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Solution of Third Order Nonlinear Equation by Taylor Series Expansion

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Abstract: The present paper illustrates an iterative numerical method to solve nonlinear equation f(x)=0, especially those containing the partial and nonparties involvement of transcendental term. We proposed a new iteration method by Taylor expansion to order 3 and we show by numerical results that this new iteration method is faster than Newton – Raphson method, hybrid iteration method. The new hybrid iteration method and present iteration needs lesser number of functional evaluations.

Key words: Algebraic equation · Nonlinear equation · Transcendental equation · Iteration process · Taylor expansion · Newton Method · Hybrid method · New hybrid Method

INTRODUCTION

Many of the complex problems in Science and Engineering contains the function of nonlinear and Transcendental nature in the equation of the from f(x)=0 Numerical iterative methods like Newton's method [1] are often used to obtain the approximate solution of such problems because it is not always possible to obtain its exact solution by usual algebraic process.

There are many methods develops on the improvement of quadratic ally convergent Newton's Method so as to get a superior convergence order than it. Earlier, many investigations [2-5] have made to explain the root of nonlinear algebraic and transcendental equation. For the same purpose the variants of the Newton's formulas have been discussed by Babajee and Dauhoo [6], whereas other [7, 8] suggests the multi-step iterative method for it. Multi-step iterative methods have multiple step process to follow the computation rout of each step which is generally cumbersome to deal with.

The usual expectation from a mathematical method of such type is to obtain the fast result. Present work found these expectations by converting the given nonlinear problems to a well defined numerical iterative scheme through the use of Taylor's theorem; thereby obtaining a well efficient, highly convergent method that not only works faster than conventional methods but also takes lesser iteration step and lesser number of functional evaluations. Then those of recently proposed iterative methods like hybrid iteration method [9], new hybrid iteration method [10] and Newton's method.

Here, the principle of present iteration method is given in section 2. The result and discussion are present in section 3 and conclusion in the last section.

Preliminary Principle of the New Iteration Method: Consider the following nonlinear algebraic equation:

$$f(x)=0. \tag{1}$$

Let α be the root of this equation in the open interval *I*, in which the function is continuous and well defined f', f'', f'''. Following the basic assumption of Abbasbandy and maheshweri [2, 9] and also others(See [11, 12]), we also take Taylor expansion of f(x) one step ahead of Newton,

$$f(x) = f(x_n) + \frac{(x-x_n)}{1!} f'(x_n) + \frac{(x-x_n)^2}{2!} f''(x_n) + \frac{(x-x_n)^3}{3!} f^{(3)}(x_n) + \ldots \tag{2}$$

Where x_n is the n-th approximation to the root of Eq.(1)
Since, α is the root of Eq.(2), so

$$f(\alpha) = f(x_n) + \frac{(\alpha-x_n)}{1!} f'(x_n) + \frac{(\alpha-x_n)^2}{2!} f''(x_n) + \frac{(\alpha-x_n)^3}{3!} f^{(3)}(x_n) + \ldots \tag{3}$$

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Using Eq.(1), we can write

$$0 = f(x_n) + \frac{(\alpha - x_n)}{1!} f'(x_n) + \frac{(\alpha - x_n)^2}{2!} f''(x_n) + \frac{(\alpha - x_n)^3}{3!} f^{(3)}(x_n) + ... \quad (4)$$

Considering three the terms of Eq.(4), the value of the root of Eq.(1) can be obtained if let $\alpha = x_{n+1}$,

$$0 = f(x_n) + \frac{(x_{n+1} - x_n)}{1!} f'(x_n) + \frac{(x_{n+1} - x_n)^2}{2!} f''(x_n) + \frac{(x_{n+1} - x_n)^3}{3!} f^{(3)}(x_n) + ... \quad (5)$$

Which can solve by x_{n+1}.

$$x_{n+1} = \text{solve}\left(f(x_n) + (x_{n+1} - x_n) f'(x_n) + (x_{n+1} - x_n)^2 \frac{f''(x_n)}{2} + (x_{n+1} - x_n)^3 \frac{f^{(3)}(x_n)}{6} = 0\right) \quad (6)$$

RESULT AND DISCUSSION

Following example illustrate the result obtained by present method to solve nonlinear equations. At the same time, the results of Newton's method and some other recently proposed methods are presented to compare the efficiency and accuracy of the method by MATLAB software and show the following tables:

Example 1: Consider the following equation [10, 12]

$$f(x) = x^3 - e^{-x} = 0.$$

Starting by initial point $x_0 = 1$, we will obtain the solution x=0.7729 after few iterations.

The results obtained by Newton iteration, hybrid iteration [12], new hybrid iteration [10] and present iteration method are shown in table 1.

Table 1: Comparison of the results obtained by different method for solving $f(x) = x^2 - e^{-x} = 0$.

Formula	n	x_n	$\lvert f(x_n)\rvert$
Newton	5	0.77288295914921012	6.5 E – 17
Hybrid	6	0.77288295914921012	6.5 E – 17
New hybrid	6	0.772882959149210113	1.62630325872826 E -19
Present iteration	3	0.772882959149210120	0.0000000000000000

Example 2: Consider the following equation [10, 12]

$$f(x)=\sin x - 0.5x=0.$$

We start x_0=1.6. The result obtained by Newton iteration, Hybrid iteration [12], New hybrid iteration [10] and present iteration method are show in table 2.

Table 2: Comparison of the results obtained by different methods for solving f(x)=sinx-0.5x=0.

