Assessment of Aerial Images Usage in DTM Generation for Earthworks Computations

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Abstract: The use of non-contact techniques and methods, such as aerial images and laser scanners, in earthworks calculation has been greatly proved as a superior tool to offer more productivity and efficiency over classical geodetic surveying methods. This is due to their potential ability to even potentially require less labor, cost and time besides gaining much safety on projects in risk or inaccessible areas as the case of unstable areas and landslides. The main aim of the current research is to investigate some factors that affect the quality and accuracy of the generated photogrammetric DTM that can be efficiently used in the field of earthworks quantity calculations. In this context, an assessment of a DTM acquired by a stereo aerial image used later in the earthworks application of volume calculation is presented. This assessment encompasses many criteria concerning both image acquisition and processing, including for example the required scanning resolution, the used mathematical method for volume computation and the influence of the corresponding extrinsic orientation parameters, in order to minimize the required number of ground control points.

Key words: Earthwork volumes • Stereo-pair • Collinearity • GPS/IMU

INTRODUCTION

Accurate earthworks calculations is an important issue in many engineering applications such as erosion studies, mining activities, and estimation of ore removed from all project fields for construction. The accuracy of such calculations are in direct proportional with the best form of land surface presentations which logically depend on the density of 3D points, point distributions and interpolation methods [1]. These requirements need a convenient well-distributed and much more points in order to provide better representation of land surface. However, these requirements yield a considerable increase in both time and cost and therefore are impractical to be acquired using classical ground surveying approaches in risky and/or inaccessible areas [2]. On the other hand, aerial photogrammetry, and other modern techniques offer an alternative tools for Earthworks computations in particularly large and fast track projects.

This impact of photogrammetry is achieved by the complete definition of the position and orientation of the image bundles relative to the ground through the Global Positioning Systems and Inertial Measurement Systems onboard the imaging platform [3], in addition to the privilege of the digital photogrammetric workstation (DPW) in bundle adjustment solution of the acquired images. The first challenge, which is the direct observation of the extrinsic orientation using integrated GPS/IMU systems, has been become more and more important in aerial photogrammetry [4, 5]. This may help in reducing the measured number of required ground control points [6] which contribute in the main obstacle of using classical geodetic surveying methods in this field.

Increasing the accuracy of these parameters to be even considered as fixed values in the photogrammetric bundle solution minimize the number of needed ground control points. In this case, control points are used for checking of system calibration and quality control purposes rather than for datum definition. This requirement is the basic motivation behind the current study, which entails the required achieved accuracy of these parameters to certain insignificant error in the final computed earthworks volume, even without the need of ground control points.

Therefore, the main objective of the present paper is to investigate the influence of some considered factors, mainly the extrinsic orientation parameters of an aerial stereo-pair on the ground positional accuracy of certain
points within a tested area, in order to improve the final accuracy of the computed 3D coordinates and consequently earthworks volumes. In this context, an explicit model, derived from the original collinearity condition equations, has been manipulated. The parameter, which gives the significant discrepancies in the computed volume after adding certain errors in its original value, will be the main factor to contribute in the final accuracy. In other words, the parameter that gives significant error in the computed volume, compared with the other parameters all having the same measured accuracy, will be the most critical one, which requires more care in its precise measurements. In addition, it can be verified by checking the accuracy of the computed volumes by applying the error propagation law to the derived explicit model of the 3D ground coordinates. The largest discrepancies in the corresponding partial derivatives will be related to the most influenced parameter.

In the above terminology, and in order to achieve the required objective, the used mathematical model for expressing the ground coordinates of any object point, as a function only in the orientation parameters, will be presented first. Then, the methodology of investigation and criteria of assessment will be described. In addition, a brief description of the field experiment, along with the acquisition of the involved stereo-pair, will be given. Moreover, the effect of all investigated factors on the final computed earthworks volume, including the suitable mathematical interpolation method, best scanning resolution and the influence of the extrinsic orientation parameters, provided with the clear analysis and discussion, will be outlined. Finally, the main conclusions will be extracted and commented upon.

