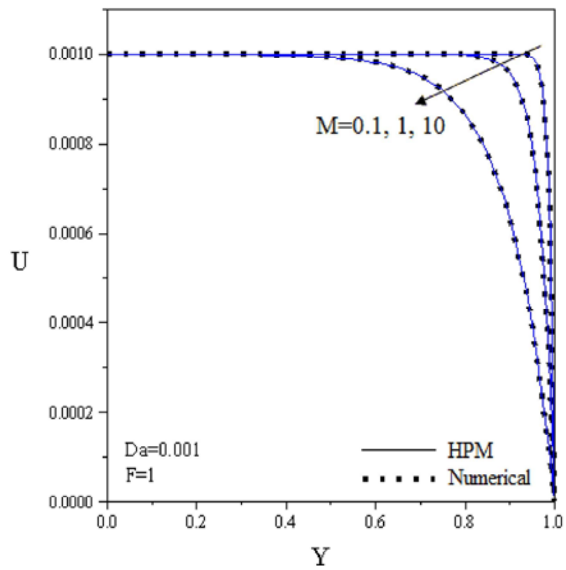

Fig. 3: Effect of *M* on Dimensionless velocity profile for Da=0.001 and F=1

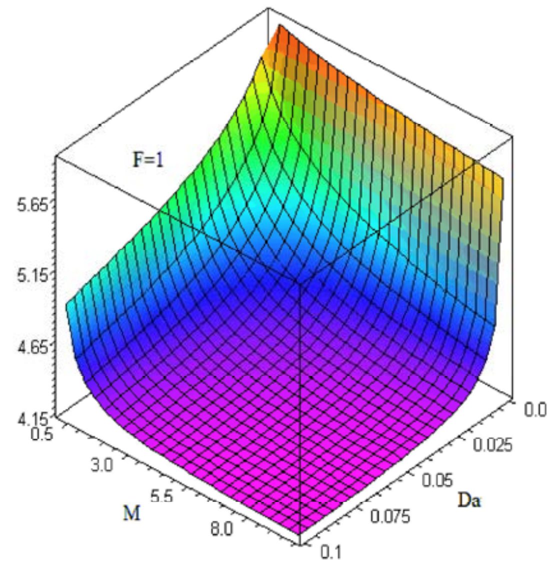

Fig. 5: Variation of Nusselt number versus *Da* and *M* at F=1

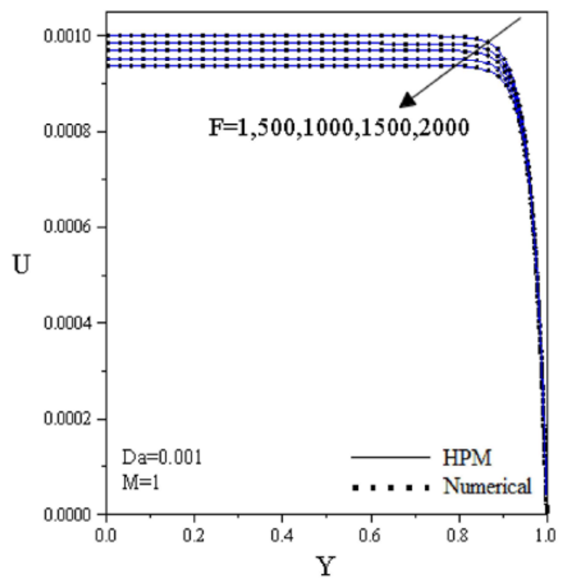

Fig. 4: Effect of *F* on Dimensionless velocity profile for Da=0.001 and M=1

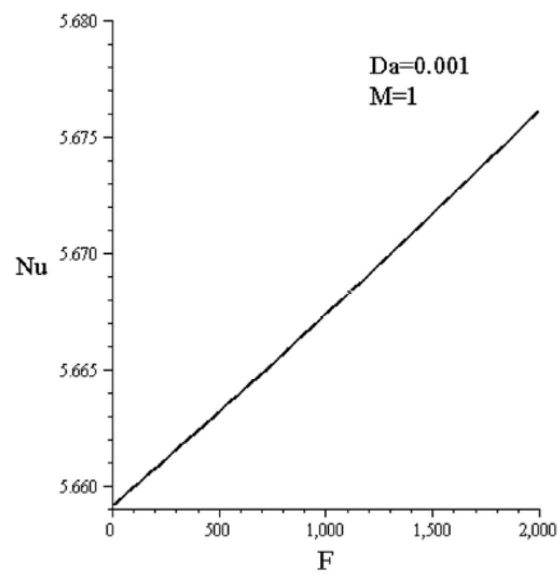

Fig. 6: Variation of Nusselt number versus *F* when M=1, Da=0.001.

thickness of the momentum boundary layer, which leads to decreasing the shear stress on the walls. In the Figure 4, the effect of F is shown on the velocity. It is clear that increasing the value of F leads to more slug-like flow pattern, which leads flattening the fluid velocity and smaller mean non-dimensional velocity profile. Furthermore, with decreasing the value of F the fluid velocity increases and fluid penetrate faster through the porous media which results in decreasing hydrodynamic boundary layer thickness on the walls of channel.

