Complex Solution for Konopelchenko-Dubrovsky System

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Abstract: In this paper, the direct algebraic method is used for complex solutions of the Konopelchenko-Dubrovsky system. The methods employed here can also be used to solve a large class of nonlinear equations.

Key words: Konopelchenko-Dubrovsky system ⋅ Direct Algebraic method

INTRODUCTION

It is well known that traveling wave solutions of nonlinear partial differential equations (NPDEs) play an important role in the study of nonlinear phenomena. Up to now, there exist many powerful methods to construct exact solutions (especially soliton solutions) of nonlinear partial differential equations. For example, the Bäcklund transformation [1], Darboux transformation [2], Hirota’s bilinear transformation [3], the truncated Painlevé expansion [4], variable separation approach [5], the extended tanh function method [6, 7], homogeneous balance method [8], tanh method [9, 10], sine-cosine method [11, 12] and its extension [13, 14], F-expansion method [15], inverse scattering method [16] and so on.

In this paper, we establish new complex solutions to the Konopelchenko-Dubrovsky system.

\[
\begin{align*}
    u_t - u_{xxx} - 6\beta uu_x + \frac{3}{2}\alpha^2 u^2 u_x - 3w_x + 3\alpha uu_x w &= 0 \\
    w_x &= u_y
\end{align*}
\]

Where \( \alpha, \beta \) are arbitrary constants.

Our method mainly consists of four steps:

Step 1: We seek complex solutions of Eq. (1) as the following form:

\[
    u = u(z), \quad z = ik(x - ct),
\]

Where \( k \) and \( c \) are real constants. Under the transformation (2), Eq. (1) becomes an ordinary differential equation:

\[
    N(u, iku', -iku', -k^2u', ...),
\]

Where \( u' = \frac{du}{dz} \).

Step 2: We assume that the solution of Eq. (3) is of the form.

\[
    u(z) = \sum_{i=0}^{n} a_i F_i(z),
\]

Where \( a_i(i = 1, 2, ..., n) \) are real constants to be determined later. \( F(z) \) expresses the solution of the auxiliary ordinary differential equation.

\[
    F'(z) = b + F(z),
\]

Eq. (5) admits the following solutions:

This Paper Is Organized as Follows: In Section 2, we briefly reviewed the direct algebraic method. In Section 3, the method is used to seek explicit complex solutions to the Konopelchenko-Dubrovsky system. In Section 4, the conclusion is given.

Description the Method: For a given partial differential equation.

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\[
F(z) = \begin{cases} 
\sqrt{-b} \tanh(\sqrt{-b} z), & b < 0 \\
\sqrt{-b} \coth(\sqrt{-b} z), & b > 0 
\end{cases}
\]

(6)

\[
F(z) = \frac{1}{z}, \quad b = 0 
\]

Integer \(n\) in (4) can be determined by considering homogeneous balance [3] between the nonlinear terms and the highest derivatives of \(u(z)\) in Eq. (3).

**Step 3:** Substituting (4) into (3) with (5), then the left hand side of Eq. (3) is converted into a polynomial in \(F(z)\), equating each coefficient of the polynomial to zero yields a set of algebraic equations for \(a, k, c\).

**Step 4:** Solving the algebraic equations obtained in Step 3 and substituting the results into (4), then we obtain the exact travelling wave solutions for Eq. (1).

**Application to the Konopelchenko-Dubrovsky Equation:**

Konopelchenko-Dubrovsky equation reads:

\[
\begin{align*}
\partial_t u + u_{xxx} - 6\beta uu_x + \frac{3}{2} \partial_x^2 u^2 u_x - 3w_x + 3\alpha uu_x w &= 0 \\
w_x &= u_y 
\end{align*}
\]

(6)

Where \(\alpha, \beta\) are arbitrary constants. Permits us converting Eq. (6) into an ODE for \(u = u(z)\), and \(z = ik(x + \lambda y - ct)\) we have

\[
\begin{align*}
-ikcu' - ik^3u'' - 6ik\beta uu' + \frac{3}{2} ik\alpha^2 u^2 u' - 3ik\lambda w' + 3ik\lambda cu'w &= 0 \\
(i\lambda w)' &= i\lambda cu' 
\end{align*}
\]

(7)

(8)

With substituting the Eq. (8) into Eq. (7) we obtain

\[
-ikcu' - ik^3u'' - 6ik\beta uu' + \frac{3}{2} ik\alpha^2 u^2 u' - 3ik\lambda^2 u' + 3ik\lambda cu'u &= 0 
\]

