

A Model for the Optimal Allocation of Resources after Earthquakes by Means of Goal Programming

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Abstract: Fast and carefully-planned rescue operations are of paramount importance after natural disasters such as flooding, earthquakes, ... , unfortunately, nowadays rescue operations in many parts of the world are conducted without any prior planning or specific order. Since rescue facilities after each natural events are limited, therefore, the optimal allocation of the extant facilities are of significance. This paper tries to offer a model for the goal programming of rescue workers and vehicles with the least cost in order to increase the number of survivors.

Key words: Earthquakes • Genetic Algorithm • Goal programming • Optimal allocation of resources

INTRODUCTION

After each natural event such as earthquake, flooding, etcetera... numerous crises may break out. This paper assumes an imaginary scenario in which an earthquake has occurred, though its findings can be applied in other natural occurring disasters as well. The lack of a pre-planned program to deal with after-earthquake consequences in some countries has increased the number of casualties and damages of natural events, the earthquakes of Turkey and Iran in the past few decades are a good example. These two countries suffered much more casualties and damages compared with those countries which are well-prepared and enjoy a program for such events like Japan and China [1].

The studies conducted by Chinese engineers show that during the first 24 hours after the earthquake the chance of survival decreases from 93% to 50% , therefore, this time period is called a golden period and it is essential that the maximum capabilities and capacities be used optimally [2]. After each event, summoning up all doctors and medical staff is neither logical nor possible, so it is better to have a rough estimate of the survivors to summon the required medical staff.

In this paper, an attempt will be made to propose a model on the basis of goal programming for the optimal allocation of extant resources in order to rescue a greater

number of people. This model allows us to estimate the number of survivors and base the invitation call for the medical staff accordingly. To solve this model, we make use of Genetic Algorithm. Here we take for granted that all the buildings in the area are of the same stability. The paper is arranged as follows: part 2 provides a short introduction to goal programming, part 3 elaborates on Genetic Algorithm, while model designing is done in part 4 and part 5 offers the numerical results and finally part 6 discusses the findings and results.

Goal Programming: There is no consensus concerning the definition of goal programming. Goal programming is mostly used for decision making of multi-purpose issues. The term ‘goal programming’ was first coined by Charnes and Cooper, who originally developed the mathematical model to address the problem of infeasibilities caused by incompatible constraints.

The overall form of goal programming is as follows:

$$\begin{aligned} \text{Min } Z_i &= P_i (\bar{w}_i \bar{d}_i + w_i^+ d_i^+) \quad (i = 1, 2, \dots, m) \\ \text{st:} \quad & \sum_{j=1}^n C_{ij} X_j + d_i^- - d_i^+ = b_i \\ & \sum_{j=1}^n C_{ij} X_j \leq b_r \end{aligned}$$

$$X_j, d_i^+, d_i^- \geq b_r$$

Where:

X_j : refers to the model's decision variables which can be any non-negative real number

d_i^+, d_i^- : stands for deviation variables from positive or negative i^{th} ideal

b_r : refers to the right side number or the tendency level of the ideal i^{th}

P_k : shows the priority of k^{th} of the ideal ($k=1, 2, \dots, m$)

a_{ij} : stands for the technical coefficients of the model

C_{ij} : shows the variable factors of j^{th} in the i^{th} ideal

w_{ik}^-, w_{ik}^+ : are positive numbers which indicate the allocated weights to the deviation variables from positive / negative ideal within the i^{th} domain for the k^{th} ideal

This model contains n variable, m ideal, k priority, s as application limit and all of its mathematical relations are of first degree type.

The Genetic Algorithms: GAS are searching techniques using the mechanics of natural selection and natural genetics for efficient global searches [3]. In comparison to the conventional searching algorithms, GAS has the following characteristics : (a) GAS work directly with the discrete points coded by finite length strings (chromosomes), not the real parameters themselves; (b) GAS consider a group of points (called a population size) in the search space in every iteration, not a single point; (c) GAS use fitness function information instead of derivatives or other auxiliary knowledge; and (d) GAS use probabilistic transition rules instead of deterministic rules. Generally, a simple GA consists of the three basic genetic operators: (a) Reproduction; (b) Crossover; and (c) Mutation. They are described as follows.

Reproduction: Reproduction is a process to decide how many copies of individual strings should be produced in the mating pool according to their fitness value. The reproduction operation allows strings with higher fitness value to have larger number of copies and the strings with lower fitness values have a relatively smaller number of copies or even none *at al.* This is an artificial version of natural selection (strings with higher fitness values will have more chances to survive).

Crossover: Crossover is a recombined operator for two high-fitness strings (parents) to produce two offsprings by matching their desirable qualities through a random

process. In this paper, the uniform crossover method is adopted. The procedure is to select a pair of strings from the mating pool at random, then, a mark is selected at random. Finally, two new strings are generated by swapping all characters correspond to the position of the mark where the bit is "1". Although the crossover is done by random selection, it is not the same as a random search through the search space. Since it is based on the reproduction process, it is an effective means of exchanging information and combining portions of high-fitness solutions.

