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Ant Colony Optimization for Locating the Critical Failure Surface in Slope Stability Analysis

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Abstract: Locating the critical slip surface with the minimum factor of safety of a slope is a difficult NP type optimization problem. In recent years, some modern global optimization methods have been developed with success in treating various types of problems, but very few of these methods have been applied the geotechnical problems. In this paper an ant colony optimization algorithm has been used to solve this complicated problem which is known as one the most important problems in geotechnical engineering. The proposed algorithm is demonstrated to be efficient in solving complicated problems with a high level of confidence.

Key words: Heuristic · Ant Colony Optimization · Critical · Factor of Safety

INTRODUCTION

An essential step in the design of earth dams and road embankments is the design of stable slopes for such structures. In practice, an experienced engineer selects an initial geometry for a proposed embankment dam or road embankment according to site conditions, material properties and external load conditions. A stability analysis is then performed to evaluate the safety of the slopes under various loading conditions. The safety of a slope is expressed in terms of the factor of safety and the limit equilibrium approach has been the most popular method for computing this factor. It makes use of the plastic limit theorem of solid mechanics to analyze the stability of a given mass of soil under the assumption that the mass will fail along a potential slip surface. The safety factor is expressed as a ratio of restoring forces to disturbing forces. In analyzing a problem as part of a design process, a number is usually determined as an acceptable minimum value for the factor of safety, usually ranging between 1.0 and 1.5 depending on loading conditions and reliability of the soil parameters used in the calculations. The use of the limit equilibrium method for earth dams and road embankments requires the selection of trial failure surfaces in order to locate the minimum factor of safety. The process of identifying the failure surface with the lowest factor of safety is traditionally performed by manual control of analysis programs, or by a brute force approach in which very many surfaces are defined and analyzed automatically [1]. Alternatively, various classic optimization methods have been considered in the literature for locating the critical slip surface in slope stability analysis by limit equilibrium methods. Baker and Garber [2] used the variation method, which was later questioned by Luceno and Castillo [3] who concluded that their variation relation was incorrectly formulated. Celestino and Duncan [4] and Li and White [5] used the alternating variable method for locating the critical noncircular failure surface in slope stability. This method was also disapproved as it became complicated even for simple slope stability problems. Baker [6] used dynamic programming to locae the critical slip surface using Spencer's [7] method of slope stability analysis. Other methods such as the simplex method, steepest descent and Davidson-Fletcher-Powell (DFP) method have also been considered in the literature [8-10]. Cheng *et al.* [11] pointed out at least two broad demerits for the above mentioned classical methods of optimization for slope stability analysis: (1) Classical methods are applicable mainly to continuous functions and are limited by the presence of the local minimum; (2) The global minimum within the solution domain may not be given by the condition the gradient of the objective function $\nabla f' = 0$. To the above two mentioned drawbacks, one may

