

## Design Artificial Nonlinear Controller Based on Computed Torque like Controller with Tunable Gain

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**Abstract:** One of the most active research areas in the field of robotics is robot arms control, because these systems are multi-input multi-output (MIMO), nonlinear, time variant and uncertainty. An artificial non linear robust controller design is major subject in this work. At present, robot manipulators are used in unknown and unstructured situation and caused to provide complicated systems, consequently nonlinear classical controllers are used in artificial intelligence control methodologies to design nonlinear robust controller with satisfactory performance (e.g. minimum error, good trajectory, disturbance rejection). Computed torque controller (CTC) is the best nonlinear robust controllers which can be used in uncertainty nonlinear. Computed torque controller works very well when all nonlinear dynamic parameters are known. This research is focused on the applied non-classical method (e.g. Fuzzy Logic) in robust classical method (e.g. computed torque controller) in the presence of uncertainties and external disturbance to reduce the limitations. Applying the Mamdani's error based fuzzy logic controller with minimum rules is the first goal that causes the elimination of the mathematical nonlinear dynamic CTC. Second target focuses on the elimination of oscillation with regard to the variety of uncertainty and external disturbance in computed torque like controller by optimization the tunable gain. Therefore computed torque like controller with tunable gain (GTCTLC) will be presented in this paper.

**Key words:** Robot manipulator • Nonlinear robust controller • Classical controller • Minimum error • Good trajectory • Disturbance rejection • Computed torque controller • Computed torque like controller and tunable gain

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### INTRODUCTION

Most of robot manipulators which work in industry are usually controlled by linear PID controllers. But the robot manipulator dynamic functions are, nonlinear with strong coupling between joints (low gear ratio), structure and unstructured uncertainty and multi- inputs multi-outputs (MIMO) which, design linear controller is very difficult especially if the velocity and acceleration of robot manipulator be high and also when the ratio between joints gear be small [2]. To eliminate above problems in physical systems most of control researcher go toward to select nonlinear robust controller.

Computed torque controller (CTC) is a powerful nonlinear controller which it widely used in control robot manipulator. It is based on Feed-back linearization and

computes the required arm torques using the nonlinear Feed-back control law. This controller works very well when all dynamic and physical parameters are known but when the robot manipulator has variation in dynamic parameters, in this situation the controller has no acceptable performance [6]. In practice, most of physical systems (e.g. robot manipulators) parameters are unknown or time variant, therefore, computed torque like controller used to compensate dynamic equation of robot manipulator[1,3]. Research on computed torque controller is significantly growing on robot manipulator application which has been reported in [1-3].

Some researchers had applied fuzzy logic methodology in computed torque controller (CTLC) in order to eliminate the nonlinear part in pure computed torque controller [1, 3].

This paper is organized as follows: In section 2, main subject of modelling robot manipulator formulation are presented. This section covers the following details, introducing the dynamic formulation of robot manipulator. In section 3, the main subject of computed torque controller and formulation are presented. The main subject of designing computed torque like controller with tuneable gain is presented in section 4. This section covers self tuning computed torque like controller. This method is used to reduce the output oscillation and estimate the nonlinear part in CTC controller. In section 5, the simulation result is presented and finally in section 6, the conclusion is presented.

**Robotic Manipulator Formulation:** Dynamic modelling of robot manipulators is used to describe the behaviour of robot manipulator, design of model based controller and simulation results. The dynamic modelling describe the relationship between joint motion, velocity and accelerations to force/torque or current/voltage and also it can be used to describe the particular dynamic effects (e.g. inertia, coriolios, centrifugal and the other parameters) to behaviour of system. It is well known that the equation of an *n-DOF* robot manipulator governed by the following equation [1,3]:

$$M(q)\ddot{q} + N(q, \dot{q}) = \tau \tag{1}$$

Where  $\tau$  is actuation torque,  $M(q)$  is a symmetric and positive definite inertia matrix,  $N(q, \dot{q})$  is the vector of nonlinearity term. This robot manipulator dynamic equation can also be written in a following form [12]:

