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Estimating Stochastic Malmquist Productivity Index

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Abstract: Nowadays the extent of productivity of Decision Making Units (DMUs) in future has a significant importance. One of the different types of calculating profitability of units is Malmquist productivity index. All approaches presented so far calculate this index in present or past time using deterministic data. In this paper, a new approach based on DEA is presented for estimating Malmquist index in future. We calculate this index with the contribution of stochastic data and this approach provides the possibility of recognition of the probable progress or regress of units. This issue is a warning for units with probable regress to alter their performance.

Key words: Chance constrained approach . data envelopment analysis . malmquist productivity index . normal distribution

INTRODUCTION

DEA is a multiple input-output efficient technique that measures the relative efficiency of DMUs by using a linear programming based model. Since there are no assumptions about weights of the underlying production function, this technique is non-parametric. DEA was originally proposed by Charnes *et al*. [1] and this model is commonly referred to as a CCR model. The DEA frontier DMUs are those with maximum output levels for given input levels or with minimum input levels for given output levels. DEA provides efficiency scores for individual units as their technical efficiency measure, with a score of one assigned to the efficient units.

For the first time Caves *et al*. [2] introduced the Malmquist index [3] in productivity analysis and showed that it corresponds to the ratio of two distance functions. These functions could not be directly estimated because they were assumed to be parametric and specific for each firm and period. Therefore, Caves *et al*. [2], based on usual perfect competition assumptions, proposed an index number approach which allows the estimation of the same Malmquist productivity index using normal discrete data on output and input quantities and prices. Simultaneously, Nishimizu and Page [4] showed that productivity growth could also be estimated in a frontier approach framework. Based on the estimation of a parametric production frontier and on the Solow [5] theory of growth, Nishimizu and Page [4] showed that by using time derivatives, productivity growth could be estimated and decomposed into two elements. On the one hand, technical change corresponding to shifts at the frontier level and, also, efficiency change corresponding to individual productivity displacements with respect to the frontier. However, the use of time derivatives at one point to estimate discrete changes will generally be misleading since economic data do not come in the form of continuous records. The original ideas contained in the Caves and Nishimizu papers were combined by Färe *et al*. [6] to give life to a new type of decomposable Malmquist productivity index. For this purpose, they came back to Caves *et al*. [2] and showed that the Malmquist index can be directly obtained as the ratio of two distance functions previously estimated using a DEA frontier approach. Also, as in Nishimizu *et al*. [4], they showed that this index could be decomposed into two components: a technical change and an efficiency change. By the technical change, it is possible to measure the development or decline of all DMUs. Efficiency change is used to measure the change in technical efficiency. It is also a measure of how close the DMU is to the frontier, when crossing the two consecutive times.

All previous subjects in calculation of the productivity gain, productivity loss, progress and regress of units have been under consideration in present time in accordance with present and past data. Whereas, in the current study we consider the importance of determining the units which may have probable progress or regress in future. In addition, we sift through this subject and present a method for

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estimating Malmquist index. For this purpose, the distribution of each input and output of DMUs can be estimated with the use of sampling from inputs and outputs of DMUs in past successive times. By using these distributions, we proposed stochastic DEA based models for calculating stochastic Malmquist index.

The remainder of the paper is organized as follows: first DEA Malmquist productivity index is introduced. Section 3 provides an approach for estimating Malmquist index in future. Sections 4 and 5 provide an example and conclusion.

PRELIMINARIES

Färe *et al*. [6] have constructed Malmquist productivity index by using DEA concepts as the geometric mean of the two Malmquist productivity indexes which are defined by Caves *et al*. [2] by a distance function. Färe *et al*. [6] decompose their Malmquist productivity index into two components, measuring the change in efficiency and measuring the change in the frontier technology. The frontier technology determined by the efficient frontier is estimated using DEA for a set of DMUs. However, the frontier technology for a specific DMU under evaluation is only represented by a segment of the frontier.

