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Self Centered Intuitionistic Fuzzy Graph

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Abstract: In this paper, the concept of $\mu\nu$-length, distance, radius, eccentricity, path cover and edge cover of an intuitionistic fuzzy graph(IFG) is defined. The definition of a self centered intuitionistic fuzzy graph and the necessary and sufficient condition for an IFG to be self centered are given. Also the necessary and sufficient conditions for a complete IFG to have an IF bridge is established.

Key words: Eccentricity, radius, diameter, path cover, edge cover, central vertex, self centered IF graph

INTRODUCTION

The first definition of a fuzzy graph was given by Kauffmann [1] in 1973, based on Zadeh's fuzzy relations [2][3][4]. But Rosenfeld [5] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. During the same time Yeh and Bang [6] have also introduced various connectedness concepts in fuzzy graphs. Krassimir introduced the concept of intuitionistic fuzzy(IF)relations and intuitionistic fuzzy graphs(IFG) in [7][8][9]. Nagoor Gani and Shajitha Begum discussed the various types of degrees and some properties of IFGs in [10]. Sunitha analysed the properties of self centered fuzzy graph in [11][12]. Parvathi and Karunambigai[13][14] introduced the concept of minmax IFG and analyzed its properties, also they analyzed the concept of operations, complements of intuitionistic fuzzy graphs. Akram and Davvaz discussed the properties of strong intuitionistic fuzzy graphs and also they introduced the concept of intuitionistic fuzzy line graphs in [15]. In [16], Akram introduced the concept of bipolar fuzzy graphs and studied some properties of interval-valued fuzzy graphs in [17]. These concepts lead us to analyze self centered IFGs and its properties. Section II consists of the basic definitions which are used to analyze the properties of a self centered IFG. In section III, we defined μ-length, ν- length, $\mu\nu$-length, μ-distance, ν-distance, distance, μ-eccentricity, ν-eccentricity, eccentricity, μ-radius, ν-radius, radius, μ-diameter, ν-diameter, diameter, central vertex, path cover, edge cover of IFGs and self centered intuitionistic fuzzy graph. Also we derived the main theorems, that is, the necessary and sufficient conditions for an IFG to be self centered IFG and the necessary and sufficient conditions for a complete IFG to have an IF bridge. Throughout this paper we consider $G : \langle \mu, \nu, V, E\rangle$ as minmax IFG and all the properties are analyzed only for minmax IFG.

PRELIMINARIES

Definition 1. [7] *An Intuitionistic Fuzzy Graph (IFG) is of the form $G : \langle \mu, \nu, V, E\rangle$ said to be a minmax IFG if*

(1) $V = \{v_0, v_1, ..., v_n\}$ *such that* $\mu_1 : V \to [0, 1]$ *and* $\nu_1 : V \to [0, 1]$, *denotes the degree of membership and non-membership of an element* $v_i \in V$ *respectively and* $0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$, *for every* $v_i \in V, (i = 1, 2, ..., n)$,

(2) $E \subseteq V \times V$ *where* $\mu_2 : V \times V \to [0, 1]$ *and* $\nu_2 : V \times V \to [0, 1]$ *are such that*
$\mu_2(v_i, v_j) \le \min(\mu_1(v_i), \mu_1(v_j))$
$\nu_2(v_i, v_j) \le \max(\nu_1(v_i), \nu_1(v_j))$,
denotes the degree of membership and non-membership of an edge $(v_i, v_j) \in E$ *respectively, where,* $0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$, *for every* $(v_i, v_j) \in E$.

Definition 2. [13] *The complement of an IFG* $G : \langle \mu, \nu, V, E\rangle$ *is an IFG* $\bar{G} : \langle \bar{\mu}, \bar{\nu}, \bar{V}, \bar{E}\rangle$, *where,*

- $\bar{V} = V$
- $\bar{\mu}_{1i} = \mu_{1i}$ *and* $\bar{\nu}_{1i} = \nu_{1i}, \forall i = 1, 2, ..., n$,
- $\bar{\mu}_{2ij} = \min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}$ *and* $\bar{\nu}_{2ij} = \max(\nu_{1i}, \nu_{1j}) - \nu_{2ij}$, $\forall i, j = 1, 2, ..., n$.

Definition 3. [13] *An IFG, $G : \langle \mu, \nu, V, E \rangle$ is said to be a complete IFG if*

$$\mu_{2ij} = \min(\mu_{1i}, \mu_{1j}) \text{ and}$$
$$\nu_{2ij} = \max(\nu_{1i}, \nu_{1j}),\ \forall\, v_i, v_j \in V.$$

Example 4. *An IFG $G : \langle \mu, \nu, V, E \rangle$ given in Figure 0.1 is a complete IFG. Consider an IFG, $G : \langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3, v_4\}$,*

$E = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_2, v_3), (v_2, v_4), (v_3, v_4)\}$, where,

$$\mu_{2ij} = \min(\mu_{1i}, \mu_{1j}) \text{ and}$$
$$\nu_{2ij} = \max(\nu_{1i}, \nu_{1j}),\ \forall\, e_{ij} \in E.$$

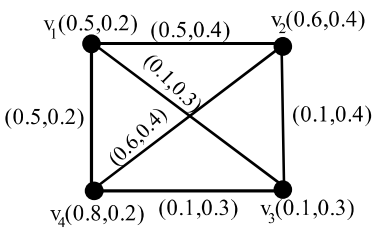

Figure 0.1: Complete Intuitionistic Fuzzy Graph

Definition 5. [13] *An IFG, $G : \langle \mu, \nu, V, E \rangle$ is said to be a strong IFG if*

$$\mu_{2ij} = \min(\mu_{1i}, \mu_{1j}) \text{ and}$$
$$\nu_{2ij} = \max(\nu_{1i}, \nu_{1j}),\ \text{for all } (v_i, v_j) \in E.$$

Example 6. *The following example shows that $G : \langle \mu, \nu, V, E \rangle$ is a strong IFG but not complete. Consider an IFG, $G : \langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_1, v_4), (v_2, v_3), (v_3, v_4)\}$, where,*

$$\mu_{2ij} = \min(\mu_{1i}, \mu_{1j}) \text{ and}$$
$$\nu_{2ij} = \max(\nu_{1i}, \nu_{1j}),\ \forall\, e_{ij} \in E.$$

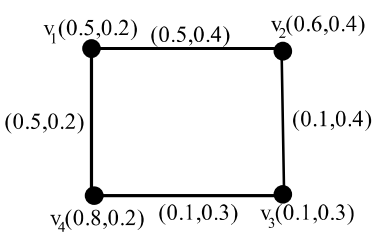

Figure 0.2: Strong Intuitionistic Fuzzy Graph

Definition 7. [13] *A path P in an IFG $G : \langle \mu, \nu, V, E \rangle$ is a sequence of distinct vertices $v_1, v_2, ..., v_n$ such that either one of the following conditions is satisfied. 1) $\mu_{2ij} > 0$ & $\nu_{2ij} = 0$, for some i & j. 2) $\mu_{2ij} = 0$ & $\nu_{2ij} > 0$, for some i & j.*

Definition 8. [13] *The length of a path $P : v_1 v_2 ... v_{n+1}$ $(n > 0)$ in $G : \langle \mu, \nu, V, E \rangle$ is n.*

Definition 9. [13] *A path $P : v_1 v_2 ... v_{n+1}$ in $G : \langle \mu, \nu, V, E \rangle$ is called a cycle if $v_1 = v_{n+1}$ and $n \geq 3$.*

Definition 10. [13] *An IFG $G : \langle \mu, \nu, V, E \rangle$ is connected if any two vertices are joined by a path.*

Definition 11. [13] *The μ- strength of a path $P : v_1 v_2 ... v_n$ is defined as $\min(\mu_2(v_i, v_j))$, for all i&j and is denoted by S_μ.*
The ν- strength of a path $P : v_1 v_2 ... v_n$ is defined as $\max(\nu_2(v_i, v_j))$, for all i&j and is denoted by S_ν.