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also add that many classical optimization methods usually rely on a good initial estimate of the failure surface in order to find the global minimum, which is often difficult to estimate for the general case. With the advent of fast computers, modern heuristic optimization based techniques have been developed to effectively overcome the drawbacks and limitations of the classical optimization methods in searching for the critical slip surface in slope stability analysis. In heuristic optimization, the solution is found among all the possible ones and while there is no guarantee that the best solution is found, solutions close to the best is often obtained quite effectively. Monte - Carlo based techniques have been successfully adopted for slope stability analysis through limit equilibrium methods. This method is essentially a randomized hunt within the search space and finding the lowest factor of safety becomes a matter of pure chance. Greco [12] and Malkawi *et al.* [13, 14] used random walk type Monte-Carlo technique for locating the critical factor of safety in a slope. Monte Carlo based methods are simply structured optimization techniques, in which large numbers of random trial surfaces are generated to ensure that the minimum factor of safety is found. This is advantageous in that the possibility of finding a failure surface which is different from what the designer originally expected will be greater if the search space is not too tightly defined. However, the process involves the analysis of a large number of solutions, whereas the method does not guarantee the location of the minimum factor of safety. Fuzzy logic has also been used for locating the critical failure surface several simple slope stability problems [15-17]. Metaheuristic optimization algorithms have evolved rapidly in the past years. These algorithms drive some basic heuristic in order to escape from local optima. Metaheuristic implies that low-level heuristics in the global optimization algorithm are allowed to obtain solutions better than those they could have achieved alone, even if iterated. The heuristic approach is usually controlled by one of two general mechanisms: (1) Constraining or randomizing the set of local neighbor solutions to consider in the local search; (2) combining elements taken by different solutions. Many metaheuristic algorithms have been developed in recent years which loosely imitate natural phenomena. The simulated annealing method [18] which is based on the simulation of very slow cooling process of heated metals is perhaps one of the first methods used for the location of the critical failure surface in slope stability analysis. Cheng [19] applied the mentioned algorithm to slope stability analysis. The Genetic Algorithm (GA) developed by Holland [20] is one of the most popular metaheuristic methods used in slope stability analysis and is based on the concepts of genetics and evolution of living creatures. The optimum solution in GA evolves through a series of generations. Genetic algorithm-based solutions have been reported in the literature by Zolfaghari *et al.* [21], MacCombie and Wilkinson [1] and Sengupta and Upadhyay [22], among others. Particle swarm optimization (PSO), first developed by Kennedy and Eberhart [23], is another method which has attracted attention in slope stability analysis in recent years. As described by Cheng *et al.* [24] who successfully applied the method and a modified form of the algorithm, modified particle swarm optimization (MPSO), to locating the critical non-circular failure surface in slope stability analysis, PSO is based on the simulation of simplified social models, such as bird flocking, fish schooling and the swarming theory. Cheng *et al.* [25] also used the fish swarm algorithm for determining the critical slip surface in slope stability analysis. Other methods include the harmony search algorithm (Geem *et al.* [26], Lee and Geem, [27]) which is based on the musical process of finding the state of perfect harmony, Tabu search (Glover [28]) and the leap-frog algorithm by Bolton *et al.* [29].

This paper describes the application of ant colony optimization (ACO) for locating the critical failure surface in slope stability analysis. The ant colony algorithm (ACA) was originally inspired by the behavior of real ants. Dorigo [30] first developed ant ACO and successfully applied it to the travelling salesman problem (TSP). Several variations of the original ACA including ant system (AS) [30], elitist ant system (ASelite) [30], max-min ant system (MMAS) [31-33] and ranked ant system (ASrank) [34, 35] have been introduced recently. ACO has enjoyed important success in various fields of engineering. Despite its success, however, the method has found little success in geotechnical engineering applications. As a frontier in application of modern metaheuristic optimization algorithms to slope stability analysis, Cheng et al. [11] have evaluated the performance of six heuristic global optimization methods in the location of critical slip surface in slope stability analysis, including ACA. As investigated by Cheng *et al.* [11], ACO performed efficiently for simple problems, while relatively poor performance was observed in cases where soft bands existed in the problem. It was also mentioned that all six methods studied, i.e. simulated annealing, genetic algorithm, particle swarm optimization, simple harmony search, modified harmony search, tabu search and ant colony algorithm could work efficiently and effectively provided that the domain transformation technique as suggested by Cheng [19] is adopted in the optimization algorithms.

Despite the valuable information provided by Cheng *et al*. [11] regarding the performance of ACO in locating the critical failure surface, a further assessment of the various ACAs seems necessary before a general conclusion can be derived on the performance of ACO in slope stability analysis. In this paper, the authors have investigated the application of ant colony algorithms, for locating the critical non-circular failure surface in limit equilibrium slope stability analysis. Janbu's simplified method is used for calculating the factor of safety of the non-circular failure surface, while Bishop's simplified method was adopted for the simple circular failure surface. The performance of this algorithm mentioned above is evaluated through several illustrative examples.

Generation of Trial Slip Surfaces: Several methods exist for the generation of trial slip surface in slope stability analysis. The slip surface generation method developed by Zolfaghari *et al*. [21] was used in this study, since it contains simple control variables which can easily be introduced into to the ACA. Consider the non-circular failure surface for the slope shown in Fig. 1. The non-circular failure surface is formed by a series of lines, each having a slope related to the slope of the previous line, connected to form a failure surface beginning from point A and ending at point B. Point A is first chosen randomly and is specified by coordinates X1 and Y1. The slope of the first line at point A, αf_1, is determined randomly in the vicinity of the Rankine failure angle range. Since the Rankine failure angle is 45 + ϕ/2 to the horizontal plane, αf_2 will be 45-ϕ/2. Therefore a soil having ϕ ranging between 0-30° would have αf_1 in the range of 30-45°. The slope of the rest of the line components comprising the failure surface, $\Delta\alpha f_1$, is then calculated randomly in a reasonable range of 0-15°, as suggested by Zolfaghari *et al*. [21]. In order to effectively form the failure surface by these lines of varying αf_1, Zolfaghari *et al*. [21] suggested that $\Delta\alpha f_1$ be grouped into different categories of different slopes, including rapid slopes lines (αf_1 between 5° and 15°), gentle lines (αf_1 between 0° and 5°) and lines with very gentle slopes (αf_1 between 0° and 3°). Since the ACA used in this study for locating the critical non-circular failure surface is a meta heuristic optimization method which will explore the search space in a random nature, additional constraints need to be introduced to the method described above in order to prevent the ACA from tending towards unrealistic failure surfaces.