So by integrating of equation above we obtain

\[
u_1 = \pm i \frac{2k}{\alpha} \sqrt{-b} \tanh(\sqrt{-b}k(x + \lambda y - \frac{1}{\alpha^2} \left(6\lambda\alpha\beta - 6\beta^2 + 2k^2\alpha^2\beta^2 - \frac{9}{2}\lambda^2\alpha^2\right))) + \frac{2\beta - \alpha\lambda}{\alpha^2}
\]

Where \(b < 0\) and \(k\) is an arbitrary real constant.

\[
u_2 = \pm \frac{2k}{\alpha} \sqrt{-b} \coth(\sqrt{-b}k(x + \lambda y - \frac{1}{\alpha^2} \left(6\lambda\alpha\beta - 6\beta^2 + 2k^2\alpha^2\beta^2 - \frac{9}{2}\lambda^2\alpha^2\right))) + \frac{2\beta - \alpha\lambda}{\alpha^2}
\]

(9)

\[
C \text{ is integrating constant determined later.}
\]

Considering the homogeneous balance between \(u'\) and \(u''\) in Eq. (7), we required that, \(3m = m + 2 \Rightarrow m = 1\) so we can write (4) as

\[
u = a_1F + a_0
\]

(10)

By substituting (10) into Eq. (9) and collecting all terms with the same power of \(F\) together, the left-hand side of Eq. (9) is converted into another polynomial in \(F\) equating each coefficient of this polynomial to zero, yields a set of simultaneous algebraic equations for \(a_1, a_0, \lambda\) as follows.

\[
\begin{align*}
2k^3a_1 + \frac{1}{2} ik\alpha^2 a_0^2 &= 0 \\
(-3ik\beta + \frac{3}{2} ik\lambda\alpha) a_1^2 + \frac{3}{2} ik\alpha^2 a_0 a_1 &= 0 \\
(-6k\beta - 3ik\lambda\alpha) a_1 + 2\eta k\beta - 3\eta k\alpha (3k\beta + \frac{3}{2} ik\lambda\alpha) + \frac{3}{2} ik\alpha^2 a_0^2 &= 0 \\
(-6k\beta - 3ik\lambda\alpha) a_0 + a_0 (3k\beta + \frac{3}{2} ik\lambda\alpha) + \frac{1}{2} ik\alpha^2 a_0^2 &= 0
\end{align*}
\]

Solving algebraic equations above we have

\[
\begin{align*}
a_1 &= \pm \frac{2k}{\alpha} \\
a_0 &= \frac{2\beta - \alpha\lambda}{\alpha^2} \\
c &= \frac{1}{\alpha^2} \left(6\lambda\alpha\beta - 6\beta^2 + 2k^2\alpha^2\beta^2 - \frac{9}{2}\lambda^2\alpha^2\right) \\
C &= \frac{1}{\alpha^2} \left(2k^2\alpha^2\beta^2 - 2\beta^2 + \lambda\alpha\beta - \frac{1}{2}\lambda^2\alpha^2\right)
\end{align*}
\]

Substituting (10) into (9) with (6), respectively, we obtain new exact complex solutions for Eq. (7) as follows:
Where $b < 0$ and $k$ is an arbitrary real constant.

$$u_3 = \pm \frac{2k}{\alpha} \left[ \sqrt{b} \cot^2 \sqrt{bk} \frac{x + \lambda y}{\alpha^2} + \frac{1}{\alpha^2} \left( 6 \lambda a^2 \beta - 6 \beta^2 + 2k^2 \alpha^2 \beta^2 - \frac{9}{2} \lambda^2 a^2 \right) \right] + \frac{2b - \alpha \lambda}{\alpha^2}$$

Where $b < 0$ and $k$ is an arbitrary real constant.

$$u_4 = \pm \frac{2k}{\alpha} \left[ -\sqrt{b} \cot \sqrt{bk} \frac{x + \lambda y}{\alpha^2} + \frac{1}{\alpha^2} \left( 6 \lambda a^2 \beta - 6 \beta^2 + 2k^2 \alpha^2 \beta^2 - \frac{9}{2} \lambda^2 a^2 \right) \right] + \frac{2b - \alpha \lambda}{\alpha^2}$$

Where $b < 0$ and $k$ is an arbitrary real constant.

$$u_5 = \pm \frac{2k}{\alpha} \left[ \frac{1}{ik(x + \lambda y - \frac{1}{\alpha^2} \left( 6 \lambda a^2 \beta - 6 \beta^2 + 2k^2 \alpha^2 \beta^2 - \frac{9}{2} \lambda^2 a^2 \right) \right]} + \frac{2b - \alpha \lambda}{\alpha^2}$$

Where $b = 0$

CONCLUSION

In this paper we have seen the complex solutions of the regularized Konopelchenko-Dubrovsky system are successfully found out by using the direct algebraic method. The obtained results in this work clearly demonstrate this method is simple and powerful as well as can be carried out in computer by the aid of symbolic computation.

REFERENCES