Note 12. *If the same edge possesses both μ-strength and ν-strength value, then it is the strength of a path P. In other words, the strength of a path is defined to be the weight of the weakest edge of the path. That is the strength of a path $= (\mu_{2ij}, \nu_{2ij}) = (S_\mu, S_\nu)$.*

Definition 13. [13] *(v_i, v_j) is said to be a bridge in $G : \langle \mu, \nu, V, E \rangle$, if either of the conditions is satisfied $\mu'^{\infty}_{2ij} < \mu^{\infty}_{2ij}$ and $\nu'^{\infty}_{2ij} \geq \nu^{\infty}_{2ij}$ or $\mu'^{\infty}_{2ij} \leq \mu^{\infty}_{2ij}$ and $\nu'^{\infty}_{2ij} > \nu^{\infty}_{2ij}$, for some $v_i, v_j \in V$. In other words, deleting an edge (v_i, v_j) reduces the strength of connectedness between some pair of vertices or (v_i, v_j) is a bridge, if there exists v_x, v_y such that, (v_i, v_j) is an edge of every strongest path from v_x to v_y.*

Definition 14. [16] *An arc (v_i, v_j) is said to be a strong arc, if $\mu_2(v_i, v_j) \geq \mu_2^{\infty}(v_i, v_j)$, $\nu_2(v_i, v_j) \geq \nu_2^{\infty}(v_i, v_j)$.*

Definition 15. [16] *Let $G : \langle \mu, \nu, V, E \rangle$ be an IFG on V. Let $v_i, v_j \in V$, we say that v_i dominates v_j in G, if there exists a strong arc between them.*

Definition 16. [16] *An IFG $G : \langle \mu, \nu, V, E \rangle$ is said to be a bipartite if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that*
(i) $\mu_2(v_i, v_j) = 0$ and $\nu_2(v_i, v_j) = 0$, if $v_i, v_j \in V_1$ or $v_i, v_j \in V_2$.
(ii) $\mu_2(v_i, v_j) > 0$, $\nu_2(v_i, v_j) > 0$, if $v_i \in V_1$ or $v_j \in V_2$, for some i and j, (or)
$\mu_2(v_i, v_j) = 0$, $\nu_2(v_i, v_j) > 0$, if $v_i \in V_1$ or $v_j \in V_2$, for some i and j, (or)
$\mu_2(v_i, v_j) > 0$, $\nu_2(v_i, v_j) = 0$, if $v_i \in V_1$ or $v_j \in V_2$, for some i and j.

Definition 17. [16] *A bipartite IFG, $G : \langle \mu, \nu, V, E \rangle$ is said to be complete if $\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ and $\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j))$, for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{V_{1i}, V_{2i}}$.*

SELF CENTERED INTUITIONISTIC FUZZY GRAPH

In this section, the basic definitions and theorems are given. Also, we proved the necessary and sufficient conditions for an IFG to be self centered and the necessary and sufficient condition for an IFG to have an IF bridge.

Theorem 18. [13] *In an IFG, $G : \langle \mu, \nu, V, E \rangle$ for which $\mu_2 : V \times V \to [0,1]$ and $\nu_2 : V \times V \to [0,1]$ are not constant mapping, an edge (v_i, v_j) for which μ_{2ij} is maximum and ν_{2ij} is minimum. Therefore it is a bridge of G.*

Definition 19. *Let $G : \langle \mu, \nu, V, E \rangle$ be an IFG. The μ-strength of the paths connecting any two vertices v_i, v_j is defined as $\max(S_\mu)$ and is denoted by μ^∞_{2ij}. The ν-strength of the paths connecting any two vertices v_i, v_j is defined as $\min(S_\nu)$ and is denoted by ν^∞_{2ij}.*

Note 20. *If same edge possesses both the μ-strength and ν-strength values, then it is the strength of the strongest path P and it is denoted by $S_P = (\mu^\infty_{2ij}, \nu^\infty_{2ij})$, for all $i, j = 1, 2, ..., n$.*

Example 21. *Consider an IFG, $G : \langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_1, v_4), (v_2, v_3), (v_2, v_4), (v_3, v_4)\}$.*

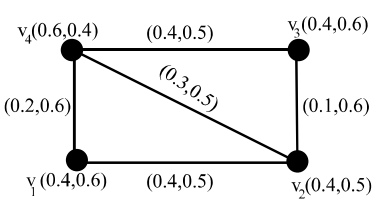

Figure 0.3: IFG

Here, v_1v_4 is a path of length 1 and the strength is $(0.2, 0.6)$, $v_1v_2v_4$ is a path of length 2 and the strength is (0.3,0.5), $v_1v_2v_3v_4$ is a path of length 3 and the strength is (0.1,0.6). A strongest path joining v_1 and v_4 is the path $P : v_1v_2v_4$ with $(\mu^\infty_2(v_1, v_4), \nu^\infty_2(v_1, v_4)) = (0.3, 0.5)$

Definition 22. *An IFG $H : \langle \mu', \nu', V', E' \rangle$ is said to be an IF subgraph of a connected IFG $G : \langle \mu, \nu, V, E \rangle$, if $\mu'_1(v_i) = \mu_1(v_i)$, $\nu'_1(v_i) = \nu_1(v_i)$, $\forall v_i \in V'$ and $\mu'_2(v_i, v_j) = \mu_2(v_i, v_j)$ and $\nu'_2(v_i, v_j) = \nu_2(v_i, v_j)$, $\forall (v_i, v_j) \in E'$.*

Example 23. *Consider an IFG, $G : \langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{(v_1, v_2), (v_1, v_4), (v_1, v_3), (v_2, v_3), (v_2, v_4), (v_3, v_4), (v_3, v_5), (v_4, v_5)\}$ and its subgraph $H{:}\langle \mu', \nu', V', E' \rangle$, such that $V' = \{v_2, v_3, v_4, v_5\}$, $E' = \{(v_2, v_3), (v_2, v_4), (v_3, v_4), (v_3, v_5), (v_4, v_5)\}$.*