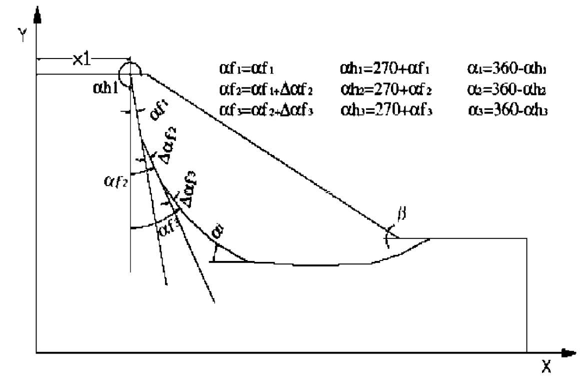

Fig. 1: Generation of admissible nonlinear failure surface

Four conditional constraints were introduced in the present study:

- $\Delta x \geq a$, where a is the horizontal distance between the crest and the toe of the slope. This constraint ensures that only general slip surfaces are considered and that very shallow slip surfaces are omitted.
- $Y_i \geq 0$, Where Yi is the y-coordinate of the endpoint of the i-th line comprising the failure surface. By specifying this constraint, failure surfaces which tend to intersect the base of model are banned.
- $X_i \leq b$, In which b is the x-coordinate of the far most right-hand line of the problem. This constraint ensures that the failure surfaces terminate in the downstream of the slope considered in the analysis.
- FS ≤ 10, Which allows the algorithm to discard failure surfaces with very high safety factors, speeding the process of searching for the critical failure surface. Any failure surface which violates the above stated constraints is heavily penalized by giving it a very high safety factor of 1000, reducing its chances of being considered in the consequent search pattern of the ACA.

Once an admissible failure surface is formed, the safety factor of the slope can be calculated using the following equation representing Janbu's simplified method [36]:

$$FS = \frac{1}{\Sigma w \tan\alpha} . f_0 . \Sigma \left[(W - ub) \left[\frac{1}{\cos\alpha + \dfrac{\tan\varphi' \sin\alpha}{FS}} \right] \tan\varphi' + c'.b \right] \cos\alpha \quad (1)$$

Review of Ant Colony Optimization Algorithms: The basic idea of ACO, inspired by the behavior of real ants, is to use artificial ants to search for *good solutions* of a combinatorial optimization problem. ACO is therefore a

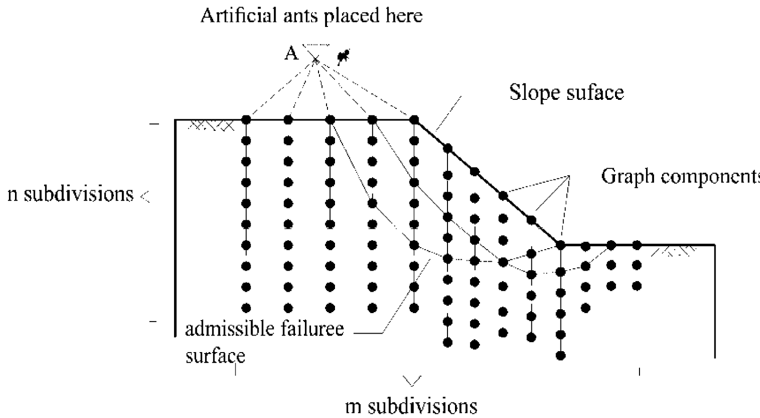

Fig. 2: Graph representation of the search space in ACA

metaheuristic in the sense that the absolute optimum solution is not found, but *good solutions* practically close enough to the optimum are found. Real ants coordinate their activities through stigmergy, which is a form of indirect communication [37]. Specifically, in the search for food, ants deposit chemicals along the path they travel which is recognized by other ants and will increase the probability of the path being followed by other ants of the colony. The chemical is called *Pheromone* [37] and it plays an important role, both, in the life of real ants and in the ACAs which are inspired by the behavior of real ants. The basic steps in ACO can be briefly stated as follows.