Suppose we have a Production Possibility Set (PPS) in time period t as well as period t+1. Here we consider constant return to scale principle for constructing PPS, variable return to scale assumption is the same. Assume that there exist n homogenous DMUs such that $x_j^k=(x_{1j}^k,\dots,x_{mj}^k)$ and $y_j^k=(y_{1j}^k,\dots,y_{sj}^k)$ represent the input and output vectors of DMU_j $j=1,\dots,n$ at time k. Calculation of Malmquist productivity index requires two single period and two mixed period measures. The two single period measures can be obtained by using the input-oriented CCR model,

$$\begin{aligned} D_o^t(x_o^t, y_o^t) = \quad & \min \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta x_{io}^t, & i = 1,\dots,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{ro}^t, & r = 1,\dots,s, \\ & \lambda_j \ge 0, & j = 1,\dots,n. \end{aligned} \tag{1}$$

The value $D_o^t(x_o^t, y_o^t)$ determines the efficiency score of DMU_o in time period t that is the amount by which observed inputs of DMU_o can be reduced proportionally, while still producing the given output level. Substituting t+1 instead of t in the above model, $D_o^{t+1}(x_o^{t+1}, y_o^{t+1})$ is obtained, the efficiency score for DMU_o in time period t+1. With the purpose of evaluating DMU_o in time t+1 by frontier of DMUs in period t which is defined as $D_o^t(x_o^{t+1}, y_o^{t+1})$, the following linear programming problem is computed:

$$\begin{aligned} D_o^t(x_o^{t+1}, y_o^{t+1}) = \quad & \min \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta x_{io}^{t+1} & i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{ro}^{t+1}, & r = 1,\dots,s, \\ & \lambda_j \ge 0 & j = 1,\dots,n. \end{aligned} \tag{2}$$

Similarly, for evaluating DMU_o in time period t by frontier of DMUs in period t+1, which is needed for computing of the input-oriented Malmquist productivity index, the following linear problem is computed and its optimal value is $D_o^{t+1}(x_o^t, y_o^t)$;

$$\begin{aligned} D_o^{t+1}(x_o^t, y_o^t) = \quad & \min \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \theta x_{io}^t, & i = 1,\dots,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \ge y_{ro}^t, & r = 1,\dots,s, \\ & \lambda_j \ge 0 & j = 1,\dots,n \end{aligned} \tag{3}$$

Färe *et al*. [6] have provided an input-oriented Malmquist productivity index, which measures the productivity change of a specific DMU_o in time t+1 and t as follows:

$$M_o = \left[\frac{D_o^t(x_o^{t+1}, y_o^{t+1})}{D_o^t(x_o^t, y_o^t)} \frac{D_o^{t+1}(x_o^{t+1}, y_o^{t+1})}{D_o^{t+1}(x_o^t, y_o^t)} \right]^{\frac{1}{2}}$$

Färe *et al*. [6] defined that $M_o>1$ indicates productivity gain, $M_o<1$ indicates productivity loss and $M_o=1$ means no change in productivity from time t to t+1. Färe *et al*. [6] decompose the Malmquist productivity index (M_o) into two components as follows:

$$M_o = \frac{D_o^{t+1}(x_o^{t+1}, y_o^{t+1})}{D_o^t(x_o^t, y_o^t)} \left| \frac{D_o^t(x_o^{t+1}, y_o^{t+1})}{D_o^{t+1}(x_o^{t+1}, y_o^{t+1})} \frac{D_o^t(x_o^t, y_o^t)}{D_o^{t+1}(x_o^t, y_o^t)} \right|^{\frac{1}{2}}$$

$$= TEC_o \times FS_o$$

TEC measures the change in technical efficiency and FS measures the technology frontier shift between time period t and t+1. Färe *et al*. [6] mentioned that a value of FS greater than one indicates technical progress, a value of FS less than one indicates technical regress and a value of FS equal to one indicates no shift in technology frontier.

STOCHASTIC MALMQUIST PRODUCTIVITY APPROACH

In this section, the aforementioned models in section 2 is developed in stochastic data envelopment analysis to calculate stochastic Malmquist productivity index.