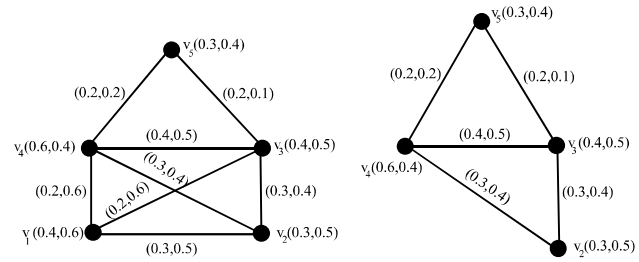

Figure 0.4: IFG and its IF subgraph

Definition 24. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The $\mu-$length of a path $P : v_1v_2...v_n$ in G, $l_\mu(P)$, is defined as $l_\mu(P) = \sum_{i=1}^{n-1} \frac{1}{\mu_2(v_i, v_{i+1})}$.*

Definition 25. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The $\nu-$length of a path $P : v_1v_2...v_n$ in G, $l_\nu(P)$, is defined as $l_\nu(P) = \sum_{i=1}^{n-1} \frac{1}{\nu_2(v_i, v_{i+1})}$.*

Definition 26. *The $\mu\nu$ - length of a path $P : v_1v_2...v_n$ in $G : \langle \mu, \nu, V, E \rangle$, $l_{\mu\nu}(P)$, is defined as $l_{\mu\nu}(P) = (l_\mu, l_\nu)$.*

Definition 27. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The μ-distance, $\delta_\mu(v_i, v_j)$, is the smallest μ-length of any $v_i - v_j$ path P in G, where $v_i, v_j \in V$. That is, $\delta_\mu(v_i, v_j) = \min(l_\mu(P))$.*

Definition 28. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The ν-distance, $\delta_\nu(v_i, v_j)$, is the smallest ν-length of any $v_i - v_j$ path P in G, where $v_i, v_j \in V$. That is, $\delta_\nu(v_i, v_j) = \min(l_\nu(P))$.*

Definition 29. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The distance, $\delta(v_i, v_j)$, is defined as $\delta(v_i, v_j) = (\delta_\mu(v_i, v_j), \delta_\nu(v_i, v_j))$.*

Definition 30. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. For each $v_i \in V$, the μ-eccentricity of v_i, denoted by $e_\mu(v_i)$ and is defined as $e_\mu(v_i) = \max\{\delta_\mu(v_i, v_j) : v_i \in V, v_i \neq v_j\}$.*

Definition 31. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. For each $v_i \in V$, the ν-eccentricity of v_i, denoted by $e_\nu(v_i)$ and is defined as $e_\nu(v_i) = \min\{\delta_\nu(v_i, v_j) : v_i \in V, v_i \neq v_j\}$.*

Definition 32. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. For each $v_i \in V$, the eccentricity of v_i, denoted by $e(v_i)$ and is defined as $e(v_i) = (e_\mu(v_i), e_\nu(v_i))$.*

Definition 33. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The μ-radius of G is denoted by $r_\mu(G)$ and is defined as $r_\mu(G) = \min\{e_\mu(v_i) : v_i \in V\}$.*

Definition 34. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The ν-radius of G is denoted by $r_\nu(G)$ and is defined as $r_\nu(G) = \min\{e_\nu(v_i) : v_i \in V\}$.*

Definition 35. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The radius of G is denoted by $r(G)$ and is defined as $r(G) = (r_\mu(G), r_\nu(G))$.*

Definition 36. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The μ-diameter of G is denoted by $d_\mu(G)$ and is defined as $d_\mu(G) = \max\{e_\mu(v_i) : v_i \in V\}$.*

Definition 37. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The ν-diameter of G is denoted by $d_\nu(G)$ and is defined as $d_\nu(G) = \max\{e_\nu(v_i) : v_i \in V\}$.*

Definition 38. *Let $G : \langle \mu, \nu, V, E \rangle$ be a connected IFG. The diameter of G is denoted by $d(G)$ and is defined as $d(G) = (d_\mu(G), d_\nu(G))$.*

Example 39. *Consider an IFG, G:$\langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_2, v_3), (v_3, v_4)\}$.*

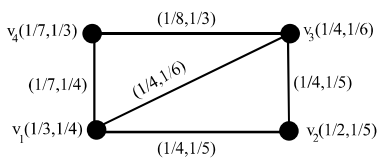

Figure 0.5: IFG

Here, v_1v_4 is a path of length 1 and $l_{\mu\nu} = (7, 4)$, $v_1v_3v_4$ is a path of length 2 and $l_{\mu\nu} = (12, 9)$, $v_1v_2v_3v_4$ is a path of length 3 and $l_{\mu\nu} = (16, 13)$. Hence $\delta_\mu(v_1, v_4) = 7$, $\delta_\nu(v_1, v_4) = 4$. That is $\delta(v_1, v_4) = (7, 4)$. Similarly, $\delta_\mu(v_1, v_2) = 4$, $\delta_\nu(v_1, v_2) = 5$, $\delta_\mu(v_1, v_3) = 4$, $\delta_\nu(v_1, v_3) = 6$, $\delta_\mu(v_2, v_3) = 4$, $\delta_\nu(v_2, v_3) = 5$, $\delta_\mu(v_2, v_4) = 11$, $\delta_\nu(v_2, v_4) = 8$, $\delta_\mu(v_3, v_4) = 8$, $\delta_\nu(v_3, v_4) = 3$, $e_\mu(v_1) = 7$, $e_\mu(v_2) = 11$, $e_\mu(v_3) = 8$, $e_\mu(v_4) = 11$, $e_\nu(v_1) = 4$, $e_\nu(v_2) = 5$, $e_\nu(v_3) = 3$, $e_\nu(v_4) = 3$, $r_\mu(G) = 7$, $r_\nu(G) = 3$, $d_\mu(G) = 11$, $d_\nu(G) = 5$.

Definition 40. *A vertex $v_i \in V$ is called a μ-central vertex of a connected IFG $G : \langle \mu, \nu, V, E \rangle$, if $r_\mu(G) = e_\mu(v_i)$.*

Definition 41. *A vertex $v_i \in V$ is called a ν-central vertex of a connected IFG $G : \langle \mu, \nu, V, E \rangle$, if $r_\nu(G) = e_\nu(v_i)$.*

Definition 42. *A vertex $v_i \in V$ is called a central vertex of a connected IFG $G : \langle \mu, \nu, V, E \rangle$, if $r_\mu(G) = e_\mu(v_i)$ and $r_\nu(G) = e_\nu(v_i)$ and the set of all central vertices of an IFG is denoted by $C(G)$.*

Definition 43. *$< C(G) > = H : \langle \mu', \nu', V', E' \rangle$ is an IF subgraph of $G : \langle \mu, \nu, V, E \rangle$ induced by the central vertices of G, is called the center of G.*

Definition 44. *A connected IFG $G : \langle \mu, \nu, V, E \rangle$ is a μ-self centered graph, if every vertex of G is a μ-central vertex, that is $r_\mu(G) = e_\mu(v_i)$, $\forall v_i \in V$.*

Definition 45. *A connected IFG $G : \langle \mu, \nu, V, E \rangle$ is a ν-self centered graph, if every vertex of G is a ν-central vertex, that is $r_\nu(G) = e_\nu(v_i)$, $\forall v_i \in V$.*