- Construct a graph of the problem. Any optimization problem to be solved by ACO is first discretized into a graph, the components of which include the variables in the search space. In the case of locating the critical failure surface, as shown in Fig. 2, this step involves dividing the two dimensions in the x and y direction into m and n discrete subdivisions, respectively. The resulting graph will have m columns, each column consisting of n nodes.
- Define the objective function and the restraints: When locating the critical slip surface, the objective function is the factor of safety, i.e. the function to be optimized. Also, some restraints are placed on the variables in this stage in order to ensure that admissible failure surfaces are found. The number of ants to participate in searching for the optimum, as well as the number of iterations for solving the problem are also specified in this step (Every time all the ants have finished moving on the graph, an iteration is said to have been completed).
- Move artificial ants on the graph in order to construct admissible solutions to the problem: In this step, artificial ants placed on the initial point start moving on the grid from left to right, randomly selecting a node on each consecutive column in order to build incremental solutions to the problem under consideration. In locating the critical failure surface, ants move from the crest of the slope towards the toe, following the rules described in section two above. Each time an ant reaches the downstream of the slope, a failure surface is formed. The more ants placed on the grid, the more failure surfaces are produced and the higher the chances are that the best solution is approached. Fig. 2 shows the paths that two ants have taken on the graph, i.e., two admissible failure surfaces that have been formed.

In selecting the nodes of a column to move to, the probability of an ant selecting the *j-th* node of the i-th column is described by the following relation:

$$P_{i,j} = \frac{\tau_{i,j}}{\Sigma\tau_{i,j}} \qquad (2)$$

In Eq. 2, $\tau_{i,j}$ is the sum of the pheromone placed on node (i, *j)* from previous iterations. In the first iteration, all nodes have an equal pheromone of To' and therefore in the first iteration, all nodes have an equal chance of being selected by the ants. This simple equation indicates that the probability rule is related directly to the amount of pheromone deposited on a path.

- Evaluate the solutions obtained by each ant in the first iteration: Once all the ants complete the first iteration, the objective function f is calculated for each path (slip surface). The objective function here, as mentioned previously, is the factor of safety.

Next, pheromone is deposited along the trail which each ant has chosen in forming an incremental solution. The amount of pheromone deposited on each node is reversely related to the objective function of the path being considered, i.e. $\tau = \frac{1}{f}$ .As the rule states, the lower the objective function (factor of safety) of a path (slip surface). The more pheromone will be deposited on the components of the path.

- Update the pheromone value of each node in the graph: After calculating the pheromone value of every node for the present iteration, the updated pheromone value of each node is obtained through the following relation:

$$\tau_{i,j}(t+1) = (1-\rho)\tau_{i,j}(t) + \Delta\tau_{i,j})t) \qquad (3)$$

In which $\Delta\tau_{i,j}$, is the difference between the deposited pheromone in the present iteration and the previous iteration, $\tau_{i,j}(t+1)$ is the updated pheromone value and $(1-\rho)$ is the evaporation index which is between zero and one. Pheromone evaporation is a useful form of forgetting, preventing the algorithm from rapidly converging towards local optima. The term $(1-\rho)$ thus determines how much of the pheromone accumulated from previous iterations is evaporated.

- Repeat steps III through IV in the next iterations in order to reach the optimum solution: in the next iterations, the decision making process of the artificial ants is no longer completely by chance; as stated by Eq. 2, nodes with more pheromone have a higher chance of being selected by the ants. After each iteration, pheromone values are updated and some pheromone is evaporated. The combined action of pheromone deposition and evaporation enables a constant exploration of the search space towards a global optimum in ACO.

The above discussed four algorithms were employed in the present study in searching for the critical failure surface in slope stability analysis. The implementation procedure is described in the next section, followed by some illustrative examples to evaluate the performance of the algorithms.