$$\tau = M(q)\ddot{q} + B(q)[\dot{q}\dot{q}] + C(q)[\dot{q}]^2 + G(q) \tag{2}$$

Where  $B(q)$  is the matrix of coriolios torques,  $C(q)$  is the matrix of centrifugal torques and  $G(q)$  is the vector of gravity force. The dynamic terms in equation (2) are only manipulator position. This is a decoupled system with simple second order linear differential dynamics. In other words, the component  $\ddot{q}$  influences, with a double integrator relationship, only the joint variable  $q_i$ , independently of the motion of the other joints. Therefore, the angular acceleration is found as to be [12]:

$$\ddot{q} = M^{-1}(q) \cdot \{\tau - N(q, \dot{q})\} \tag{3}$$

**Design of a Computed Torque Controller for Robotic Manipulator:** The central idea of Computed torque controller (CTC) is feedback linearization so, originally

this algorithm is called feedback linearization controller. It is assumed that the desired motion trajectory for the manipulator  $q_d(t)$ , as determined, by a path planner. Define the tracking error as [4-5]:

$$e(t) = q_d(t) - q_a(t) \tag{4}$$

Where  $e(t)$  is error of the plant,  $q_d(t)$ , is desired input variable, that in our system is desired displacement,  $q_a(t)$  is actual displacement. If an alternative linear state-space equation in the form  $\dot{x} = Ax + BU$  can be defined as

$$\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} U \tag{5}$$

With  $U = -M^{-1}(q)N(q, \dot{q}) + M^{-1}(q)\tau$  and this is known as the Brunousky canonical form. By equation (4) and (5) the Brunousky canonical form can be written in terms of the state  $x = [e^T \ \dot{e}^T]^T$  as:

$$\frac{d}{dt} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} U \tag{6}$$

With  $U = \ddot{q}_d + M^{-1}(q) \cdot \{N(q, \dot{q}) - \tau\}$  (7)

Then compute the required arm torques using inverse of equation (12), namely, [1, 3-5]

$$\tau = M(q)(\ddot{q}_d - U) + N(q, \dot{q}) \tag{8}$$

This is a nonlinear feedback control law that guarantees tracking of desired trajectory. Selecting proportional-plus-derivative (PD) feedback for  $U(t)$  results in the PD-computed torque controller [3];

$$\tau = M(q)(\ddot{q}_d + K_v \dot{e} + K_p e) + N(q, \dot{q}) \tag{9}$$

and the resulting linear error dynamics are

$$(\ddot{q}_d + K_v \dot{e} + K_p e) = 0 \tag{10}$$

According to linear system theory, convergence of the tracking error to zero is guaranteed [2].

Where  $K_p$  and  $K_v$  are the controller gains.

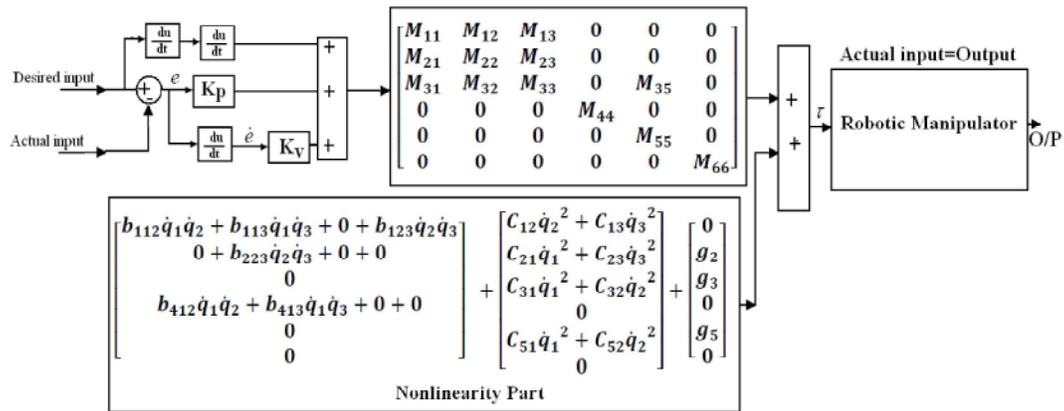


Fig. 1: Block diagram of PD-computed torque controller (PD-CTC)

The resulting schemes is shown in Figure 1, in which two feedback loops, namely, inner loop and outer loop, which an inner loop is a compensate loop and an outer loop is a tracking error loop. However, mostly parameter  $N(\dot{q}, q)$  is all unknown. So the control cannot be implementation because non linear parameters cannot be determined. In the following section computed torque like controller will be introduced to overcome the problems.