The Used Mathematical Model: The present study relies on the well-known collinearity condition mathematical model, which theoretically forces the three points, namely the image point, perspective center and the corresponding object point, to lie on the same projective straight line. Its associated equations relate both image coordinates and object ground coordinates, as a function of the intrinsic and extrinsic orientation parameters, besides all the lens distortion parameters [7]. Hence, one of the main aims of the present paper is to study the effect of the extrinsic orientation parameters on the final computed earthworks volume, it is logical to adopt the inverse of the original collinearity condition equations. This inverse form gives the ground coordinates as an explicit function of both measured image coordinates and the camera orientation [8], as follows:

\[
X = (Z - Z_L) \frac{N_x}{N_z} + X_L
\]

\[
Y = (Z - Z_L) \frac{N_y}{N_z} + Y_L
\]

Where:

\[
N_x = m_{11} \left( \bar{x} + \bar{x}r^2 \right) + m_{21} \left( \bar{y} + \bar{y}r^2 \right) + m_{31} (-f)
\]

\[
N_y = m_{12} \left( \bar{x} + \bar{x}r^2 \right) + m_{22} \left( \bar{y} + \bar{y}r^2 \right) + m_{32} (-f)
\]

\[
N_z = m_{13} \left( \bar{x} + \bar{x}r^2 \right) + m_{23} \left( \bar{y} + \bar{y}r^2 \right) + m_{33} (-f)
\]

Hence, these two equations can be reduced to one equation only, by dividing them by each other, in order to eliminate the Z-coordinate, to finally yield the required equation, as a function in X and Y-coordinates only, as:

\[
\frac{X - X_L}{Y - Y_L} = \frac{N_x}{N_y}
\]

Of course, a similar equation can be simply written for the same object point under consideration, using its (x, y) image coordinates, as measured in the second photo of the stereo-pair, besides its corresponding extrinsic orientation parameters. In this case, one will have two equations in two unknowns, namely the X and Y-ground coordinates of the same object point, which can be directly solved for them, to yield the final expression as:

\[
X_i = \left( \frac{N_{01}}{N_{02}} \right) X_{L1} - \left( \frac{N_{02}}{N_{02}} \right) X_{L2} + \left( \frac{N_{01}}{N_{01}} \right) \left( \frac{N_{02}}{N_{02}} \right) \left( Y_{L1} - Y_{L2} \right) = \frac{A_{X_i}}{B_{X_i}}
\]

(4)

\[
Y_i = \left( \frac{N_{01}}{N_{01}} \right) Y_{L1} - \left( \frac{N_{02}}{N_{01}} \right) Y_{L2} + (X_{L1} - X_{L2}) = \frac{A_{Y_i}}{B_{Y_i}}
\]

(5)

Where (i) refers to the object point of interest, and the subscript indicates the photo number in the stereo-pair.
Finally, the corresponding value of Z-coordinate can be obtained by substitution with the approximate values of X or Y-coordinate in either equation (3) or (4), that is:

\[ Z_i = (X_i - X_{L1}) \left( \frac{N_{x1}}{N_{x1}} \right)_i + Z_{L1} \]

or

\[ Z_i = (Y_i - Y_{L1}) \left( \frac{N_{x1}}{N_{x1}} \right)_i + Z_{L1} \]

(6)

**Methodology and Criteria of Assessment:** Many factors govern the suitability of using aerial images to generate accurate DTM for earthworks calculations. Among those factors, the suitable interpolation method for calculating volumes from field 3D data points will be investigated. In this context, the three common mathematical methods which are trapezoidal method, Simpson rule and Simpson 3/8 rule, whose formulae are well-known in many literatures such as [9], are tested. This assessment will include two criteria including the minimum difference between arithmetic computed volumes from both terrestrial and photogrammetric approaches using the three adopted methods at the same cross sections spacing; besides the sensitivity of each method in the value of computed volumes with increasing the spacing. Of course, the method gives the minimum error in the volume and its non-significant change in the computed volume from different spacing will be the best suitable one to be adopted.

Also for the analogue captured aerial images, the scanning resolution is one of the investigated factors for assessment in this study case. In this terminology, both images are scanned with different photogrammetric scanning resolutions. The choice of the most preferable resolution does not rely only on the accuracy of computed volume compared with the corresponding geodetic one, but many other inherent items are taken into account as the availability of scanners, total cost, processing time and the final file size.