Implementation of ACO in Slope Stability Analysis and Numerical Examples: The basic components of ACA applied to the problem of locating the critical failure

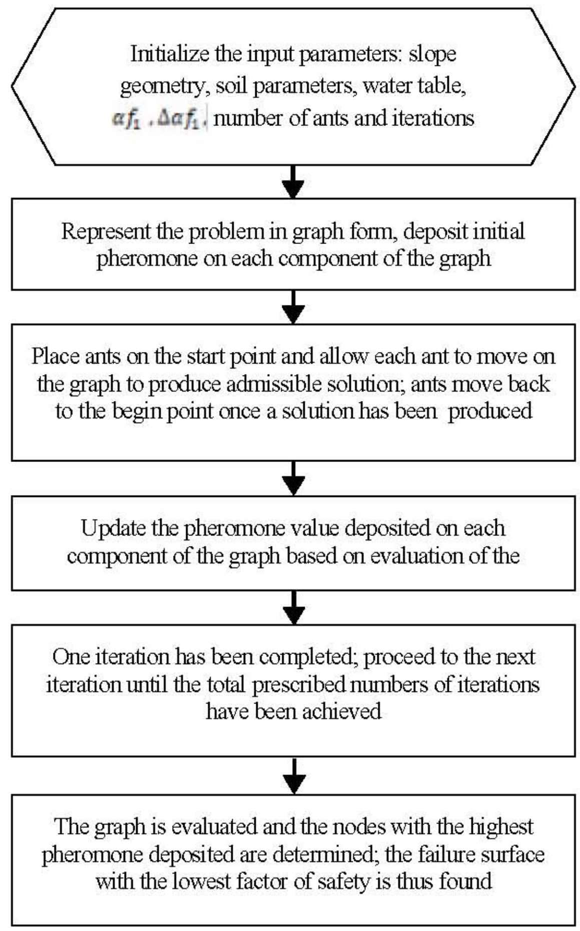

Fig. 3: Basic components of the ACA flowchart

surface is shown as a flowchart in Fig. 3. The algorithm initiates with reading the set of variables $[X_1, \alpha f_1 \Delta\alpha f]$ and $[X_1, \alpha f_1 \Delta\alpha f_1]$ for circular and non-circular failure surface, respectively and terminates when all the ants have participated in searching the solution space in the number of iterations specified. The aim of the present study was to assess the ability of the four aforementioned ACAs in locating the critical failure surface in slope stability analysis. Several previously published examples were chosen in order to compare the results of ACAs with other methods reported in the literature. The effectiveness of a global optimization method in locating the critical surface relies on its ability to escape local minima [24]. In choosing the examples discussed herein, attention was given to cases where layers of soft soil exist within the stratification, which makes it more challenging for the optimization algorithm to identify the precise location of the critical failure surface. Comparison has been made between the four ACO algorithms in terms of both, accuracy as well as efficiency of the results.

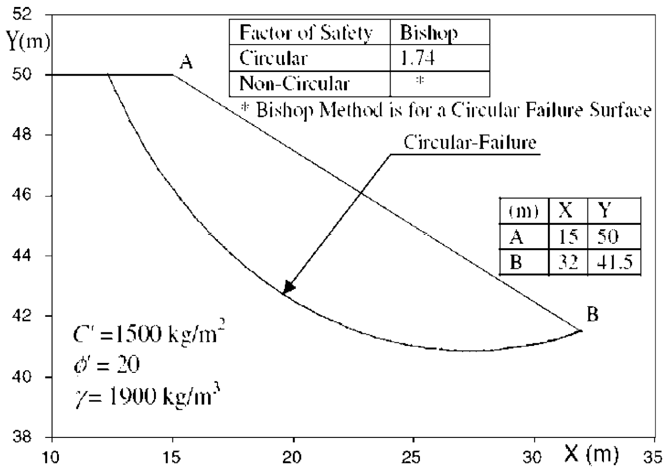

Fig. 4: General layout and specifications of example 1

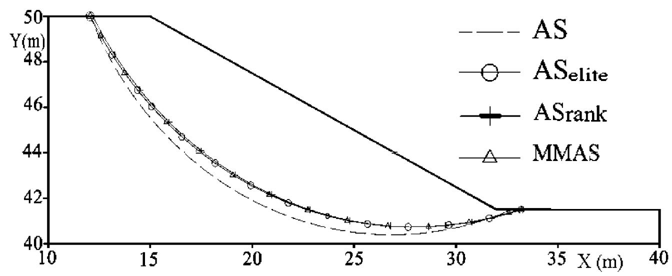

Fig. 5: Failure surfaces from AS, ASelite, ASrank and MMAS for example 1

Table 1: Results for example 1 (Bishop's simplified method)