Definition 46. *A connected IFG $G : \langle \mu, \nu, V, E \rangle$ is a self centered graph, if every vertex of G is a central vertex, that is $r_\mu(G) = e_\mu(v_i)$ and $r_\nu(G) = e_\nu(v_i)$, $\forall v_i \in V$.*

Example 47. *Consider an IFG, $G : \langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3\}$, $E = \{(v_1, v_2), (v_1, v_3), (v_2, v_3)\}$.*

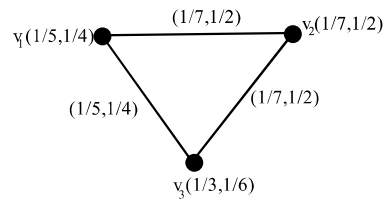

Figure 0.6: Self Centered IFG

Here, $\delta_\mu(v_1, v_2) = 7$, $\delta_\nu(v_1, v_2) = 2$, $\delta_\mu(v_1, v_3) = 5$, $\delta_\nu(v_1, v_3) = 4$, $\delta_\mu(v_2, v_3) = 7$, $\delta_\nu(v_2, v_3) = 2$, $e_\mu(v_i) = 7$, $e_\nu(v_i) = 2$, where $i = 1, 2, 3$, $r(G) = (7, 2)$, $d(G) = (7, 2)$. Here, $e_\mu(v_i) = r_\mu(G)$, $e_\nu(v_i) = r_\nu(G)$, $\forall v_i \in V$.

Definition 48. *A path cover of an IFG $G : \langle \mu, \nu, V, E \rangle$ is a set P of paths such that every vertex of $G : \langle \mu, \nu, V, E \rangle$ is incident to some path of P.*

Example 49. *Consider an IFG, G:$\langle \mu, \nu, V, E \rangle$, such that V=$\{v_1, v_2, v_3, v_4, v_5\}$, E=$\{(v_1, v_2), (v_1, v_4), (v_2, v_3), (v_2, v_5), (v_3, v_4), (v_3, v_5)\}$.*

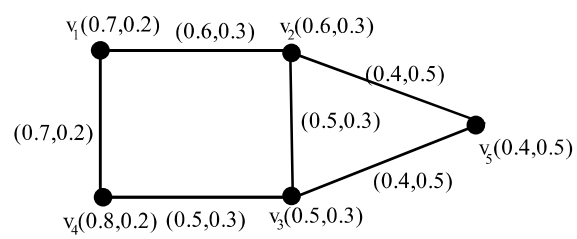

Figure 0.7: IFG

In this example, the some of the path covers of an IFG $G : \langle \mu, \nu, V, E \rangle$ are $M_1 = \{v_1v_2v_5, v_4v_3\}$, $M_2 = \{v_1v_2v_5, v_4v_3v_5\}$, $M_3 = \{v_1v_2v_5, v_2v_3v_4\}$, $M_4 = \{v_1v_2, v_3v_4, v_3v_5\}$, $M_5 = \{v_1v_4, v_2v_3, v_2v_5\}$, $M_6 = \{v_4v_3v_5, v_1v_2\}$, $M_7 = \{v_4v_3v_5, v_3v_2v_1\}$, $M_8 = \{v_2v_1v_4, v_3v_5\}$, $M_9 = \{v_1v_4, v_2v_3, v_3v_5\}$, $M_{10} = \{v_4v_1v_2, v_5v_3v_2\}$.

Definition 50. *An edge cover of an IFG $G : \langle \mu, \nu, V, E\rangle$ is a set L of edges such that every vertex of $G : \langle \mu, \nu, V, E\rangle$ is incident to some edge of L.*

Example 51. *In Figure 0.7, the some of the edge covers of an IFG $G{:}\langle \mu, \nu, V, E\rangle$ are*
L_1=$\{(v_1, v_2), (v_4, v_3), (v_3, v_5)\}$,
L_2=$\{(v_1, v_2), (v_2, v_5), (v_4, v_3)\}$,
L_3= $\{(v_1, v_4), (v_2, v_3), (v_2, v_5)\}$,
$L_4 = \{(v_1, v_4), (v_2, v_3), (v_3, v_5)\}$,
L_5=$\{(v_1, v_4), (v_1, v_2), (v_4, v_3), (v_3, v_5)\}$,
L_6=$\{(v_1, v_4), (v_4, v_3), (v_2, v_5)\}$.

Definition 52. *A maximal connected subgraph of an IFG $G : \langle \mu, \nu, V, E\rangle$ is a subgraph that is connected and is not contained in any other connected subgraph of $G : \langle \mu, \nu, V, E\rangle$.*

Definition 53. *The components of an IFG $G : \langle \mu, \nu, V, E\rangle$ is its maximal connected subgraphs, where $G : \langle \mu, \nu, V, E\rangle$ is a disconnected IFG.*

Example 54. *In the following Figure 0.8, the components of an IFG $G{:}\langle \mu, \nu, V, E\rangle$ are $G_1{:}\langle \mu', \nu', V_1, E_1\rangle$, $G_2{:}\langle \mu'', \nu'', V_2, E_2\rangle$ and $G_3{:}\langle \mu''', \nu''', V_3, E_3\rangle$, such that V_1=$\{v_1, v_2, v_3, v_4\}$, E_1=$\{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$, V_2=$\{v_5, v_6, v_7\}$, E_2=$\{(v_5, v_6), (v_5, v_7), (v_6, v_7)\}$,$V_3$=$\{v_8, v_9\}$, E_3=$\{(v_8, v_9)\}$.*

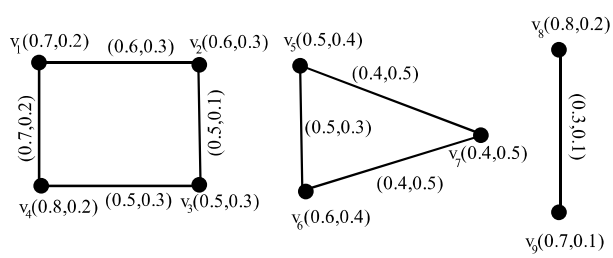

Figure 0.8: IFG with 3 components

Definition 55. *An IFG $H{:}\langle \mu', \nu', V', E'\rangle$ is said to be a intuitionistic fuzzy subgraph(IFSG) of $G{:}\langle \mu, \nu, V, E\rangle$ induced by V' if $V' \subseteq V$ and $\mu'_1(v_i) = \mu_1(v_i)$ & $\nu'_1(v_i) = \nu_1(v_i)$, $\forall v_i \in V'$, $\mu'_2(v_i, v_j) = \mu_2(v_i, v_j)$, $\nu'_2(v_i, v_j) = \nu_2(v_i, v_j)$, $\forall v_i, v_j \in V'$.*

Example 56. *Consider an IFG, $G{:}\langle \mu, \nu, V, E\rangle$, such that V=$\{v_1, v_2, v_3, v_4, v_5\}$, E=$\{(v_1, v_2), (v_1, v_4), (v_2, v_3), (v_2, v_5), (v_3, v_4), (v_3, v_5)\}$ and its induced IF subgraph $H{:}\langle \mu', \nu', V', E'\rangle$, such that V'=$\{v_2, v_3, v_5\}$, E'=$\{(v_2, v_3), (v_2, v_5), (v_3, v_5)\}$.*