Finally and in order to study the effect of the extrinsic rotation angles orientation parameters (\(\alpha\), \(\phi\), \(\kappa\)) on the positional accuracy of object points and hence the final computed volumes, a small set of errors will be introduced to each one of the three rotation angles. The biggest percentage of computed volume error will be contributed to the rotation angle that should be measured more precisely. Also to verify this finding, the accuracy of the computed 3D ground coordinates of all field data points is assessed by applying the variance law to each one of the three previous equations (4), (5) and (6), respectively. In other words, each equation will be partially differentiated related to each element of the three rotation angles separately and the values of all partial derivatives using the original and the incorrect values of rotation angles will be determined. Accordingly, the statistical parameters (max., mean, min., RMS) of the resulted discrepancies between both values, particularly the (RMS) value, will be the base on which the assessment will be performed. For instance, the bigger the (RMS) value after introducing an error to a certain rotation angle, the more and the significant the effect of this element on the final accuracy of the computed 3D coordinates.

The final expressions for the associated partial derivatives relative to the three ground coordinates \(X\), \(Y\) and \(Z\), respectively for the first photo, for example, will be:

\[
\begin{align*}
\frac{\partial X_i}{\partial \alpha_1} &= C_{X_{01}} C_{X_i}, \\
\frac{\partial X_i}{\partial \phi_1} &= C_{X_{01}} C_{X_i}, \\
\frac{\partial X_i}{\partial \kappa_1} &= C_{X_{01}} C_{X_i}, \\
\frac{\partial Y_i}{\partial \alpha_1} &= C_{Y_{01}} C_{Y_i}, \\
\frac{\partial Y_i}{\partial \phi_1} &= C_{Y_{01}} C_{Y_i}, \\
\frac{\partial Y_i}{\partial \kappa_1} &= C_{Y_{01}} C_{Y_i},
\end{align*}
\]

\[
\frac{\partial Z_i}{\partial \alpha_1} = (X_i - X_{L1}) \left( \frac{N_{x1}}{N_{x1}} \right)_i + \frac{N_{y1}}{N_{y1}} \left( \frac{\partial Y_i}{\partial \alpha_1} \right)_i \\
\frac{\partial Z_i}{\partial \phi_1} = (X_i - X_{L1}) \left( \frac{N_{y1}}{N_{y1}} \right)_i + \frac{N_{x1}}{N_{x1}} \left( \frac{\partial X_i}{\partial \phi_1} \right)_i \\
\frac{\partial Z_i}{\partial \kappa_1} = (X_i - X_{L1}) \left( \frac{N_{y1}}{N_{y1}} \right)_i + \frac{N_{x1}}{N_{x1}} \left( \frac{\partial Y_i}{\partial \kappa_1} \right)_i
\]

\[
\frac{\partial Z_i}{\partial \alpha_1} = (X_i - X_{L1}) \left( \frac{N_{y1}}{N_{y1}} \right)_i + \frac{N_{x1}}{N_{x1}} \left( \frac{\partial X_i}{\partial \alpha_1} \right)_i \\
\frac{\partial Z_i}{\partial \phi_1} = (X_i - X_{L1}) \left( \frac{N_{x1}}{N_{x1}} \right)_i + \frac{N_{y1}}{N_{y1}} \left( \frac{\partial Y_i}{\partial \phi_1} \right)_i \\
\frac{\partial Z_i}{\partial \kappa_1} = (X_i - X_{L1}) \left( \frac{N_{x1}}{N_{x1}} \right)_i + \frac{N_{y1}}{N_{y1}} \left( \frac{\partial Y_i}{\partial \kappa_1} \right)_i
\]

Where:
Field Experiment: The test field area was selected as a sandy land in 6th October city, Egypt, in order to carry out such study. It is composed of a nearly flat part with a variation in height of approximately 3.0 m besides a huge natural hill of height differences reaching up to 15.0 m. The whole test area extends as an ellipse of size 420.0 m and 760.0 m. Fig. 1 depicts the selected area as illustrated from Google Earth and close range view. It is an accessible and easily reachable area since it is bounded by three roads.

Four well-distributed ground control points have been established within the area. The coordinates of these control points have been measured by GPS observations, using a Trimble R3 precise GPS geodetic receiver. In addition, the test area has been surveyed by both techniques, which are the geodetic classical surveying and aerial photogrammetry. The former uses a Topcon 712 GTS total station with nearly 260 field data points, whereas the latter captures a stereo-pair of aerial images from a flying height of approximately 1750 m. One of the captured aerial images is shown in Fig. 2.
Table 1: The adjusted orientation parameters of the associated aerial stereo-pair