Models with stochastic data: Recently stochastic inputs and outputs in DEA field have been studied by a number of researchers including Thore [7], Land *et al*. [8, 9], Cooper *et al*. [10, 11], Huang and Li [12], Khodabakhshi and Asgharian [13], khodabakhshi [14], Behzadi *et al*. [15] and Hosseinzadeh Lotfi [16]. Here we consider This type of data for estimating Malmquist productivity index. Assume that input and output vector of each DMU_j j=1,…,n is deterministic in time t and following Cooper *et al*. [11], let

$$\tilde{x}_j^{t+1} = \left(\tilde{x}_{1j}^{t+1}, \ldots, \tilde{x}_{mj}^{t+1}\right) \quad and \quad \tilde{y}_j^{t+1} = \left(\tilde{y}_{1j}^{t+1}, \ldots, \tilde{y}_{sj}^{t+1}\right)$$

be random input and output vectors of DMU_j in time t+1. These components have been deemed to be normally distributed such that,

$$\tilde{x}_{ij}^{t+1} \sim N\left(x_{ij}^{t+1}, \sigma_{ij}^2\right) \quad and \quad \tilde{y}_{rj}^{t+1} \sim N\left(y_{rj}^{t+1}, \sigma_{rj}^2\right)$$

On the basis of what has been discussed, model (1) is a deterministic model and other aforementioned models in section (2) are stochastic models. The chance constrained form of these models are as follows:

$$\begin{aligned} \tilde{D}_o^t(\tilde{x}_o^{t+1}, \tilde{y}_o^{t+1}) = \quad & \min \theta \\ s.t. \quad & p\left\{\sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta \tilde{x}_{io}^{t+1}\right\} \ge 1-\alpha, \quad i = 1, \ldots, m, \\ & p\left\{\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge \tilde{y}_{ro}^{t+1}\right\} \ge 1-\alpha, \quad r = 1, \ldots, s, \\ & \lambda_j \ge 0, \quad j = 1, \ldots, n. \end{aligned} \tag{4}$$

$$\begin{aligned} \tilde{D}_o^{t+1}(x_o^t, y_o^t) = \quad & \min \theta \\ s.t. \quad & p\left\{\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}^{t+1} \le \theta x_{io}^t\right\} \ge 1-\alpha, \quad i = 1, \ldots, m, \\ & p\left\{\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj}^{t+1} \ge y_{ro}^t\right\} \ge 1-\alpha, \quad r = 1, \ldots, s, \\ & \lambda_j \ge 0, \quad j = 1, \ldots, n \end{aligned} \tag{5}$$

$$\begin{aligned} \tilde{D}_o^{t+1}(\tilde{x}_o^{t+1}, \tilde{y}_o^{t+1}) = \quad & \min \theta \\ s.t. \quad & p\left\{\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}^{t+1} \le \theta \tilde{x}_{io}^{t+1}\right\} \ge 1-\alpha, \quad i = 1, \ldots, m, \\ & p\left\{\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj}^{t+1} \ge \tilde{y}_{ro}^{t+1}\right\} \ge 1-\alpha, \quad r = 1, \ldots, s \\ & \lambda_j \ge 0, \quad j = 1, \ldots, n. \end{aligned} \tag{6}$$

In the above models, p means “probability” and α is a level of error between 0 and 1, which is a predetermined number.

Deterministic equivalent: In order to transform model (4) into deterministic form, positive slack variables can be used to convert the inequality constraints of this model into equality form as follows:

$$\begin{aligned}
\tilde{D}_o^t(\tilde{x}_o^{t+1}, \tilde{y}_o^{t+1}) = \quad & \min \theta \\
s.t. \quad & p\left\{\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}^t - s_i \le \theta \tilde{x}_{io}^{t+1}\right\} = 1 - \alpha \qquad i = 1, \ldots, m, \\
& p\left\{\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj}^t - s_r^+ \ge \tilde{y}_{ro}^{t+1}\right\} = 1 - \alpha, \qquad r = 1, \ldots, s \\
& \lambda_j \ge 0, \qquad j = 1, \ldots, n.
\end{aligned} \tag{7}$$