Optimization Algorithm	Minimum factor of safety	NOFs (total)
AS	1.754	400
AS_{elite}	1.745	400
AS_{rank}	1.740	100
MMAS	1.740	200
SGA by Zolfaghari *et al.* [21]	1.740	N/A

The efficiency of an ACO algorithm was evaluated by means of the number of cycles, defined as the product of three terms: the number of ants, the number of iterations and the number of runs. For example, if 10 ants were used to produce solutions in 5 iterations and the problem was solved 3 times, a total of 150 cycles are generated. This is loosely comparable with the number of objective function evaluations, denoted as NOF by Cheng *et al.* [24].

The first example considered is taken from Zolfaghari *et al.* [21] and represents a simple case of a homogeneous soil slope with physical and mechanical parameters defined in Fig. 4. This example was chosen in order to evaluate the efficiency and accuracy of different ACA's algorithms in locating the critical failure surface in simple problems where strong local minima do not exist. As depicted in Fig. 4 Zolfaghari *et al.* [21] calculated the circular factor of safety of the slope by means of Bishop's simplified method and by employing Simple Genetic Algorithm (SGA), as 1.74. The Problem was solved using the four previously discussed ACA's and the resulting critical failure surface obtained by each algorithm is drawn in Fig. 5. The factor of safety obtained by each algorithm is also tabulated in Table 1. This very simple example was chosen to illustrate that even with the simplest problems, the AS is weak at finding the critical factor of safety, compared to the other algorithms. Moreover, a graph of the number of cycles against the factor of safety for this problem is drawn in Fig. 6, suggesting that AS is not efficient in searching for the optimum solution, compared to the other ACO algorithms. The authors attribute this behavior to the fact that in the pheromone depositing process, AS does not support the optimum solutions. Therefore, it seems critical to point out that in order to properly assess ACO in solving geotechnical optimization problems such as locating the critical failure surface in slope stability, it is necessary to evaluate at least several of the available ACAs before deducing a general judgment. In the consequent examples, more complicated problems are considered to further evaluate the performance of the ACAs.