**Fuzzy Logic and its Application to Computed Torque Controller (CTLTC):** As mention previously, computed torque like controller (CTLTC) is fuzzy controller based on computed torque method for easy implementation, stability and robustness. The main drawback of CTLTC is the value of gain updating factor  $K_p$  and  $K_v$  must be pri-defined very carefully and the most important advantage of CTLTC compare to pure CTC is a nonlinearity dynamic parameter. It is basic that the system performance is sensitive to the gain updating factors for both computed torque controller and computed torque like controller application. For instance, if large value of  $K_v$  is chosen the response is very fast but the system is very unstable and conversely, if small value of  $K_v$  considered the response of system is very slow but the system is very stable. Therefore, calculate the optimum value of gain updating factors for a system is one of the most important challenging works. However most of time the control performance for FLC and CTLTC is similar to each other, but CTLTC has two most important advantages [7-11]:

- The number of rule base is smaller
- Increase the robustness and stability

In this method the control output can be calculated by

$$\tau = \hat{\tau} + \tau_{fuzzy(s)} \tag{11}$$

Where  $\hat{\tau}$  the nominal compensation is term and  $\tau_{fuzzy(s)}$  is the output of computed torque fuzzy controller.

The most important target in computed torque like controller (CTLTC) is design computed torque control combined to fuzzy logic systems to solve the problems in classical computed torque controller. To compensate the nonlinearity of nonlinear dynamic part several researchers used model base fuzzy controller instead of classical nonlinear dynamic part that was employed to obtain the desired control behaviour and a fuzzy switching control was applied to reinforce system performance [6]. In proposed fuzzy computed torque controller the author design fuzzy rule base to estimate the dynamic nonlinear part. A block diagram for proposed fuzzy computed controller is shown in Figure 2.

The sliding surface is defined as follows:

$$L = M(q)(\ddot{q}_d + K_v\dot{e} + K_p e) \tag{12}$$

Based on classical computed torque controller for a multi DOF robot manipulator:

$$\hat{\tau} = \hat{\tau}_{nonlinear} + \tau_{lin} \tag{13}$$

where, the model-based component  $\hat{\tau}_{nonlinear}$  compensate for the nominal dynamics of systems. So  $\hat{\tau}_{nonlinear}$  can calculate as follows:

