New Travelling Wave Solutions to the Perturbed Nonlinear Schrodinger’s Equation

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Abstract: In this present work we applied new applications of direct algebraic method to Drinfel’d-Sokolov-Wilson system and new application of \((G'/G)\) -expansion method to the perturbed nonlinear Schrodinger’s equation. Then new types of complex solutions are obtained to the Drinfel’d-Sokolov-Wilson system. In \((G'/G)\) -expansion method the balance number of it is not positive integer. Then new types of exact travelling wave solutions are obtained to this equation.

Keywords: Direct algebraic method \cdot \((G'/G)\) -expansion method \cdot Perturbed nonlinear Schrodinger \cdot S equation \cdot Drinfel’d-Sokolov-Wilson system

INTRODUCTION

The \((G'/G)\) -expansion method was developed by Wang et al. [1]. The method is now used by many researchers in a variety of scientific fields. The method has been proved by many authors [2, 3, 4]. Recently, many powerful methods have been established and improved. Among these methods, we cite the, the hyperbolic tangent expansion method [7, 8], the trial function method [9], the homogeneous balance method [5, 6], the tanh-method [10-14], the inverse scattering transform [15], the Backlund transform [16, 17], the Hirota’s bilinear method [18, 19]. The motivation of the present paper is to explore the possibilities of solving such equations, the balance numbers of which are not positive integers, using the \((G'/G)\) -expansion method. The \((G'/G)\) -expansion method is based on the assumptions that the travelling wave solutions can be expressed by a polynomial in \((G'/G)\) and that \(G = G(\xi)\) satisfies a second order linear ordinary differential equation (ODE).

Description of the \((G'/G)\) -expansion Method: Considering the nonlinear partial differential equation in the form

\[ P(u, u_x, u_t, u_{xx}, u_{tt}, u_{xxt}) = 0 \]

Where \(u = u(x, t)\) an unknown function is \(P\) is a polynomial in \(u = u(x, t)\) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the \((G'/G)\) -expansion method.

**Step 1:** Combining the independent variables \(x\) and \(t\) into one variable \(\xi = x - vt\), we suppose that

\[ u(x, t) = u(\xi), \quad \xi = x - vt \tag{2} \]

The travelling wave variable (2) permits us to reduce Eq. (1) to an ODE for \(G = G(\xi)\), namely

\[ P(u, u_x, u_t, u_{xx}, u_{tt}, \ldots) = 0 \tag{3} \]

**Step 2:** Suppose that the solution of ODE (3) can be expressed by a polynomial in \((G'/G)\) as follows

\[ u(\xi) = \alpha_m \frac{G'}{G} + \ldots, \tag{4} \]

Where \(G = G(\xi)\) satisfies the second order LODE in the form

\[ G'' + \lambda G' + \mu G = 0 \tag{5} \]

\(\alpha_m, \ldots, \lambda, \mu\) and \(\lambda\) are constants to be determined later \(\alpha_m \neq 0\), the unwritten part \(\alpha_m(\frac{G'}{G})\) is also a polynomial in \((G'/G)\), but the degree of which is generally equal to or less than \(m - 1\), the positive integer \(m\) can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).

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Step 3: By substituting (4) into Eq. (3) and using the second order linear ODE (5), collecting all terms with the same order \( \frac{G'}{G} \) together, the left-hand side of Eq. (3) is converted into another polynomial in \( \frac{G'}{G} \). Equating each coefficient of this polynomial to zero yields a set of algebraic equations for \( \alpha_n \), \( \lambda \) and \( \mu \).

Step 4: Assuming that the constants \( \alpha_n \), \( \lambda \) and \( \mu \) can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (5) have been well known for us, then substituting \( \alpha_n \), \( \lambda \) and the general solutions of Eq. (5) into (4) we have more travelling wave solutions of the nonlinear evolution equation (1).

Application to the Perturbed Nonlinear Schrodinger's Equation: The perturbed nonlinear Schrodinger's equation reads

\[
\frac{\partial u}{\partial t} + u_{xx} + \alpha |u|^2 u + i \left[ \gamma_1 u_{xxx} + \gamma_2 |u|^2 u_x + \gamma_3 (|u|^2)_x \right] = 0
\]

We may choose the following travelling wave transformation:

\[
u = \phi(\xi) e^{(sx-Ct)}, \quad \xi = k(x-ct)
\]

Where \( s, \Omega, c \) are constants to be determined later. Using the traveling wave solutions (7) we have the nonlinear ordinary differential equation:

\[
\begin{align*}
\eta \kappa^3 \phi'' + (2\kappa i - 3\gamma_1 \kappa^2 k - cik) \phi' + (k^2 + 3\gamma_1 \kappa^2) \phi' + (ik\kappa_2 + 2ik\gamma_3) \\
\phi'' + (\alpha - \gamma_3 \kappa^2) \phi' + (\Omega - s + \gamma_1 \kappa^2) \phi = 0
\end{align*}
\]

Suppose that the solution of ODE (8) can be expressed by a polynomial in \( \frac{G'}{G} \) as follows:

\[
u(\xi) = \alpha_m \left( \frac{G'}{G} \right)^m + \ldots
\]