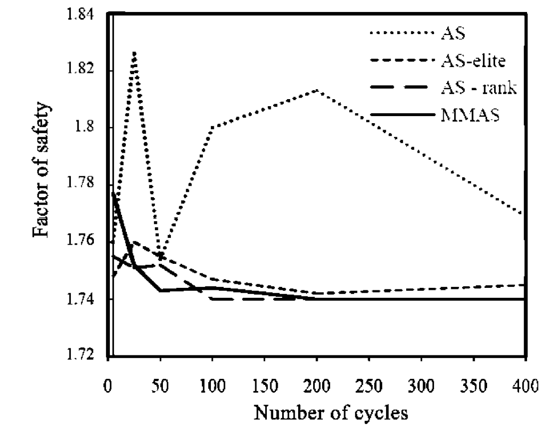

Fig. 6: Comparison of the efficiency of the ACAs

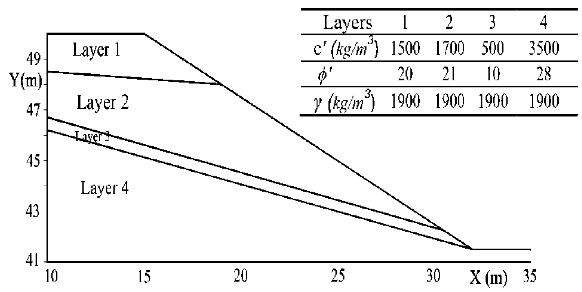

Fig. 7: Geometrical layout and material properties for example 2

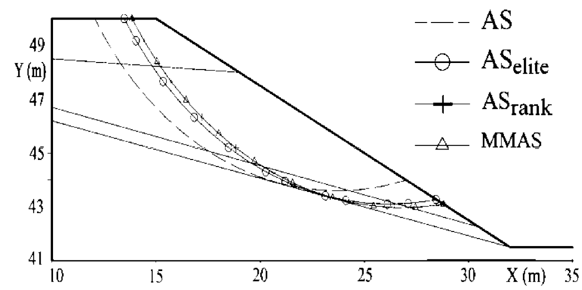

Fig. 8: Critical failure surface predicted by different ACAs for example 2

Example 2 is also taken from Zolfaghari *et al.* [21] and presents a case of relatively complex soil layering with a weak soil layer within the stratification. The problem geometry and soil conditions are presented in Fig. 7. Layer 3 in this example is a weak layer and it is expected that the critical failure surface moves within this layer. For this example Zolfaghari *et al.* [21] calculated the critical factor of safety for a circular failure surface using Bishop's simplified method as 1.475. The problem was solved using the four ACAs discussed in this paper and the critical failure surface of each algorithm is depicted in Fig. 8. Table 2 lists the minimum factor of safety obtained by each algorithm. As evident from Fig. 8, the critical failure

Table 2: Results for example 1 (Bishop's simplified method)

Optimization Algorithm	Minimum factor of safety	NOFs (total)
AS	1.615	400
AS_{elite}	1.491	400
AS_{rank}	1.425	100
MMAS	1.425	200
SGA by Zolfaghari *et al.* [21]	1.475	N/A

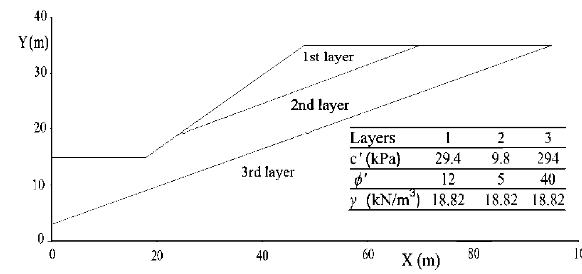

Fig. 9: Geomertical layout and material properties for example 3

surface of MMAS, ASrank and ASelite are similar while AS predicts a critical failure surface that deviates from the other algorithms. Comparison of the minimum factor of safety obtained by the four algorithms also reveals that MMAS is most effective while AS once again shows a poor performance. Moreover, the results suggest that MMAS and ASrank algorithms are even more effective than the SGA.

The first two examples considered involved circular failure surfaces using Bishop's simplified method. In the next two illustrative examples, a noncircular failure surface is considered using Janbu's Simplified method of analysis for locating the critical failure surface. Both examples will involve zones of weakness which presents difficult optimization situations in slope stability analysis. The problem geometry, soil properties and the results obtained by different optimization methods for the two mentioned problems are presented in this section.

The third example considers a slope including layered soil with geometrical configuration and properties shown in Fig. 9. The second layer is a soft layer overlying a hard layer as shown in the figure. The problem was analyzed by Arai and Tagyo [38] using conjugate gradient method and several researchers have used this example in evaluating the efficiency of their proposed method in locating the non-circular critical failure surface, some of which include Sridevi and Deep [39] who used RST-2 method, Greco [12] and Malkawi *et al.* [13] who used Monte Carlo methods and Cheng *et al.* [24] who employed particle swarm optimization (PSO) and modified particle swarm optimization (MPSO) methods.

Table 3: Results for example 1 (Bishop's simplified method)

Optimization Algorithm	Minimum factor of safety	NOFs (total)
AS	0.4130	15000
AS_{elite}	0.4110	7600
AS_{rank}	0.4000	38000
MMAS	0.3970	38000
Conjugate gradient	0.4050	N/A
RST-2	0.4010	N/A
Monte Carlo	0.3880	N/A
Monte Carlo	0.4010	N/A
PSO	0.3944	29877
MPSO	0.3963	14832