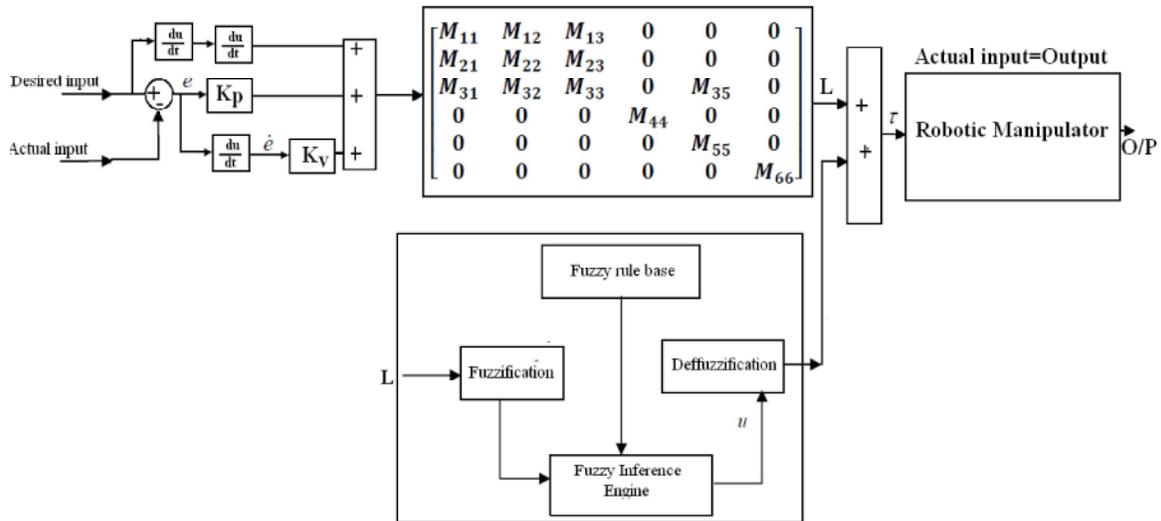


Fig. 2: Block diagram of proposed fuzzy computed torque controller with minimum rule base

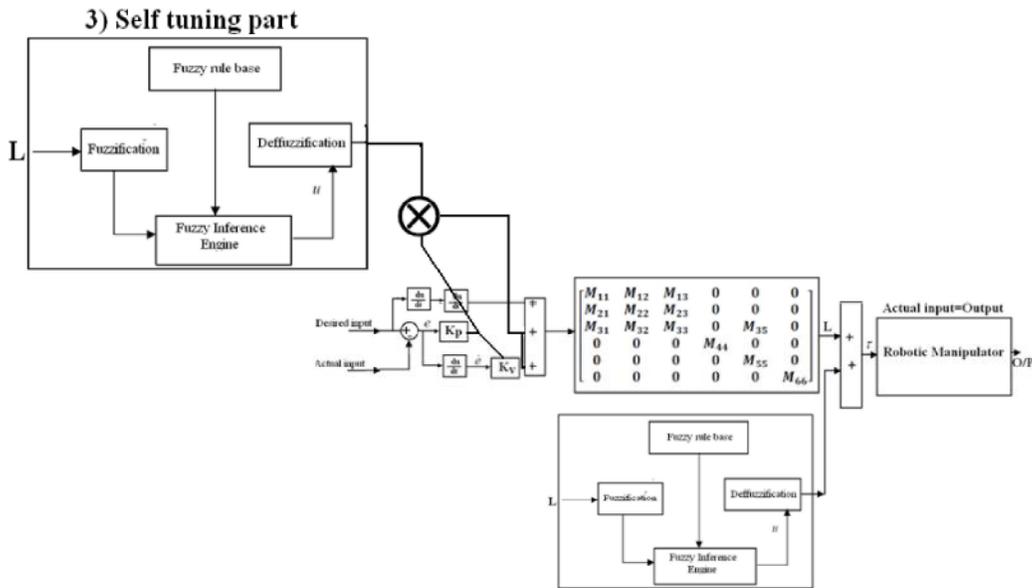


Fig. 3: Block diagram of proposed gain tuning fuzzy computed torque like controller with minimum rule base in fuzzy nonlinear part and fuzzy supervisory.

$$\hat{\tau}_{nonlinear} = B(q)\dot{q}\dot{q} + C(q)\dot{q}^2 + g(q) \quad (14)$$

and  $\tau_{lin}$  can calculate as follows:

$$\tau_{lin} = (q)(\ddot{q}_d + K_v\dot{e} + K_p e) \quad (15)$$

**Gain Tuning Computed Torque like Controller (GTCTLC):** This section focuses on, self tuning gain updating factor for most important factor in self tuning computed torque controller (GTCTLC) namely, nonlinear equivalent part (nonlinear term of controller). The block diagram for this method is shown in Figure 3.

It is a basic fact that the system performance in CTLC is sensitive to gain updating factor,  $K$ . Thus, determination of an optimum  $K$  value for a system is an important problem. If the system parameters are unknown or uncertain, the problem becomes more highlighted. This problem is solved by adjusting the proportional and derivative gain updating factor of the computed torque controller continuously in real-time. In this way, the performance of the overall system is improved with respect to the classical computed torque controller. Therefore this section focuses on, self tuning gain