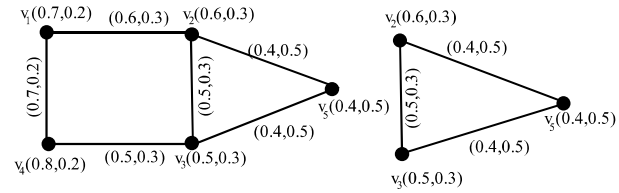

Figure 0.9: IFG and its induced IF subgraph

Theorem 57. *If $G : \langle \mu, \nu, V, E\rangle$ is an IF bipartite graph then it has no strong cycle of odd length.*

Proof. Let G be an IF bipartite graph with IF bipartition V_1 and V_2. Suppose that it contains a strong cycle of odd length say $v_1, v_2, ..., v_n, v_1$ for some odd n. Without loss of generality, let $v_1 \in V_1$. Since (v_i, v_{i+1}) is strong for $i = 1, 2, ..., n-1$ and the nodes are alternatively in V_1 and V_2, we have v_n and $v_1 \in V_1$. But this implies that (v_n, v_1) is an edge in V_1, which contradicts the assumption that G is an IF bipartite. Hence IF bipartite graph has no strong cycle of odd length. □

Theorem 58. *Every complete intuitionistic fuzzy graph $G : \langle \mu, \nu, V, E\rangle$ is a self centered IFG and $r_\mu(G) = \frac{1}{\mu_{1i}}, r_\nu(G) = \frac{1}{\nu_{1i}}$, where μ_{1i} is the least and ν_{1i} is the greatest.*

Proof. Let $G : \langle \mu, \nu, V, E\rangle$ be a complete IFG. To prove that $G : \langle \mu, \nu, V, E\rangle$ is self centered IFG. That is we have to show that every vertex is a central vertex. First we claim that $G : \langle \mu, \nu, V, E\rangle$ is a μ- self centered IFG and $r_\mu(G) = \frac{1}{\mu_{1i}}$, where μ_{1i} is the least. Now fix a vertex $v_i \in V$ such that μ_{1i} is least vertex membership value of G.
Case (i): Consider all the $v_i - v_j$ paths P of length n in $G, \forall v_j \in V$.
Subcase(i): If $n = 1$, then μ - length of $P = l_\mu(P) = \frac{1}{\mu_{1i}}$.
Subcase(ii): If $n > 1$, then one of the edges of P possesses the μ-strength μ_{1i} and hence, $\mu-$ length of a $v_i - v_j$ path will exceed $\frac{1}{\mu_{1i}}$. That is, $\mu-$ length of (P) $= l_\mu(P) > 1/\mu_{1i}$. Hence,

$$\delta_\mu(v_i, v_j) = \min(l_\mu(P)) = \frac{1}{\mu_{1i}}, \forall\, v_j \in V. \quad (1)$$

Case (ii): Let $v_k \neq v_i \in V$. Consider all $v_k - v_j$ paths Q of length n in $G, \forall v_j \in V$.
Subcase(i): If $n = 1$, then $\mu_2(v_k, v_j) = \min(\mu_{1k}, \mu_{1j}) \geq \mu_{1i}$, since μ_{1i} is the least. Hence μ - length$(Q) = l_\mu(Q)$ $= \frac{1}{\mu_2(v_k, v_j)} \leq \frac{1}{\mu_{1i}}$.
Subcase(ii): If $n = 2$, then $l_\mu(Q) = \frac{1}{\mu_2(v_k, v_{k+1})} + \frac{1}{\mu_2(v_{k+1}, v_j)} \leq \frac{2}{\mu_{1i}}$, since μ_{1i} is the least.
Subcase(iii): If $n > 2$, then $l_\mu(Q) \leq \frac{n}{\mu_{1i}}$, since μ_{1i} is

the least. Hence,

$$\delta_\mu(v_k, v_j) = \min(l_\mu(Q)) \leq 1/\mu_{1i},\ \forall v_k, v_j \in V. \quad (2)$$

From Equation (1) and (2), we have,

$$e_\mu(v_i) = \max(\delta_\mu(v_i, v_j)) = \frac{1}{\mu_{1i}}, \forall v_i \in V \quad (3)$$

Hence $G : \langle \mu, \nu, V, E \rangle$ is a μ- self centered IFG.

$$\begin{aligned} Now,\ r_\mu(G) &= \min(e_\mu(v_i)) \\ &= \frac{1}{\mu_{1i}},\ \text{since by equation(3)} \\ r_\mu(G) &= \frac{1}{\mu_{1i}},\ \text{where } \mu_1(v_i) \text{ is least} \end{aligned}$$

Next, we claim that $G : \langle \mu, \nu, V, E \rangle$ is a ν- self centered IFG and $r_\nu(G) = \frac{1}{\nu_{1i}}$, where ν_{1i} is the greatest. Now, choose some vertex $v_i \in V$ such that ν_{1i} is the greatest vertex membership value of G .

Case (i): Consider all the $v_i - v_j$ path P of length n in $G, \forall v_j \in V$.

Subcase(i): If $n = 1$, then $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j}) = \nu_{1i}$. Therefore, the ν - length of $P = l_\nu(P) = \frac{1}{\nu_{1i}}$.

Subcase(ii): If $n > 1$, then one of the edges of P possesses the ν-strength ν_{1i} and hence $\nu-$ length of P will exceed $\frac{1}{\nu_{1i}}$. That is, $\nu-$ length of $P = l_\nu(P) > 1/\nu_{1i}$. Hence,

$$\delta_\nu(v_i, v_j) = \min(l_\nu(P)) = \frac{1}{\nu_{1i}}, \forall\ v_j \in V. \quad (4)$$

Case (ii): Let $v_k \neq v_i \in V$. Consider all $v_k - v_j$ paths Q of length n in $G, \forall v_j \in V$.

Subcase(i): If n= 1, then $\nu_2(v_k, v_j)$= $\max(\nu_{1k}, \nu_{1j}) \leq \nu_{1i}$, since ν_{1i} is the greatest. Therefore ν - length of $Q = l_\nu(Q) = \frac{1}{\nu_2(v_k, v_j)} \geq \frac{1}{\nu_{1i}}$.

Subcase(ii): If $n = 2$, $l_\nu(Q) = \frac{1}{\nu_2(v_k, v_{k+1})} + \frac{1}{\nu_2(v_{k+1}, v_j)} \geq \frac{2}{\nu_{1i}}$, since ν_{1i} is the greatest.