By using properties of normal distribution and model (7) the deterministic form of model (4) can be obtained as follows:

$$\begin{aligned}
\tilde{D}_c^t(\tilde{x}_o^{t+1}, \tilde{y}_o^{t+1}) = \quad & \min \theta \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^t - \theta \sigma_{io} \Phi^{-1}(\alpha) \le \theta x_{io}^{t+1}, \qquad i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^t + \sigma'_{ro} \Phi^{-1}(\alpha) \ge y_{ro}^{t+1}, \qquad r = 1, \ldots, s, \\
& \lambda_j \ge 0, \qquad j = 1, \ldots, n.
\end{aligned} \tag{8}$$

Here, Φ is the cumulative distribution function of the standard normal distribution and $\Phi^{-1}(\alpha)$ is its inverse in level of α. Similarly the deterministic form of models (5) and (6) can be obtained as follows:

$$\begin{aligned}
\tilde{D}_c^{t+1}(x_o^t, y_o^t) = \quad & \min \theta \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} - \theta \sigma_i^I(\theta, \lambda) \Phi^{-1}(\alpha) \le \theta x_{io}^t, \qquad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} + \sigma_r^o(\lambda) \Phi^{-1}(\alpha) \ge y_{ro}^t \qquad r = 1, \ldots, s, \\
& \lambda_j \ge 0, \qquad j = 1, \ldots, n.
\end{aligned} \tag{9}$$

where

$$\left(\sigma_i^I(\theta, \lambda)\right)^2 = \sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_j \lambda_k cov\left(\tilde{x}_{ij}^{t+1}, \tilde{x}_{ik}^{t+1}\right), \quad \left(\sigma_r^o(\lambda)\right)^2 = \sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_j \lambda_k cov\left(\tilde{y}_{rj}^{t+1}, \tilde{y}_{rk}^{t+1}\right)$$

$$\begin{aligned}
\tilde{D}_c^{t+1}(\tilde{x}_o^{t+1}, \tilde{y}_o^{t+1}) = \quad & \min \theta \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} - \theta \bar{\sigma}_i^I(\theta, \lambda) \Phi^{-1}(\alpha) \le \theta x_{io}^{t+1}, \qquad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} + \bar{\sigma}_r^o(\lambda) \Phi^{-1}(\alpha) \ge y_{ro}^{t+1}, \qquad r = 1, \ldots, s, \\
& \lambda_j \ge 0, \qquad j = 1, \ldots, n.
\end{aligned} \tag{10}$$

where

Table 1: Inputs and outputs at present time (time t)

DMU	Input1	Input2	Input3	Output1	Output2	Output3	Output4	Input5
DMU1	9.107	18.789	7.276	149.849	49.596	4.721	4.784	30.094
DMU2	10.589	44.304	1.109	50.773	73.125	1.853	3.054	5.829
DMU3	6.714	19.729	19.203	259.913	108.038	6.065	10.057	2.714
DMU4	11.915	17.435	59.471	137.507	44.967	4.898	4.217	63.612
DMU5	7.020	10.379	12.233	95.899	31.626	2.776	9.038	64.552
DMU6	18.994	16.666	568.631	112.583	71.980	13.189	41.896	291.359
DMU7	11.164	25.464	552.846	192.970	78.053	7.790	15.890	7.485
DMU8	15.052	123.064	14.777	724.375	219.690	35.298	23.977	361.596
DMU9	8.873	36.160	361.882	548.151	86.254	17.636	86.229	565.194
DMU10	19.876	46.412	12.812	1229.139	194.583	25.901	86.600	600.600
DMU11	18.919	36.883	24.435	11557.450	155.196	1662.667	8.416	119.881
DMU12	20.445	100.869	115.196	1132.223	248.160	46.880	31.852	96.208
DMU13	12.415	20.188	78.017	438.389	104.410	10.678	30.218	331.987
DMU14	8.053	33.208	115.333	260.798	87.387	8.452	6.105	36.926
DMU15	18.478	45.357	57.520	11190.920	166.441	65.025	132.774	919.015
DMU16	10.349	11.162	43.322	709.847	159.478	36.811	12.148	79.663
DMU17	9.513	31.494	173.372	308.108	107.031	11.699	13.628	342.237
DMU18	13.707	40.323	10.875	259.209	81.788	5.201	8.009	107.986
DMU19	11.689	26.443	31.222	381.331	72.933	5.153	50.323	577.235
DMU20	7.816	17.740	13.056	399.247	40.950	11.058	6.319	82.127