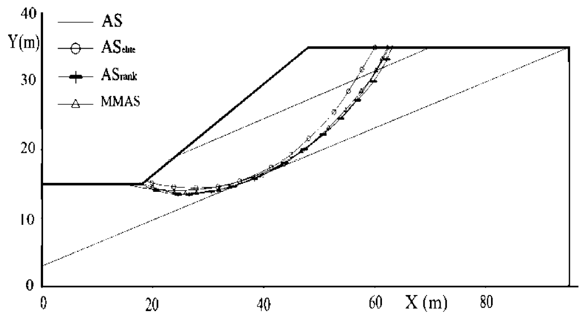

Fig. 10: Critical failure surface predicted by different ACAs for example 3

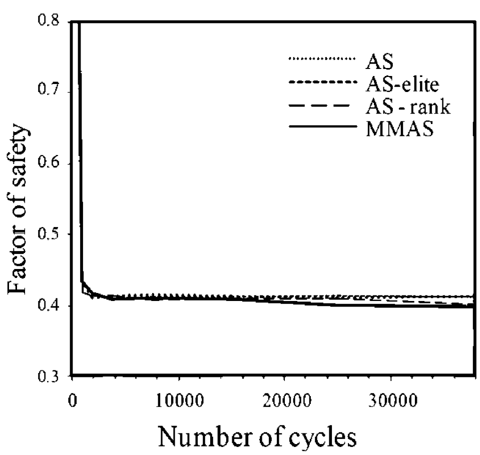

Fig. 11: Critical failure surface predicted by different ACAs for example 3

In the present study, the problem was solved by means of the four ACAs under consideration, i.e. AS, ASelite, ASrank and MMAS and the results are compared with those available from the literature. Fig. 10 shows the critical failure surface obtained by each of the algorithms and Table 3 lists the minimum factor of safety of the ACAs as well as those from previous studies. Careful evaluation of the failure surface developed indicates that a greater portion of the failure surface developed by AS and AS_{elite} pass into the first, stronger layer while the failure surface of AS_{rank} and MMAS cut less into this layer. It is expected that a lower factor of safety would result if the failure surface was to move completely into the second layer. Cheng *et al.* [24] mentioned that the failure surface developed by PSO and MPSO were able to move fully into the weak layer at the right end. Table 3 shows the NOFs required by different methods in order to reach the optimum solution. Comparison of PSO, MPSO and the ACAs indicates that among the ACAs, AS_{rank} and MMAS are particularly efficient and are generally comparable with PSO, but are relatively less efficient compared to MPSO. Fig. 11 shows the convergence rate of the four ACAs employed. The graph shows that the four algorithms are very similar in efficiency for this example, all converging at a rapid rate to their optimum solution. The results presented for this example were achieved after many initial runs, in which the number of ants, iterations and iterations were varied in each run and the most efficient combination was selected for each algorithm. The sensitivity of the results to the mentioned parameters was found to be very high. The improved convergence rate observed in Fig. 11, compared to Fig. 6, is perhaps due to the mentioned sensitivity analysis performed, suggesting that the success rate of ACAs is dependent partially on selecting appropriate combinations of the three aforementioned parameters involved in the number of cycles. Once again, the success of AS_{rank} and particularly MMAS is attributed to the fact that these algorithms effectively support the best solution in their pheromone deposition process. In general, the solutions of MMAS and AS_{rank} are believed to be satisfactory in the examples considered.

CONCLUSIONS

This paper dealt with the evaluation of the effectiveness and accuracy of ant colony optimization (ACO) algorithms in locating the critical failure surface and the corresponding minimum factor of safety. Four ant colony algorithms were studied, including ants system (AS), elite ants system (ASelite), ranked ants system (AS_{rank}) and maximum-minimum ants system (MMAS). Several illustrative examples were considered in order to evaluate the performance of the four mentioned algorithms. The following conclusions were drawn from the results of this study:

- The failure surface generation algorithm affects the results obtained by the metaheuristic optimization methods. The Failure surface generation algorithm suggested by Zolfaghari *et al.* [21] was found to be effective for simple circular failure surfaces. However, in the case of noncircular failure surfaces, particularly in problems where weak soil layers exist, the mentioned algorithm shows poor performance. For such problems, the method proposed by Cheng et al. [11] is suggested.
- The illustrative examples showed that all ant colony algorithms discussed showed an acceptable performance for simple problems involving homogeneous soil slopes. The MMAS algorithm and the AS_{rank} performed better than the two other algorithms, with the AS showing the poorest performance.
- For complex problems involving layers of weak soil, AS performed poorly, while MMAS was able to generate relatively acceptable results. The success of the ACAs, however, was generally limited to the effectiveness of the failure surface generation method used, i.e., the method proposed by Zolfaghari *et al.* [21]. It is believed that MMAS would be able to perform much better if it was used in conjunction with the failure surface generation algorithm suggested by Cheng *et al.* [11].
- The performance of the ACAs was found to be affected by the number of ants, iterations and runs used to produce the results. It is suggested that ACAs be employed only after careful sensitivity analysis, taking into account the effect of the three mentioned parameters on the results.

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