Subcase(iii): If $n > 2$, then $l_\nu(Q) \geq \frac{n}{\nu_{1i}}$, since ν_{1i} is the greatest. Hence,

$$\delta_\nu(v_k, v_j) = \min(l_\nu(Q)) \geq 1/\nu_{1i},\ \forall v_k, v_j \in V. \quad (5)$$

From Equation (4) and (5), we have,

$$e_\nu(v_i) = \min(\delta_\nu(v_i, v_j)) = \frac{1}{\nu_{1i}}, \forall v_i \in V \quad (6)$$

Hence $G : \langle \mu, \nu, V, E \rangle$ is a ν- self centered IFG.

$$\begin{aligned} Now,\ r_\nu(G) &= \min(e_\nu(v_i)) \\ &= \frac{1}{\nu_{1i}},\ \text{since by equation(6)} \\ r_\nu(G) &= \frac{1}{\nu_{1i}},\ \text{where } \nu_1(v_i) \text{ is greatest} \end{aligned}$$

From equation(3) and (6), every vertex of G:$\langle \mu, \nu, V, E \rangle$ is a central vertex. Hence G is a self -centered IFG. □

Example 59. *The following example shows that the converse of the above theorem is not true. Consider an IFG, $G : \langle \mu, \nu, V, E \rangle$, such that $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_1, v_4), (v_2, v_3), (v_3, v_4)\}$, which is not complete.*

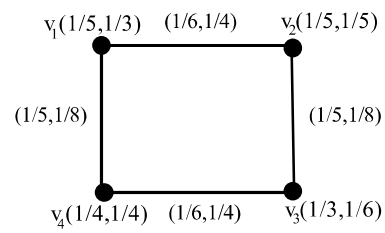

Figure 0.10: Self Centered IFG but not complete

Here, $\delta_\mu(v_1, v_2) = 6$, $\delta_\nu(v_1, v_2) = 4$, $\delta_\mu(v_1, v_3) = 11$, $\delta_\nu(v_1, v_3) = 12$, $\delta_\mu(v_1, v_4) = 5$, $\delta_\nu(v_1, v_4) = 8$, $\delta_\mu(v_2, v_3) = 5$, $\delta_\nu(v_2, v_3) = 8$, $\delta_\mu(v_2, v_4) = 11$, $\delta_\nu(v_2, v_4) = 12$, $\delta_\mu(v_3, v_4) = 6$, $\delta_\nu(v_3, v_4) = 4$, $e_\mu(v_i) = 11$, $e_\nu(v_i) = 4$, where $i = 1, 2, 3, 4$, $r(G) = (11, 4)$, $d(G) = (11, 4)$. Here, $e_\mu(v_i) = r_\mu(G)$, $e_\nu(v_i) = r_\nu(G)$, $\forall v_i \in V$. Hence $G : \langle \mu, \nu, V, E \rangle$ is self centered IFG but not complete.

Lemma 60. *An IFG $G : \langle \mu, \nu, V, E \rangle$ is a self centered IFG if and only if $r_\mu(G) = d_\mu(G)$, $r_\nu(G) = d_\nu(G)$.*

Proof. Proof follows from the definitions (29), (32), (35), (38), (42) and (46). □

Theorem 61. *If $G : \langle \mu, \nu, V, E \rangle$ is a complete IFG then for at least one edge $\mu_2^\infty(v_i, v_j) = \mu_2(v_i, v_j)$ and $\nu_2^\infty(v_i, v_j) = \nu_2(v_i, v_j)$*

Proof. If $G : \langle \mu, \nu, V, E \rangle$ be a complete IFG. Consider a vertex v_i whose membership value is μ_{1i} and non-membership value is ν_{1i}.

Case (i): Let μ_{1i} be the least and ν_{1i} be the greatest in the vertex $v_i \in V$. Let $v_i, v_j \in V$, then $(\mu_{2ij}, \nu_{2ij}) = (\mu_{1i}, \nu_{1i})$ and $(\mu_{2ij}^\infty, \nu_{2ij}^\infty) = (\mu_{1i}, \nu_{1i})$. The strength of all the edges which are incident on the vertex v_i is (μ_{1i}, ν_{1i}). Since G is a complete IFG.

Case (ii): Let μ_{1i} be the least and ν_{1k} be the greatest, where $v_i \neq v_k$. Then $(\mu_{2ik}, \nu_{2ik}) = (\mu_{1i}, \nu_{1k})$. Since it is a complete IFG, there will be an edge between v_i and v_k, therefore $\mu_{2ik}^\infty = \mu_{1i}$ and $\nu_{2ik}^\infty = \nu_{1k}$. □

Theorem 62. *Let $G : \langle \mu, \nu, V, E \rangle$ be a complete IFG with n vertices. Then G has an intuitionistic fuzzy bridge if and only if there exist an increasing sequence $\{t_1, t_2, ..., t_{n-1}, t_n\}$ such that $t_{n-2} < t_{n-1} \leq t_n$ where $t_i = \mu_1(v_i)$, for every i = 1,2,...,n and a decreasing sequence $\{s_1, s_2, ..., s_{n-1}, s_n\}$ such that $s_{n-2} > s_{n-1} \geq s_n$ where $s_i = \nu_1(v_i)$, $\forall\ i = 1, 2, ..., n$. Also the arc (v_{n-1}, v_n) is an IF bridge of G.*

Proof. Assume that $G : \langle \mu, \nu, V, E\rangle$ is a complete IFG and that G has an intuitionistic fuzzy bridge(v_i, v_j). Then we claim that there exist an increasing sequence $\{t_1, t_2, ..., t_{n-1}, t_n\}$ such that $t_{n-2} < t_{n-1} \leq t_n$, where $t_i = \mu_1(v_i)$, for every $i = 1, 2, ..., n$ and a decreasing sequence $\{s_1, s_2, ..., s_{n-1}, s_n\}$ such that $s_{n-2} > s_{n-1} \geq s_n$ where $s_i = \nu_1(v_i)$,$\forall\, i = 1, 2, ..., n$.
Without loss of generality, let $\mu_1(v_i) \leq \mu_1(v_j)$, that is $t_{n-1} \leq t_n$, where, $t_{n-1} = \mu_1(v_i)$, $t_n = \mu_1(v_j)$ and $\nu_1(v_i) \geq \nu_1(v_j)$, that is, $s_{n-1} \geq s_n$, where, $s_{n-1} = \nu_1(v_i)$, $s_n = \nu_1(v_j)$, so that $\mu_2(v_i, v_j) = \mu_1(v_i)$, $\nu_2(v_i, v_j) = \nu_1(v_i)$. Assume, to the contrary, that there is at least one vertex $v_i \neq v_k$ such that $\mu_1(v_i) \leq \mu_1(v_k)$, that is $t_{n-1} \leq t_k$, where $k \neq n$, $t_{n-1} = \mu_1(v_i)$, $t_k = \mu_1(v_k)$ and $\nu_1(v_i) \geq \nu_1(v_k)$, $s_{n-1} \geq s_k$, where $k \neq n$, $s_{n-1} = \nu_1(v_i)$, $s_k = \nu_1(v_k)$. Now consider the cycle $C : v_iv_jv_kv_i$ in G, then $\mu_2(v_i, v_j) = \mu_2(v_i, v_k) = \mu_1(v_i)$ and $\nu_2(v_i, v_j) = \nu_2(v_i, v_k) = \nu_1(v_i)$

$$\mu_2(v_j, v_k) = \begin{cases} \mu_{1j} & \text{if } \mu_{1i} = \mu_{1j} \text{ or } \mu_{1i} < \mu_{1j} \leq \mu_{1k} \\ \mu_{1k} & \text{if } \mu_{1i} < \mu_{1k} \leq \mu_{1j} \end{cases}$$

$$\nu_2(v_j, v_k) = \begin{cases} \nu_{1j} & \text{if } \nu_{1i} = \nu_{1j} \text{ or } \nu_{1i} > \nu_{1j} \geq \nu_{1k} \\ \nu_{1k} & \text{if } \nu_{1i} > \nu_{1k} > \nu_{1j} \end{cases}$$