$$\left(\tilde{\sigma}_i^I(\theta,\lambda)\right)^2 = \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_j\lambda_k cov\left(\tilde{x}_{ij}^{t+1},\tilde{x}_{ik}^{t+1}\right) + \theta^2 v_{io}^2 - 2\theta\sum_{k=1}^{n}\lambda_j cov\left(\tilde{x}_{ij}^{t+1},\tilde{x}_{io}^{t+1}\right).$$

$$\left(\sigma_r^O(\lambda)\right)^2 = \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_j\lambda_k cov\left(\tilde{y}_{rj}^{t+1},\tilde{y}_{rk}^{t+1}\right) + \sigma_{ro}^2 - 2\sum_{k=1}^{n}\lambda_j cov\left(\tilde{y}_{rj}^{t+1},\tilde{y}_{ro}^{t+1}\right).$$

Models (9) and (10) are nonlinear problems which can be converted into quadratic programming models by considering nonnegative variables $u_r=\sigma_r^o(\lambda)$ and $v_i=\sigma_i^I(\theta,\lambda)$.

Stochastic Malmquist index (SM) in α level of error can be used to forecast Malmquist index in the future is as follows:

$$SM_o = \frac{\hat{D}_o^{t+1}(x_o^{t+1},y_o^{t+1})}{D_o^t(x_o^t,y_o^t)}\left[\frac{\hat{D}_o^t(x_o^{t+1},y_o^{t+1})}{\hat{D}_o^{t+1}(x_o^{t+1},y_o^{t+1})}\frac{D_o^t(x_o^t,y_o^t)}{\hat{D}_o^{t+1}(x_o^t,y_o^t)}\right]^{\frac{1}{2}}$$

$$= \widetilde{TEC}_o \times \widetilde{FS}_o \tag{11}$$

In expression (11), TEC measures the probabilistic change in technical efficiency and FS measures the probabilistic frontier shift between time period t. Finally, FS>1 shows the probabilistic progress and FS<1 shows the probabilistic regress with confidence of $100\times(1-\alpha)$ percent.

EMPIRICAL EXAMPLE

In this section, an empirical example about the application of the present approach into banking industry is given. Here the data sources consist of 20 branches of an Iranian bank monthly reports for the period between May 2008 and February 2009. We consider three types of inputs: personal rate (weighted combination of personal qualifications, quantity, education and others), payable benefits (of all deposits) and delayed requisitions (delay in returning ceded loans and other facilities). There are five types of banking outputs: facilities (sum of business and individual loans), amount of deposits (of current, short duration and long duration accounts), received benefits (of all ceded loans and facilities), received commission (on banking operations, issuance guaranty, transferring money and others) and other resources of deposits. The bank central management wants to know which branches have productivity gain or loss in future. Relevant data of February 2009 (time t) are deterministic and gathered in Table 1. We also consider inputs and outputs as random variables and by pertaining to these ten successive months estimate their distributions at time t+1. By using goodness of fit tests normal distributions have been fit on random variables. Predicted data are gathered in Table 2 and 3. Data in Table 1-3 are multiplied in a multiplier of 10 in order to make the tables small.

We assume a confidence level of at least 95 percent, $\alpha = 0.05$, for estimating Malmquist index. Stochastic Malmquist index, which is obtained by solving models (1), (8), (9) and (10) using GAMS software and applying expression(11), is presented in Table 4.

From Table 4 it is observed that the stochastic Malmquist indexes of DMU_2, DMU_4, DMU_5, DMU_6 and DMU_{20} are less than one. It means that by definition we can predict that these DMUs will have productivity loss with a probability of at least 90 percent, while other DMUs will have productivity gain. DMU_{15} with SM_{15}=3.083 has the most productivity gain among others. Additionally this DMU has the most technical progress with FS_{15}=3.083, also it has been efficient in both periods t and t+1. Although DMU_2 has been efficient in both periods t and t+1, it has the most productivity loss among others with SM_2 = 0.506. Furthermore, DMU_3, DMU_7, DMU_{14} and DMU_{18} are inefficient DMUs in both periods, but their changes are such that they have been estimated as productive.