Formula	n	x_n	$\lvert f(x_n)\rvert$
Newton	5	1.895494267033980950	0.0000000000000000
Hybrid	6	0.0000000000000000	0.0000000000000000
New hybrid	5	1.895494267033999400	0.0000000001509903
Present iteration	4	1.895494267033980900	0.0000000000000000

Example 3: Consider the following equation [10, 12]

$$f(x)=x^3-2x-5=0.$$

We start x_0=2. The result obtained by Newton iteration, Hybrid iteration [12], New hybrid iteration [10] and present iteration method are shoe in table 3.

Table 3: Comparison of the results obtained by different methods for solving $f(x) = x^3-2x-5=0$.

Formula	n	x_n	$\lvert f(x_n)\rvert$
Newton	5	2.0945514815185277800	0.0000000000088818
Hybrid	9	2.094551481518527000	0.0000000000355271
New hybrid	5	2.094551481542326500	0.0000000000088818
Present iteration	2	2.094551481542326500	0.0000000000088818

Example 4: Consider the following equation [10, 12]

$$f(x) = x \log x - 1.2 = 0.$$

Starting with x_0=2, we will obtain the solution x=1.88808 after few iteration. The results obtained are in table 4.

Example 5: Consider the following equation [10, 12]

$$f(x)=x^2-5=0.$$

We start with x_0=1. The results obtained by Newton iteration, Hybrid iteration method [12], New hybrid iteration method [10] and present iteration method are shown in table 5.

Table 4: Comparison of the results obtained by different methods for solving f(x)=x logx – 1.2 =0.

Formula	n	x_n	$\lvert f(x_n)\rvert$
Newton	5	1.888086753028343 40	0.0000000000022204
Hybrid	10	1.888086753028343400	0.0000000000022204
New hybrid	6	1.888086753028343400	0.0000000000022204
Present iteration	4	1.888086753028343600	0.0000000000022204

Table 5: Comparison of the results obtained by different methods for solving $f(x)=x^2-5=0$.

Formula	n	x_n	$\lvert f(x_n)\rvert$
Newton	7	2.236068988499789 80	0.0000000000088818
Hybrid	4	-2.236067977499789800	0.0000000000088818
New hybrid	6	2.236067977499789800	0.0000000000088818
Present iteration	2	2.236067977499789800	0.0000000000088818

Table 6: Comparison of the results obtained by different methods for solving $f(x) = \tan^{-1}x = 0$.

Formula	n	x_n	$\lvert f(x_n)\rvert$
Newton	11	Failure	-----
Hybrid	6	0.0000000000000000	0.0000000000000000
New hybrid	7	0.0000000000000000	0.0000000000000000
Present iteration	4	0.0000000000000000	0.0000000000000000

Example 6: Consider the following equation [10, 12]

$$f(x)=\tan^{-1}x=0.$$

We start with x_0 =2. The results obtained by Newton iteration, Hybrid iteration method [12], new hybrid iteration [10] and present iteration method on taking the same starting root as taken in Example 4 is show in table 6.

Example 7: Consider the following equation [10, 12]

$$f(x)=x^3+4x^2-10=0.$$

We start with x_0 =1.5. The results obtained by Newton iteration, Hybrid iteration method [12], New hybrid iteration [10] and present iteration method are shown in table 7.

Example 8: Reconsider the following equation of Example 7:

$$f(x)=x^3+4x^2-10=0.$$

The results obtained by Newton iteration, Hybrid iteration method [12], New hybrid iteration [10] and present iteration method on taking the starting condition of Example 7 are shown in table 8.

Table 7: Comparison of the results obtained by different methods for $(x) = x^3+4x^2-10=0$.

Formula	n	x_n	$\lvert f(x_n)\rvert$
Newton	5	1.365230013414096850	0.0000000000000000
Hybrid	6	1.365230013893928000	0.00000000792364218682
New hybrid	4	1.36523001344889900	0.0000000055653792658
Present iteration	2	1.36523003414096900	0.0000000000000000

Table 8: Comparison of the cost effecting parameter for $f(x)=x^3+4x^2-10=0$.

Method	No. of iterations	No. of functional
Newton iteration	5	10
Hybrid iteration	6	24
New hybrid iteration	4	12
Present iteration	2	8

Table 9: Comparison of the cost effecting parameter for $f(x)=x^3-2x-5$

Method	No. of iterations	No. of functional
Newton iteration	5	10
Hybrid iteration	14	56
New hybrid iteration	5	15
Present iteration	2	8

Example 9: Reconsider the following equation of Example 3:

$$f(x)=x^3-2x-5=0.$$

The results obtained by Newton iteration, Hybrid iteration method [12], New hybrid iteration [10] and present iteration method on taking the starting condition of Example 3 are shown in table 9.

The Main Observations Are as Follows:

- Present method takes lesser number of iteration than the others compared here.
- Examples show that the present method requires lesser number of functional evaluations, as compared to other methods.

Now, One Question Can Be Asked:

Question: In your present method, you use "solve" in software, so it can use for basic function *f* and then find the root it?

Answer: We mast note that we use "solve" in our method for a polynomial dagger 3 and we know that it always have a real root.

CONCLUSION

In the paper, we proposed a new iteration method by Taylor expansion to order 3 to solve nonlinear equations f(x) =0., that this new iteration method is faster than Newton – Raphson method, hybrid iteration method, new hybrid iteration method and present iteration needs lesser number of functional evaluations.

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