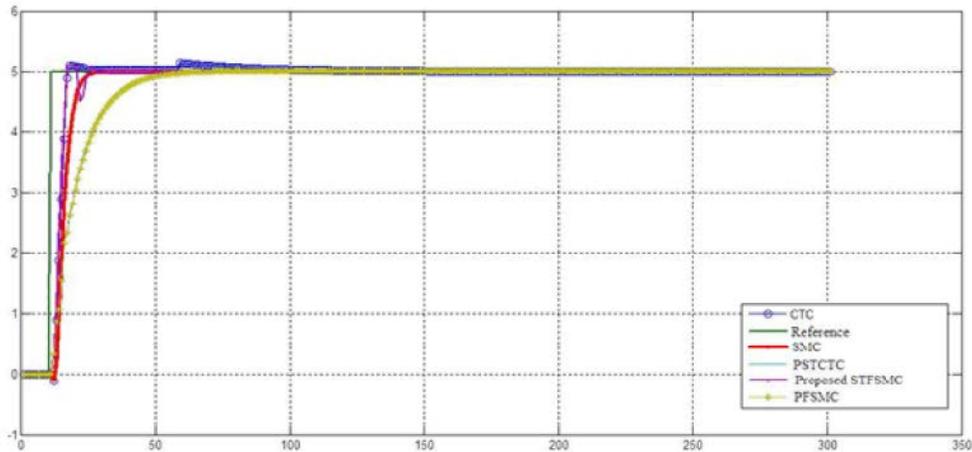


Fig. 4: First link step trajectory without disturbance

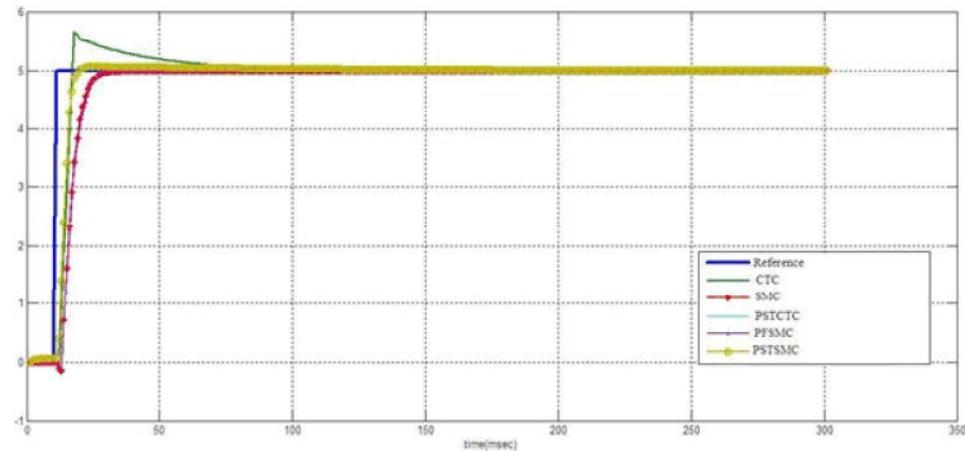


Fig. 5: Second link step trajectory without disturbance

updating factor for two type most important factor in CTLC, namely, proportional gain updating factor ( $K_p$ ) and derivative gain updating factor ( $K_v$ ). Gain tuning-CTLC has strong resistance and solves the uncertainty problems.

In this controller the actual gain updating factor ( $K_{new}$ ) is obtained by multiplying the old gain updating factor ( $K_{old}$ ) with the output of supervisory fuzzy controller ( $\alpha$ ). The output of fuzzy supervisory controller ( $\alpha$ ) is calculated on-line by fuzzy dynamic model independent which has sliding surface (S) as inputs.. The value of  $\alpha$  is not longer than 1 but it calculated on-line from its rule base. The scale factor,  $K_v$  and  $K_p$  are updated by equations (16) and (17),

$$K_v^{new} = K_v^{old} \times \alpha \tag{16}$$

$$K_p^{new} = K_p^{old} \times \alpha \tag{17}$$

**Simulation Result:** Computed torque controller (CTC), classical sliding mode control (SMC), fuzzy sliding mode control (FSMC), gain tuning computed torque like controller (GTCTLC) and gain tuning fuzzy sliding mode controller (GTFSMC) are implemented in Matlab/Simulink environment for 3 DOF robot manipulator. Tracking performance and robustness are compared.