Table 2: Estimated distribution of inputs

DMU_j	x_{1j}~$N(\mu,\sigma^2)$	x_{2j}~$N(\mu,\sigma^2)$	x_{3j}~$N(\mu,\sigma^2)$
1	N(9.131,0.05)	N(18.79,8.81)	N(7.228,0.58)
2	N(10.59,0.53)	N(44.32,24.1)	N(1.121,0.02)
3	N(6.712,0.86)	N(19.73,27.7)	N(19.21,0.47)
4	N(11.91,0.31)	N(17.43,12.2)	N(59.47,0.85)
5	N(7.012,0.02)	N(10.38,2.12)	N(12.23,59.9)
6	N(18.99,0.88)	N(16.67,10.8)	N(568.6,28.1)
7	N(11.16,0.01)	N(25.46,18.6)	N(552.8,43.2)
8	N(15.05,0.48)	N(123.1,42.6)	N(14.78,0.06)
9	N(8.787,0.38)	N(36.16,38.4)	N(361.8,23.2)
10	N(19.88,0.25)	N(46.41,53.1)	N(12.81,0.38)
11	N(18.92,0.17)	N(36.88,54.5)	N(24.43,0.01)
12	N(20.45,0.42)	N(100.8,31.8)	N(115.2,19.4)
13	N(12.41,0.12)	N(20.19,10.6)	N(78.02,24.1)
14	N(8.051,0.79)	N(33.21,24.3)	N(115.3,15.6)
15	N(18.48,0.92)	N(45.36,92.6)	N(57.52,12.8)
16	N(10.35,0.27)	N(11.16,3.32)	N(43.32,36.1)
17	N(9.511,0.01)	N(31.49,38.5)	N(173.3,3.13)
18	N(13.71,0.18)	N(40.32,51.4)	N(10.88,0.12)
19	N(11.69,0.26)	N(26.44,26.2)	N(31.22,0.05)
20	N(7.823,0.58)	N(17.74,10.1)	N(13.06,8.88)

Table 3: Estimated distribution of outputs

DMU_j	y_{1j}~$N(\mu,\sigma^2)$	y_{2j}~$N(\mu,\sigma^2)$	y_{3j}~$N(\mu,\sigma^2)$	y_{4j}~$N(\mu,\sigma^2)$	y_{5j}~$N(\mu,\sigma^2)$
1	N(149.8548.51)	N(49.621,48.01)	N(4.701,0.41)	N(4.748,0.41)	N(30.09,24.34)
2	N(50.7728.011)	N(73.132,13.11)	N(1.815,0.02)	N(3.035,0.68)	N(5.823,0.689)
3	N(259.91295.9)	N(108.04,225.6)	N(6.016,0.01)	N(10.06,2.98)	N(2.721,4.392)
4	N(137.5121.65)	N(44.972,13.78)	N(4.923,1.65)	N(4.212,3.84)	N(63.61,33.73)
5	N(95.9012.521)	N(31.633,38.94)	N(2.718,0.44)	N(9.024,7.23)	N(64.55,31.25)
6	N(112.583.562)	N(71.958,70.05)	N(13.19,9.15)	N(41.89,13.2)	N(291.3,92.52)
7	N(192.97145.6)	N(78.015,195.6)	N(7.791,3.56)	N(15.89,2.34)	N(7.499,12.68)
8	N(724.38660.2)	N(219.69,375.6)	N(35.32,15.3)	N(23.98,10.4)	N(361.6,48.35)
9	N(548.15418.6)	N(86.225,48.31)	N(17.64,3.67)	N(86.23,17.2)	N(565.2,175.1)
10	N(1229.169.02)	N(194.58,17.35)	N(25.91,12.3)	N(86.76,22.1)	N(600.6,86.34)
11	N(11557718.1)	N(155.32,49.13)	N(166.6,14.1)	N(8.142,31.1)	N(119.9,14.89)
12	N(1132.1353.5)	N(248.16,238.9)	N(46.88,26.3)	N(31.85,45.1)	N(96.21,44.47)
13	N(438.39174.1)	N(104.41,257.1)	N(10.68,10.4)	N(30.22,37.7)	N(331.9,171.6)
14	N(260.8215.32)	N(87.369,234.4)	N(8.415,3.52)	N(6.101,2.86)	N(36.93,5.777)
15	N(111901214)	N(166.44,106.9)	N(65.12,16.1)	N(132.7,13.9)	N(919.1,133.1)
16	N(709.8542.19)	N(159.48,327.8)	N(36.89,6.28)	N(12.15,25.1)	N(79.66,105.2)
17	N(308.1143.72)	N(107.03,99.39)	N(11.87,1.76)	N(13.63,15.7)	N(342.3,134.9)
18	N(259.21342.1)	N(81.779,51.65)	N(5.212,19.4)	N(8.021,3.52)	N(107.9,130.7)
19	N(381.33573.8)	N(72.993,69.26)	N(5.165,1.79)	N(50.32,34.7)	N(577.2,113.4)
20	N(399.2510.41)	N(40.985,25.93)	N(11.51,0.99)	N(6.432,4.25)	N(82.13,98.91)