<table>
<thead>
<tr>
<th>Orientation Type</th>
<th>The Corresponding Adjusted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intrinsic Orientation Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$x_i$</td>
<td>$1.01135x10^{-5}$ m</td>
</tr>
<tr>
<td>$y_i$</td>
<td>$2.3696x10^{-5}$ m</td>
</tr>
<tr>
<td>$f$</td>
<td>$0.15284$ m</td>
</tr>
<tr>
<td>$k_i$</td>
<td>$28.4592$ m</td>
</tr>
<tr>
<td><strong>Extrinsic Orientation Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Rotation Angles</td>
<td>$\omega = [-0.470502, 0.403135]^{\text{deg}}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$[0.151399, 0.366778]^{\text{deg}}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$[89.04964, 87.375493]^{\text{deg}}$</td>
</tr>
<tr>
<td>Exposure Coord.</td>
<td>$X = [302892.577, 302007.453]$ m</td>
</tr>
<tr>
<td>Coord.</td>
<td>$Z = [1755.249, 1750.720]$ m</td>
</tr>
</tbody>
</table>

The photogrammetric solution was done using Delta Photogrammetric Station Software. The adjusted output values of both intrinsic and extrinsic orientation parameters of the involved stereo-pair are fully listed in Table 1.

**Analysis of the Obtained Results:** This section will be devoted to the presentation and discussion of the obtained results, concerning the effect of all considered factors on the final computed earthworks volume from aerial photogrammetry, according to the methodology and criteria of assessment stated before.

**Influence of Used Interpolation Method:** Since there is no exact value of the final earthworks volume of the tested area from both geodetic and photogrammetric approaches, the choice of the best interpolation method among the three used ones previously-mentioned will not depend only on the minimum percentage error in computed photogrammetric volume compared with the corresponding geodetic volume using all involved methods. It also includes the sensitivity of each method to the computed volumes with the increase of cross sections spacing. It should be noted that, the output computed volume from each approach using each interpolation method with spacing of 5.0 m and scanning resolution of 8 µm was taken as a datum to which all the volume error percentage will be determined. Accordingly, Table 2 and Fig. 3 illustrate the corresponding results, from which one can easily conclude the suitability of trapezoidal method to be used in this field of interest according to its minimum error in the computed photogrammetric volume besides its non-significant effect of increasing spacing.

Table 2: Comparison between computed earthwork volume by both geodetic and photogrammetric approaches using all considered interpolation methods

<table>
<thead>
<tr>
<th>Interpolation Method</th>
<th>Geodetic</th>
<th>Photogramm.</th>
<th>Error in Volume %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapezoidal</td>
<td>1601125</td>
<td>16653968</td>
<td>3.86</td>
</tr>
<tr>
<td>Simpson</td>
<td>16051344</td>
<td>17101581</td>
<td>6.14</td>
</tr>
<tr>
<td>3/8 Simpson</td>
<td>16043366</td>
<td>16829294</td>
<td>4.67</td>
</tr>
</tbody>
</table>

Table 3: Relationship between scanning resolution and error in computed volumes

<table>
<thead>
<tr>
<th>Scanning Resolution</th>
<th>Photogrammetric Volume (m3)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 µm</td>
<td>16653968</td>
<td>3.86</td>
</tr>
<tr>
<td>12 µm</td>
<td>16686020</td>
<td>3.08</td>
</tr>
<tr>
<td>16 µm</td>
<td>17506150</td>
<td>8.54</td>
</tr>
<tr>
<td>25 µm</td>
<td>18173898</td>
<td>11.50</td>
</tr>
</tbody>
</table>

**Influence of Scanning Resolution:** Again, it is convenient to use suitable scanning resolution satisfying many criteria. In this context, the captured aerial stereo-pair was scanned with different photogrammetric scanning resolutions of 8 µm, 12 µm, 16 µm and 25 µm, whose corresponding results computed by the trapezoidal method are listed in Table 3. Of course, smaller scanning resolution gives better results concerning the error in computed volume, but, as an overall evaluation, scanning resolution up to 12 µm can be implemented.