In Figure 5 the effects of the parameters Da and M are investigated on the Nusselt number as an important feature for analyzing heat transfer phenomenon. The observations show that decreasing the value of either Da or M increases the Nusselt number. This is due to the fact that small *Da* or *M* translates to more flow near the walls.

Figure 6a depicts that the Nu increases with increasing the Forchheimer number. This is because an increase in the inertia parameter, due to a more vigorous mixing of the fluid, causes a more uniform velocity profile.

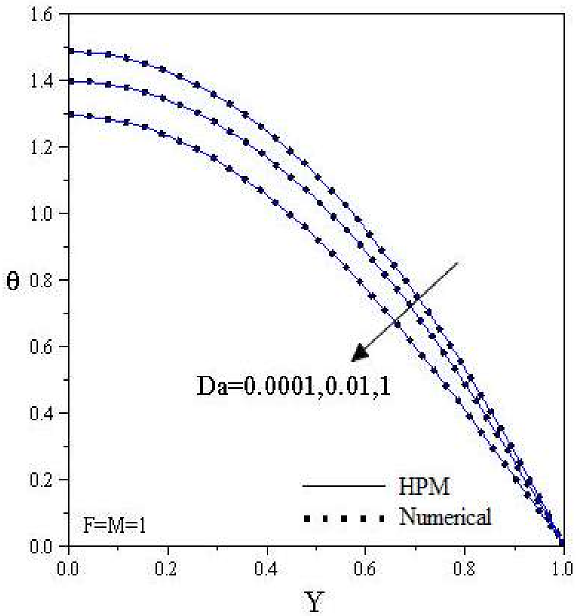

Fig. 7: Effect of *Da* on Dimensionless velocity profile for M= F=1

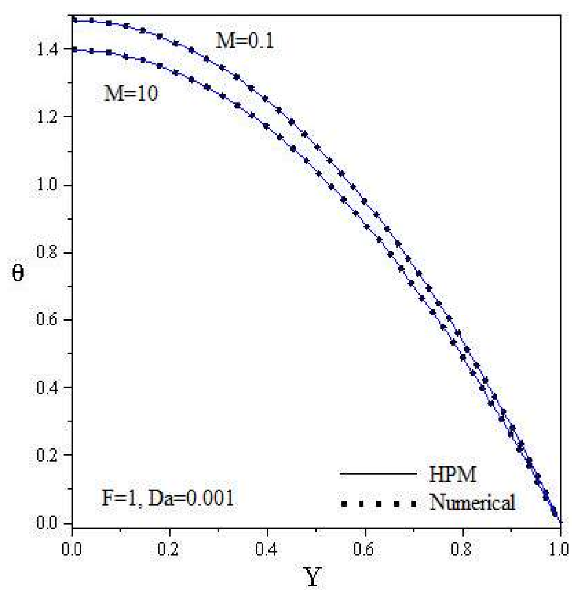

Fig. 8: Effect of *M* on Dimensionless velocity profile for Da=0.001 and F=1

At this point, it is worth mentioning that Nield *et al*. [13] found similar results for variation of Nusselt number with Forchheimer number. In their paper they wrote that "the effect of an increase in F is to produce a more slug-like flow and this directly decreases ($T_w - T_m$) and this leads, via the factor $(T_w - T_m)^{-1}$ in the definition of Nusselt number, to an increase in Nu" This is why higher rate of heat transfer can be seen at higher values of inertia parameters.

Figure 7, 8 show the dimensionless temperature distribution in various values of Da and M. As seen with increasing the value of Da and M the temperature decreases and thermal boundary layers on the walls increases. It is worth mentioning that with increasing the thermal boundary layer, the local heat transfer coefficient decreases. Therefore, increasing Darcy number and viscosity ratio lead to decreasing local heat transfer coefficients and consequently Nusselt number.

CONCLUSION

In this study, the nonlinear Darcy-Brinkman-Forchhimer equation is solved analytically using HPM and numerically using finite volume method for fully developed flow through a porous saturated channel bounded by two iso-flux parallel plates. The results show an excellent agreement between analytical and numerical solutions and indicate that HPM is a reliable solution for the studied problem. In addition, the effects of key parameters are investigated on dimensionless velocity profile, temperature distribution and Nusselt number. Based on the results of this study, it can be drawn that homotopy perturbation method is a good candidate for solving strong nonlinear differential equations like Darcy-Brinkman-Forchhimer equation upon appropriate choose of the linear operator, which leads to faster convergence of the problem and lower CPU usage.

Nomenclature

C_F	Inertial coefficient
C_p	Specific heat at constant pressure
Da	Darcy number, K/H^2
F	Forchheimer number
G	Negative pressure gradient
H	Half channel distance
k	Thermal conductivity
K	Permeability
M	μ_{eff}/μ
Nu	Nusselt number
Pe	Péclet number
q''	wall heat flux
T	temperature
T_m	bulk mean temperature
T_w	downstream wall temperature
u	filtration velocity
U	$\mu u/ GH^2$
u^*	mean velocity
$\bar{U}$	normalized velocity

x longitudinal coordinate
X x/PeH
y transverse coordinate
Y y/H
μ fluid viscosity
ρ fluid density

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