Table 4: Computational results for stochastic models (1), (8)-(10) and expression (11)

DMU	Model (1)	Model (8)	Model (9)	Model (10)	TEC	FS	SM
DMU1	0.297	0.569	0.911	1.000	3.367	0.431	1.450
DMU2	1.000	0.405	1.581	1.000	1.000	0.506	0.506
DMU3	0.859	2.194	1.459	0.941	1.095	1.171	1.283
DMU4	0.264	0.333	0.645	0.418	1.585	0.571	0.904
DMU5	0.458	0.428	0.923	0.565	1.233	0.613	0.756
DMU6	1.000	1.045	1.825	1.000	1.000	0.757	0.757
DMU7	0.397	1.692	0.608	0.629	1.585	1.325	2.100
DMU8	0.767	3.522	1.118	1.000	1.304	1.554	2.027
DMU9	1.000	3.634	1.824	1.000	1.000	1.411	1.411
DMU10	1.000	8.939	3.596	1.000	1.000	1.577	1.577
DMU11	1.000	10.000	2.339	1.000	1.000	2.067	2.067
DMU12	0.764	3.243	0.958	1.000	1.309	1.608	2.105
DMU13	0.842	5.427	1.895	1.000	1.188	1.553	1.844
DMU14	0.498	1.941	0.649	0.682	1.370	1.478	2.024
DMU15	1.000	20.000	2.104	1.000	1.000	3.083	3.083
DMU16	1.000	3.303	2.155	1.000	1.000	1.238	1.238
DMU17	0.750	4.779	0.959	1.000	1.333	1.934	2.578
DMU18	0.344	0.891	0.934	0.851	2.473	0.621	1.536
DMU19	1.000	2.950	1.644	1.000	1.000	1.340	1.340
DMU20	0.322	0.542	1.583	0.783	2.431	0.375	0.912

Also DMU_3, DMU_7, DMU_{14} have progress with FS_3=0.171, FS_7=1.325 and FS_{14}=1.478 respectively. but DMU_{18} has regress with $FS_{18}<1$.

CONCLUSION

This paper discusses Malmquist productivity index in stochastic data envelopment analysis. Prediction of progress and regress of DMUs in future can be a suitable tool from a managerial point of view. For this purpose we have proposed probabilistic models which were converted to deterministic models and stochastic Malmquist index has been introduced. The deterministic equivalent of stochastic models are nonlinear programming models which can be converted to quadratic programming models. Finally, we have demonstrated how to use the results by using a numerical example about banking industry. Results from stochastic models have a probabilistic nature and depend on the level of error that is considered in stochastic models. It means that if the level of error changes, the results may change too. In stochastic models we suppose that data have normal distributions. These models can be expanded for other distributions in future researches. Estimating Malmquist productivity index by considering fuzzy data is also a good direction for future research.

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