In either case it is seen that μ_{1i} is the least and ν_{1i} is the greatest among all other edges. But by Theorem 18, it is a contradiction to the fact that (v_i, v_j) is an IF bridge.
Conversely,
let $t_1 < t_2 < ... < t_{n-2} < t_{n-1} \leq t_n$ and $t_i = \mu_1(v_i)$ and $s_1 > s_2 > ... > s_{n-2} > s_{n-1} \geq s_n$ and $s_i = \nu_1(v_i)$.
Claim: (v_{n-1}, v_n) is an IF bridge of G.
Now $\mu_2(v_{n-1}, v_n) = \min(\mu_1(v_{n-1}), \mu_1(v_n)) = \mu_1(v_{n-1})$ and $\nu_2(v_{n-1}, v_n)= \max(\nu_1(v_{n-1}), \nu_1(v_n))= \nu_1(v_{n-1})$ and clearly by hypothesis, all other edges of G will have μ- strength strictly less than $\mu_1(v_{n-1})$ and ν-strength strictly greater than $\nu_1(v_{n-1})$. Hence the edge (v_{n-1}, v_n) is an intuitionistic fuzzy bridge by Theorem 18. □

Theorem 63. *Let $G : \langle \mu, \nu, V, E\rangle$ be a connected IFG with path covers P_1 and P_2 of G. Then the necessary and sufficient condition for an IFG to be self centered IFG is*

$$\delta_\mu(v_i, v_j) = d_\mu(G), \forall\, (v_i, v_j) \in P_1 \text{ and}$$
$$\delta_\nu(v_i, v_j) = r_\nu(G), \forall\, (v_i, v_j) \in P_2. \tag{7}$$

Proof. **Necessary Condition:** We now assume that $G : \langle \mu, \nu, V, E\rangle$ is a self centered IFG and we have to prove that equation (7) holds. Suppose equation (7) does not hold, then we have, $\delta_\mu(v_i, v_j) \neq d_\mu(G)$, for some $(v_i, v_j) \in P_1$ and $\delta_\nu(v_i, v_j) \neq r_\nu(G)$, for some $(v_i, v_j) \in P_2$. By using Lemma 60, the above inequality becomes $\delta_\mu(v_i, v_j) \neq r_\mu(G)$, for some $(v_i, v_j) \in P_1$ and $\delta_\nu(v_i, v_j) \neq r_\nu(G)$, for some $(v_i, v_j) \in P_2$. Then $e_\mu(v_i) \neq r_\mu(G)$, $e_\nu(v_i) \neq r_\nu(G)$, for some $v_i \in V$, which implies $G : \langle \mu, \nu, V, E\rangle$ is not self centered IFG, which is a contradiction. Hence $\delta_\mu(v_i, v_j) = d_\mu(G)$, $\forall (v_i, v_j) \in P_1$, $\delta_\nu(v_i, v_j) = r_\nu(G)$, $\forall\, (v_i, v_j) \in P_2$.
Sufficient Condition: We now assume that equation (7) holds and we have to prove that $G : \langle \mu, \nu, V, E\rangle$ is a self centered IFG. If equation (7) holds, then we've $e_\mu(v_i) = \delta_\mu(v_i, v_j)$, $\forall\, (v_i, v_j) \in P_1$ and $e_\nu(v_i) = \delta_\mu(v_i, v_j)$, $\forall\, (v_i, v_j) \in P_2$, which implies $e_\mu(v_i) = r_\mu(G)$, $e_\nu(v_i) = r_\nu(G)$, $\forall\, v_i \in V$. Hence $G : \langle \mu, \nu, V, E\rangle$ is self centered IFG.

□

Corollary 64. *If $G : \langle \mu, \nu, V, E\rangle$ is a connected complete IFG with edge covers L_1 and L_2 of G. Then the necessary and sufficient condition for an IFG to be self centered IFG is*

$$\delta_\mu(v_i, v_j) = d_\mu(G), \forall\, (v_i, v_j) \in L_1 \text{ and}$$
$$\delta_\nu(v_i, v_j) = r_\nu(G), \forall\, (v_i, v_j) \in L_2. \tag{8}$$

Theorem 65 (Embedding Theorem). *Let $H : \langle \mu', \nu', V', E'\rangle$ be a connected ν-self centered IFG. Then there exist a connected IFG $G : \langle \mu, \nu, V, E\rangle$ such that $< C(G) >$ is isomorphic to H. Also $d_\mu(G) = 2r_\mu(G)$, $d_\nu(G) = 2r_\nu(G)$.*

Proof. Given that $H : \langle \mu', \nu', V', E'\rangle$ be a connected ν-self centered IFG. Let $d_\mu(H) = l$ and $d_\nu(H) = m$. Then construct $G : \langle \mu, \nu, V, E\rangle$ from H as follows:
Take two vertices $v_i, v_j \in V$ with $\mu_1(v_i) = \mu_1(v_j) = \frac{1}{l}$, $\nu_1(v_i) = \nu_1(v_j) = \frac{1}{2m}$ and join all the vertices of H to both v_i and v_j with $\mu_2(v_i, v_k) = \mu_2(v_j, v_k) = \frac{1}{l}$, $\nu_2(v_i, v_k) = \nu_2(v_j, v_k) = \frac{1}{2m}$, $\forall\, v_k \in V'$. Put $\mu_1 = \mu'_1$, $\nu_1 = \nu'_1$, for all vertices in H and $\mu_2 = \mu'_2$, $\nu_2 = \nu'_2$, for all edges in H (see figure 0.11)
Claim: $G : \langle \mu, \nu, V, E\rangle$ is an IFG. First note that $\mu_1(v_i) \leq \mu_1(v_k)$, $\forall v_k \in H$. If possible, let $\mu_1(v_i) > \mu_1(v_k)$ for at least one vertex $v_k \in H$. Then $\frac{1}{l} > \mu_1(v_k)$, that is $l < \frac{1}{\mu_1(v_k)} \leq \frac{1}{\mu_2(v_k, v_l)}$, where the last inequality holds for every $v_l \in V'$, since H is an IFG. That is $\frac{1}{\mu_2(v_k, v_l)} > l, \forall v_k \in H$ which contradicts that $d_\mu(H) = l$. Therefore $\mu_1(v_i) \leq \mu_1(v_k)$, for all $v_k \in V'$ and $\mu_2(v_i, v_k) \leq \min(\mu_{1i}, \mu_{1k}) = \frac{1}{l}$, similarly, $\mu_2(v_j, v_k) \leq \min(\mu_{1j}, \mu_{1k}) = \frac{1}{l}$, for all $v_k \in V'$. Note that $\nu_1(v_i) \leq \nu_1(v_k)$, $\nu_1(v_j) \leq \nu_1(v_k)$, $\forall v_k \in V'$, since $d_\nu(H) = m$. Therefore $\nu_2(v_i, v_k) \leq \max(\nu_{1i}, \nu_{1k}) = \frac{1}{2m}$, similarly, $\nu_2(v_j, v_k) \leq \max(\nu_{1j}, \nu_{1k}) = \frac{1}{2m}$. Hence $G : \langle \mu, \nu, V, E\rangle$ is an IFG.
Also, $e_\mu(v_k) = l$, $\forall v_k \in V'$ and $e_\mu(v_i) = e_\mu(v_j) = \frac{1}{\mu_2(v_i, v_k)} + \frac{1}{\mu_2(v_k, v_l)} = 2l$, $r_\mu(G) = l$, $d_\mu(G) = 2l$.