**Tracking Performances:** Figure 4, 5 and 6 shows tracking performance for first, second and third link of robot manipulator with above controllers. By comparing step response trajectory without disturbance in above controllers, it is found that the GTCLC and GTFSMC overshoot (1.32%) are lower than CTC and SMC (6.44%), all of them have about the same rise time. Besides the Steady State and RMS error in GTCTLC and GTFSMC (Steady State error =0 and RMS error=0) are fairly lower than CTC and SMC (Steady State error  $\approx -3^{-5}$  and RMS error  $\approx -1.6 \times 10^{-5}$ ).

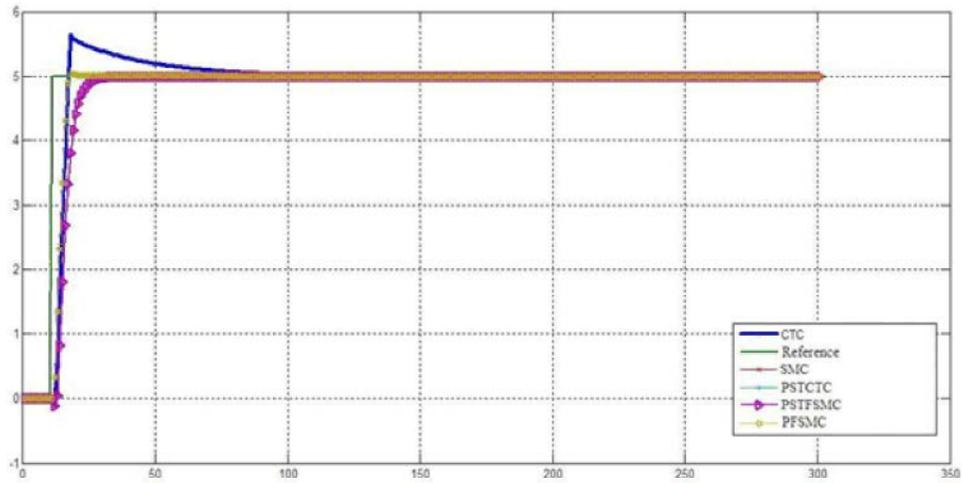


Fig. 6: Third link step trajectory without disturbance

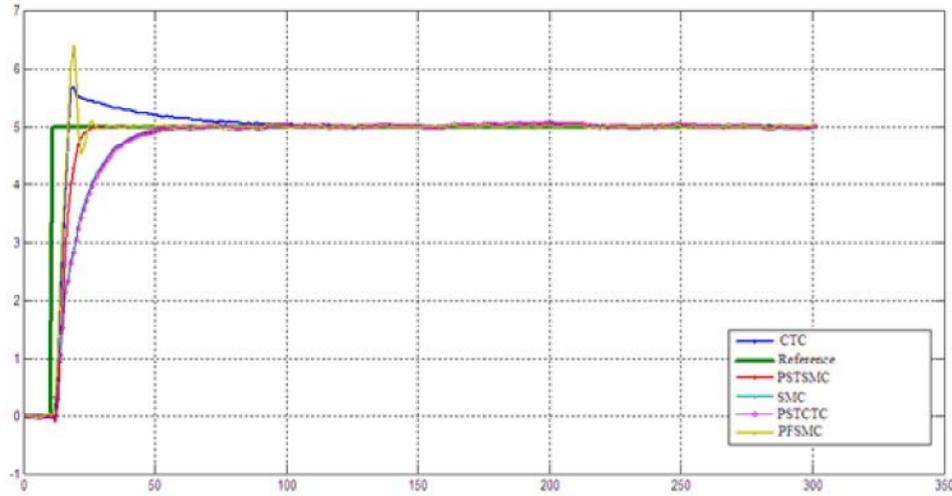


Fig. 7: First link step trajectory with disturbance

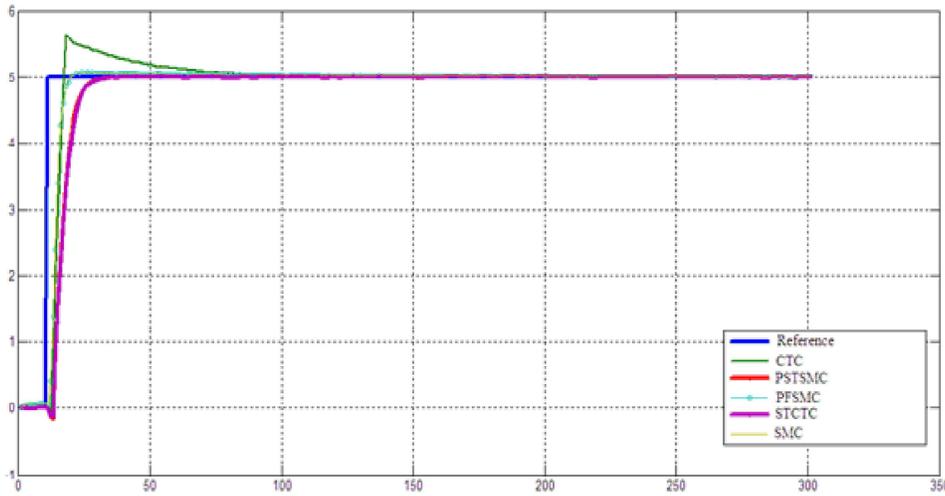


Fig. 8: Second link step trajectory with disturbance

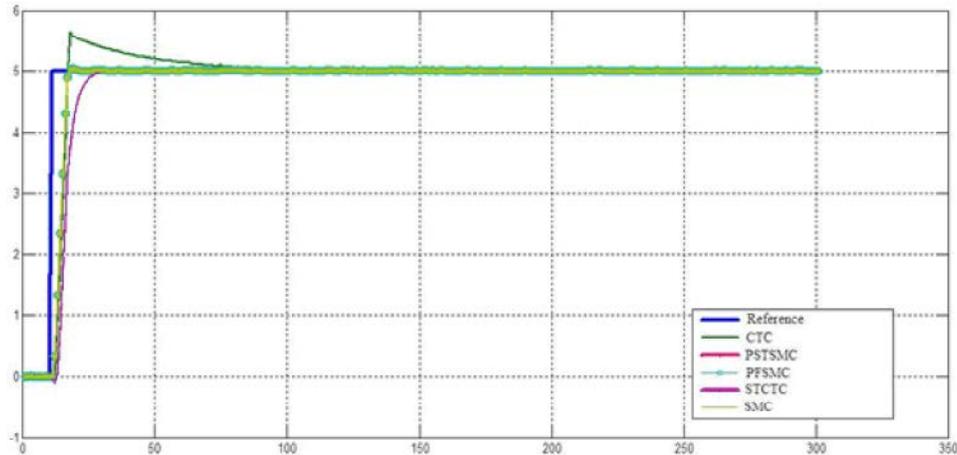


Fig. 9: Third link step trajectory with disturbance

**Disturbance Rejection:** Figure 7, 8 and 9 have shown the power disturbance elimination in above controllers. The main targets in these controllers are disturbance rejection as well as the other responses. A band limited white noise with predefined of 40% the power of input signal is applied to the step response. It found fairly fluctuations in trajectory responses. As mentioned earlier, CTC and SMC works very well when all parameters are known, this challenge plays important role to select the GTCTLC and GTFSMC as a based robust controller in this research.

### CONCLUSIONS

Refer to the research, a position artificial intelligence controller with tunable gain (GTCTLC) design and application to robot manipulator has proposed in order to design high performance nonlinear controller in the presence of uncertainties. Regarding to the positive points in computed torque controller, fuzzy logic controller and tunable method, the performance has improved. Each method by adding to the previous controller has covered negative points. The system performance in computed torque controller, computed torque like controller are sensitive to the gain updating factor. Therefore, compute the optimum value of gain updating factor for a system is the important challenge work. This problem has solved by adjusting gain updating factor of GTCTLC. In this way, the overall system performance has improved with respect to the classical computed torque controller. This method solved output oscillation as well as mathematical nonlinear equivalent part by applied fuzzy supervisory method.

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