Where \( G = G(\xi) \) satisfies (5). Considering the homogeneous balance between \( \phi'' \) and \( \phi' \) in Eq. (7), we required that \( 3m = m + 3 \). It should be noticed that \( m \) is not a positive integer. However, we may still choose the solution of Eq. (8) in the form

\[
\phi = A \xi^{\frac{3}{2}}
\]

\[
\phi' = \frac{3}{2} A \left[ -\lambda \left( \frac{G'}{G} \right)^2 - \mu \left( \frac{G'}{G} \right)^3 - \left( \frac{G'}{G} \right)^2 \right],
\]

\[
\phi'' = \frac{3}{2} A \left[ \left( \frac{3}{2} \lambda^2 + 2\mu \right) \left( \frac{G'}{G} \right)^2 + 2\lambda \mu \left( \frac{G'}{G} \right)^{\frac{1}{2}} + 4\lambda \mu \left( \frac{G'}{G} \right)^{\frac{5}{2}} + \frac{1}{2} \mu^2 \left( \frac{G'}{G} \right)^{\frac{1}{2}} \right],
\]

\[
\phi''' = \frac{3}{2} A \left[ -\frac{9}{2} \lambda^3 - \frac{11}{2} \lambda \mu \left( \frac{G'}{G} \right) + \left( -\frac{13}{4} \lambda \mu - \frac{9}{4} \lambda \mu^2 + \frac{35}{4} \lambda \mu \left( \frac{G'}{G} \right)^{\frac{1}{2}} + \left( -\frac{9}{4} \lambda^2 - \frac{53}{4} \lambda \mu \left( \frac{G'}{G} \right)^{\frac{5}{2}} - \frac{1}{4} \mu^3 \left( \frac{G'}{G} \right)^{3} - \frac{35}{4} \lambda \mu \left( \frac{G'}{G} \right)^{7} - \frac{35}{4} \lambda \mu \left( \frac{G'}{G} \right)^{9} \right) \right],
\]

\[
\phi'''' = \frac{3}{2} A \left[ -\lambda \left( \frac{G'}{G} \right)^2 - \mu \left( \frac{G'}{G} \right)^3 - \left( \frac{G'}{G} \right)^2 \right],
\]

\[
\phi'''' = \frac{3}{2} A \left[ -\lambda \left( \frac{G'}{G} \right)^2 - \mu \left( \frac{G'}{G} \right)^3 - \left( \frac{G'}{G} \right)^2 \right],
\]
On substituting (9)-(12) into (8), collecting all terms with the same powers of \((G'/G)\) and setting each coefficient to zero, we obtain the following system of algebraic equations:

\[
\begin{align*}
-\frac{9}{8} \lambda \mu + \frac{3}{4} A \mu^2 &= 0, \\
\frac{3}{2} \mu (ik \gamma_s + 2ik \gamma) - \frac{105}{8} \lambda &= 0, \\
\frac{3}{2} (2sk - 3 \gamma_s^2 k - eck) + 6k \lambda (2k + 3 \gamma_s k^2) + \frac{3}{2} \lambda &= \frac{3}{2} A \mu^2 = 0.
\end{align*}
\]

(30)

On solving the above algebraic Eq. (14) by using the Maple, we get

\[
\begin{align*}
e &= -3 \gamma_s^2 + 2s + 6ik \lambda + 12isk \lambda \gamma_1 + \frac{2}{3} \gamma_1^2 k^2 - \frac{9}{4} \lambda^2, \\
\lambda &= \frac{-2i}{\gamma k^3}, \\
\mu &= 0, \\
A &= \pm \frac{k}{2} \sqrt{\frac{-105 \gamma_1}{2\alpha - 2\gamma_s k - 3\lambda}}.
\end{align*}
\]

From (5), (7), (9) and (15), we obtain the exact travelling wave solution of (6) as follows:

\[
A = \pm \frac{k}{2} \sqrt{\frac{-105 \gamma_1}{2\alpha - 2\gamma_s k - 3\lambda}} \left( \frac{\cosh \left( \frac{3}{2} \lambda \right)}{\cosh \left( \frac{3}{2} \lambda \right) + \cosh \left( \frac{3}{2} \lambda \right)} \right) e^{i(\kappa x - \Omega t)}
\]

Hence

\[
A = \pm \frac{k}{2i \gamma k^3} \sqrt{\frac{-105 \gamma_1}{2\alpha - 2\gamma_s k - 3\lambda}} \left( \frac{\cosh \left( \frac{3}{2} \lambda \right)}{\cosh \left( \frac{3}{2} \lambda \right) + \cosh \left( \frac{3}{2} \lambda \right)} \right) e^{i(\kappa x - \Omega t)}
\]

Where \(c_1, c_2, k\) and \(x, \Omega\) are arbitrary constants. Eq. (16) is a new type of exact travelling wave solution to the perturbed nonlinear Schrödinger’s equation. Especially, if we choose \(c_1 = -c_2\) in (16), we obtain the envelope solitary wave solutions of Eq. (6),

\[
A = \pm \frac{i}{\gamma k^3} \sqrt{\frac{105 \gamma_1}{2\alpha - 2\gamma_s k - 6i}} \left( \frac{1 - \sqrt{\frac{-2i}{\gamma k^3} (x - (-3 \gamma_s^2 + 2s + 6i \lambda \gamma_1 + 2 \gamma_1 k^2 - \frac{9}{4} \lambda^2))}}{2} \right) e^{i(\kappa x - \Omega t)}
\]

**CONCLUSION**

We have noted that the \((G'/G)\) expansion method changes the given difficult problems into simple problems which can be solved easily. This paper presents a wider applicability for handling nonlinear evolution equations using the \((G'/G)\) - expansion method. The new type of exact travelling wave solution obtained in this paper might have significant impact on future research.

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