Next, $e_\nu(v_k) = m$, $\forall v_k \in V'$ and $e_\nu(v_i) = e_\nu(v_j) = \frac{1}{\nu_2(v_l, v_k)} = 2m$, $\forall v_k \in V'$. Therefore, $r_\nu(G) = m$, $d_\nu(G) = 2m$. Hence $< C(G) >$ is isomorphic to H. □

The following example shows that $< C(G) >$ is isomorphic to H. Also $d_\mu(G) = 2r_\mu(G)$, $d_\nu(G) = 2r_\nu(G)$.

Example 66. *Let $H : \langle \mu', \nu', V', E' \rangle$ be an ν-self centered intuitionistic fuzzy graph where $d_\mu(H) = 4$ and $d_\nu(H) = 4$ and $G : \langle \mu, \nu, V, E \rangle$ is constructed from $H : \langle \mu', \nu', V', E' \rangle$ as follows*

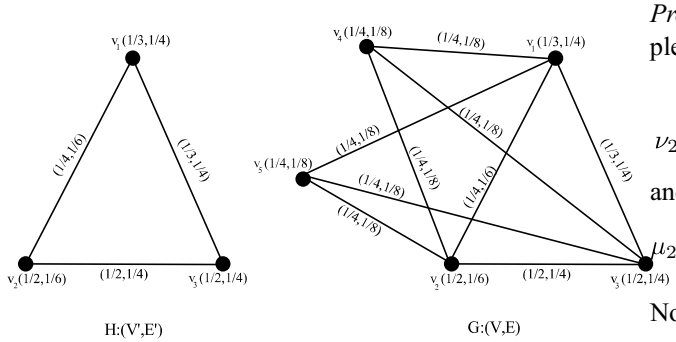

Figure 0.11: IFG

Theorem 67. *An IFG $G : \langle \mu, \nu, V, E \rangle$ is a self centered if and only if $\delta_\mu(v_i, v_j) \leq r_\mu(G)$, $\delta_\nu(v_i, v_j) \geq r_\nu(G)$, $\forall v_i, v_j \in V$.*

Proof. **Necessary Condition:** We assume that G is self-centered IFG. That is, $e_\mu(v_i) = e_\mu(v_j)$, $e_\nu(v_i) = e_\nu(v_j)$, $\forall v_i, v_j \in V$, $r_\mu(G) = e_\mu(v_i)$, $r_\nu(G) = e_\nu(v_i)$, $\forall v_i \in V$. Now we wish to show that $\delta_\mu(v_i, v_j) \leq r_\mu(G)$, $\delta_\nu(v_i, v_j) \geq r_\nu(G)$, $\forall v_i, v_j \in V$. By the definition of eccentricity, we obtain, $\delta_\mu(v_i, v_j) \leq e_\mu(v_i)$, $\delta_\nu(v_i, v_j) \geq e_\nu(v_i)$, $\forall v_i, v_j \in V$. This is possible only when $e_\mu(v_i) = e_\mu(v_j)$, $e_\nu(v_i) = e_\nu(v_j)$, $\forall v_i, v_j \in V$. Since G is self centered IFG, the above inequality becomes $\delta_\mu(v_i, v_j) \leq r_\mu(G)$, $\delta_\nu(v_i, v_j) \geq r_\nu(G)$.
Sufficient Condition: We now assume that $\delta_\mu(v_i, v_j) \leq r_\mu(G)$, $\delta_\nu(v_i, v_j) \geq r_\nu(G)$, $\forall v_i, v_j \in V$. Then we have to prove that G is self centered IFG. Suppose that G is not self centered IFG. Then $e_\mu(v_i) \neq r_\mu(G)$, $e_\nu(v_i) \neq r_\nu(G)$, for some $v_i \in V$. Let us assume that $e_\mu(v_i)$ and $e_\nu(v_i)$ is the least value among all other eccentricity. That is,

$$r_\mu(G) = e_\mu(v_i), r_\nu(G) = e_\nu(v_i), \tag{9}$$

where $e_\mu(v_i) < e_\mu(v_j)$, $e_\nu(v_i) < e_\nu(v_j)$, for some $v_i, v_j \in V$ and

$$\delta_\mu(v_i, v_j) = e_\mu(v_j) > e_\mu(v_i), \text{ and}$$

$$\delta_\nu(v_i, v_j) = e_\nu(v_j) > e_\nu(v_i), \text{for some } v_i, v_j \in V. \tag{10}$$

Hence from equation (9) and (10), we have, $\delta_\mu(v_i, v_j) > r_\mu(G)$, $\delta_\nu(v_i, v_j) < r_\nu(G)$, for some $v_i, v_j \in V$, which is a contradiction to the fact that $\delta_\mu(v_i, v_j) \leq r_\mu(G)$, $\delta_\nu(v_i, v_j) \geq r_\nu(G)$, $\forall v_i, v_j \in V$. Hence G is a self centered intuitionistic fuzzy graph. □

Theorem 68. *Let $G : \langle \mu, \nu, V, E \rangle$ be an IFG. If the graph $G : \langle \mu, \nu, V, E \rangle$ is a complete bipartite IF graph then the complement of $G : \langle \mu, \nu, V, E \rangle$ is a self-centered IFG.*

Proof. A bipartite IFG $G : \langle \mu, \nu, V, E \rangle$ is said to be complete if

$$\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j)),$$
$$\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j)), \forall\, v_i \in V_1 \text{ and } v_j \in V_2$$

and

$$\mu_2(v_i, v_j) = 0,\ \nu_2(v_i, v_j) = 0, \forall\, v_i, v_j \in V_1 \text{ or } v_i, v_j \in V_2 \tag{11}$$

Now,

$$\bar{\mu}_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j)) - \mu_{2ij},$$
$$\bar{\nu}_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j)) - \nu_{2ij} \tag{12}$$

By using equation(11)

$$\bar{\mu}_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j)), \tag{13}$$

$$\bar{\nu}_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j)), \tag{14}$$
$$\forall\, v_i, v_j \in V_1 \text{ or } v_i, v_j \in V_2$$

Hence from equation (11), (13) and (14), the complement of $G : \langle \mu, \nu, V, E \rangle$ has two components and each component is a complete IFG, which are self centered IFG by Theorem 58. Hence the proof. □

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