**Influence of Extrinsic Orientation Parameters:** In the bundle adjustment of aerial images, the accuracy of the rotation angles ($\omega$, $\phi$, $\kappa$) should be better than the accuracy of exposure station coordinates ($X_i$, $Y_i$, $Z_i$).
Table 4: Effect of rotation angles error on both positional accuracy and computed volume

<table>
<thead>
<tr>
<th>Study case</th>
<th>D</th>
<th>RMSX</th>
<th>RMSY</th>
<th>RMSZ</th>
<th>RMSP</th>
</tr>
</thead>
<tbody>
<tr>
<td>ω</td>
<td>+40°</td>
<td>0.001</td>
<td>0.316</td>
<td>0.048</td>
<td>0.320</td>
</tr>
<tr>
<td></td>
<td>+80°</td>
<td>0.001</td>
<td>0.633</td>
<td>0.096</td>
<td>0.640</td>
</tr>
<tr>
<td></td>
<td>+120°</td>
<td>0.002</td>
<td>0.949</td>
<td>0.144</td>
<td>0.960</td>
</tr>
<tr>
<td></td>
<td>+300°</td>
<td>0.004</td>
<td>2.373</td>
<td>0.359</td>
<td>2.400</td>
</tr>
<tr>
<td>φ</td>
<td>+40°</td>
<td>0.316</td>
<td>0.005</td>
<td>0.085</td>
<td>0.326</td>
</tr>
<tr>
<td></td>
<td>+80°</td>
<td>0.632</td>
<td>0.011</td>
<td>0.164</td>
<td>0.655</td>
</tr>
<tr>
<td></td>
<td>+120°</td>
<td>0.948</td>
<td>0.016</td>
<td>0.247</td>
<td>0.980</td>
</tr>
<tr>
<td></td>
<td>+300°</td>
<td>2.374</td>
<td>0.020</td>
<td>0.615</td>
<td>2.453</td>
</tr>
<tr>
<td>ϵ</td>
<td>+40°</td>
<td>0.099</td>
<td>0.178</td>
<td>0.255</td>
<td>0.325</td>
</tr>
<tr>
<td></td>
<td>+80°</td>
<td>0.199</td>
<td>0.347</td>
<td>0.588</td>
<td>0.711</td>
</tr>
<tr>
<td></td>
<td>+120°</td>
<td>0.298</td>
<td>0.727</td>
<td>0.680</td>
<td>1.039</td>
</tr>
<tr>
<td></td>
<td>+300°</td>
<td>1.237</td>
<td>1.852</td>
<td>2.454</td>
<td>3.314</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Study case</th>
<th>Volume (m³)</th>
<th>Computed</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ω</td>
<td>+40°</td>
<td>16265387</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>+80°</td>
<td>16263598</td>
<td>-0.58</td>
</tr>
<tr>
<td></td>
<td>+120°</td>
<td>16271682</td>
<td>-0.63</td>
</tr>
<tr>
<td></td>
<td>+300°</td>
<td>16410743</td>
<td>-1.49</td>
</tr>
<tr>
<td>φ</td>
<td>+40°</td>
<td>16268236</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>+80°</td>
<td>16404275</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>+120°</td>
<td>16456019</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>+300°</td>
<td>16769713</td>
<td>3.71</td>
</tr>
<tr>
<td>ϵ</td>
<td>+40°</td>
<td>16328277</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>+80°</td>
<td>16489975</td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td>+120°</td>
<td>17142298</td>
<td>5.71</td>
</tr>
<tr>
<td></td>
<td>+300°</td>
<td>18247634</td>
<td>12.85</td>
</tr>
</tbody>
</table>

effect of the rotation angles on the positional accuracy is always greater than the corresponding effect of the exposure stations coordinates [10]. Hence, the investigation here will be carried out only on the accuracy of extrinsic rotation angles. From Table 4, one can easily specify the effect of each rotation angle on the accuracy of computed 3D ground coordinates of discrete check points, and hence on the final computed earthworks volume of a certain hill. Logically, increasing the value of added error to any rotation angle will increase the error in the spatial position of check points and the computed volume. In the following subsections, the effect of each rotation angle will be investigated separately.

**Effect of Rotation Angle (ω):** Error in the measured angle (ω) has no effect on the computed X-coordinates and very small errors on the computed Z-coordinates of check points even in case of large added errors. On the other hand, a significant systematic error is occurred on the computed Y-coordinates. Accordingly, this leads to large RMS of the spatial positions of all check points, but a minor effect on the final computed volume since the whole data are shifted systematically in the direction of Y-axis. This is quite clear since only 1.49% error in the final computed volume within an error of 5.0 minutes in the measured angle (ω).

**Effect of Rotation Angle (φ):** Error in the measured angle (φ) has nearly no effect on the computed Y-coordinates and small errors on the computed Z-coordinates of check points even in case of large added errors. On the other hand, a significant systematic error is occurred on the computed X-coordinates. Accordingly, this leads to large RMS of the spatial positions of all check points, but a minor effect on the final computed volume since the whole data are shifted systematically in the direction of X-axis. This is quite clear since only 3.71% error in the final computed volume within an error of 5.0 minutes in the measured angle (φ).

**Effect of Rotation Angle (ϵ):** In this case, the effect of rotation angle (ϵ) has different trend compared with the other previous two rotation angles. In this context, any added error to the angle has considerable effect on the computed coordinates in all directions, within an error in the Z-coordinates more than the corresponding effects in the other two directions. Hence, this rotation angle (ϵ) gives the largest errors in both RMS of the spatial positions of all check points as well as the final computed volumes with an error of 12.85%, compared with the corresponding values attained from the other two rotation angles (ω and φ).

In addition and to verify the previous finding of the great effect of the rotation angle (ϵ) on the final computed earthwork volume, the study will be extended to examine the accuracy of the computed 3D coordinates, by applying error propagation law, of the selected check points used to calculate the required volumes. This will be carried also by checking the discrepancies in the values of the partial derivatives of the explicit forms of (X, Y, Z) coordinate, expressed by equations (4), (5) and (6) respectively, with respect to each rotation angle after adding certain value of error in the tested rotation angle (ϵ). The corresponding results concerning the mean and RMS of the partial derivatives discrepancies computed for all selected check points are listed in Table 5, for both left and right image.
Table 5: The Statistical Parameters of the Partial Derivatives of the 3D Ground Coordinates, when Introducing an Error of Δκ= 120°

<table>
<thead>
<tr>
<th>Discrepancies in Partial Derivatives</th>
<th>Statistical Parameters</th>
<th>Right Image</th>
<th>Left Image</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>RMS</td>
<td>Mean</td>
</tr>
<tr>
<td>X</td>
<td>0.545</td>
<td>0.653</td>
<td>0.752</td>
</tr>
<tr>
<td>∂X/∂ω</td>
<td>1.281</td>
<td>1.586</td>
<td>0.838</td>
</tr>
<tr>
<td>∂X/∂κ</td>
<td>0.315</td>
<td>0.421</td>
<td>0.393</td>
</tr>
<tr>
<td>Y</td>
<td>2.087</td>
<td>2.320</td>
<td>1.419</td>
</tr>
<tr>
<td>∂Y/∂ω</td>
<td>0.677</td>
<td>0.836</td>
<td>0.731</td>
</tr>
<tr>
<td>∂Y/∂κ</td>
<td>0.512</td>
<td>0.529</td>
<td>0.496</td>
</tr>
<tr>
<td>Z</td>
<td>4.420</td>
<td>6.771</td>
<td>3.686</td>
</tr>
<tr>
<td>∂Z/∂ω</td>
<td>6.516</td>
<td>7.103</td>
<td>3.545</td>
</tr>
<tr>
<td>∂Z/∂κ</td>
<td>0.517</td>
<td>0.719</td>
<td>0.548</td>
</tr>
</tbody>
</table>

Revealing the output results listed in table, adding a certain error to the rotation angle (κ) changes significantly the values of the partial derivatives of the Z-coordinate with respect to the three rotation angles. This is quite obvious from the large mean and RMS values of these discrepancies compared with the other corresponding ones of X-coordinates and Y-coordinates. Accordingly, any error in the rotation angle (κ) affects greatly also the accuracy of the computed Z-coordinates and hence the accuracy of the final resulted earthwork volume. It should be noted that, the same trend was tested also for the other two rotation angles, but no significant systematic discrepancies have been occurred.

**CONCLUSIONS**

According to the adopted criteria of assessment and the corresponding obtained results, the main conclusions of this investigation will be briefly enumerated below:

- Maximum applied photogrammetric scanning resolution is 12 µm to yield approximately an error of 5% in the computed earthwork volume.
- The effect of the rotation angles measured by the IMU systems on the accuracy of computed earthwork volume is always greater than the corresponding effect of the exposure stations coordinates measured by GPS, concerning the extrinsic orientation parameters.
- Any error in the measured rotation angles (ω and θ) produces a systematic shift in the planimetric directions to finally yield a non-significant effect in the final computed earthwork volume.
- The most critical factor is the rotation angle (κ) whose required accuracy should be nearly 1/3 the corresponding accuracy of the other rotation angles (ω and θ).
- Errors up to 5.0 minutes in both rotation angles (ω and θ) whereas errors up to 2.0 minutes in the rotation angle (κ) are the maximum accepted error to give accuracy of computed earthwork volume reaches up to 98%.

**REFERENCES**