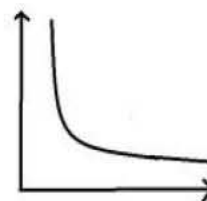
Mutation: Mutation is a process to provide an occasional random alteration of the value at a particular string position. In the case of binary string, this simply means changing the state of a bit from 1 to 0 and vice versa. In this paper we provide a uniform mutation method. This method is first to produce a mask and select a string randomly, then complement the selected string value correspond to the position of mask where the bit value is "1". Mutation is needed because some digits at particular position in all strings may be eliminated during the reproduction and the crossover operations. So the mutation plays the role of a safeguard in GAS. It can help GAS to avoid the possibility of mistaking a local optimum for a global optimum [4,5].

Model Design: In this section, we first provide an overall design for the model and then solve some cases for different scenarios.

At first, let us discuss the limitation and hypotheses which are as follows:

The Limitation and Hypotheses

Time: Needless to say that time plays a pivotal role after natural disasters such as earthquakes. With the passage of time , the number of disaster casualties increases. Seismic engineers in China have shown that in the first 22 hours after the occurrence of earthquake, the chance of survival significantly decreases from 93% to 53% , as a result, this period is called a golden period which requires maximum usage of existing capacities and facilities. The graph of survival and its mathematical relation are as follows[1]:



Since $t = 0, p(0) = .93$ then :
 $t = 1, p(1) = .5$

$$p(t) = be^{-at} \Rightarrow \begin{cases} p(0) = be^{-a \cdot 0} = b = .93 \\ p(1) = .93e^{-a \cdot 1} = .5 \Rightarrow a = .620756 \end{cases}$$

Therefore the likelihood of survival is equal to:

$$P(t) = .93e^{-a \cdot 620756}$$

Here we deploy the model for the first hour in which the chance of survival is on average 90%

Vehicles: There is a limited number of accessible vehicles. We assume that the existing vehicles with m_1 person capacity is y_1 and m_2 person capacity is y_2 .

Rescue Workers: There is a limited number of rescue workers in the area. We assume that the number of rescue workers is y_1 . Since the capacity of vehicles is m_1 and m_2 person, we divide the teams into m_1 and m_2 groups.

Budget: Budget allocation for any operation is limited, here we assume that the budget amounts to y_3 \$. Moreover, the budget for each m_1 team is equal to n_1 \$ and the cost for each m_2 team is n_2 \$.

Population: We assume that the people under rubble is g person.

Capability of Rescue Teams: The number of persons rescued by m_1 team in u^{th} hour is $k_1 e^{-.1u}$ and the number of persons rescued by m_2 team in u^{th} hour is $k_2 e^{-.2u}$.

Now we describe the model variables:

Variables:

- Number of m_1 person rescue teams x_1
- Number of m_2 person rescue teams x_2
- Number of rescue vehicles of m_1 person capacity x_3
- Number of rescue vehicles of m_2 person capacity x_4
- Number of persons rescued in the u^{th} hour t_u
- Number of redundant persons who can be deployed elsewhere d_1^-
- Number of required personnel who should be called from other areas otherwise they have to be included in the model we set $d_1^+ = 0$ d_1^+

- Extra budget not needed which can be used elsewhere d_2^-
- Required budget to be funded by officials, in case we run out of funding, it should be included in the model we set $d_2^+ = 0$ d_2^+
- Number of rescue m_1 person capacity vehicles not needed which can be used in other areas d_3^-
- Number of required m_1 person capacity vehicles which should be summoned from other areas otherwise they need to be included in the model we set $d_3^+ = 0$ d_3^+
- Number of rescue m_2 person capacity vehicles not needed which can be used in other areas d_4^-
- Number of required m_2 person capacity vehicles which should be summoned from other areas otherwise they need to be included in the model we set $d_4^+ = 0$ d_4^+
- Number of the persons who cannot be rescued in the allocated time period. d_5^-

Now let us describe the ideals in order of importance:

Ideals:

- P_1 is the maximum number of persons rescued in the proposed time period
- P_2 is the maximum use of existing rescue workers
- P_3 is the maximum use of existing m_1 person capacity vehicles
- P_4 is the maximum use of existing m_2 person capacity vehicles
- P_5 is the maximum use of existing budget

Based on the above said points, the model is designed as follows:

$$\begin{aligned} \min & p_1(d_5^-), p_2(d_1^- + d_1^+), p_3(d_3^- + d_3^+), p_4(d_4^- + d_4^+), p_5(d_2^- + d_2^+) \\ & m_1 x_1 + m_2 x_2 + d_1^- - d_1^+ = y_1, \\ & n_1 x_1 + n_2 x_2 + d_2^- - d_2^+ = y_3, \\ & x_3 + d_3^- - d_3^+ = y_2, \\ & x_4 + d_4^- - d_4^+ = y_2, \\ & k_1 e^{-.1u} x_1 + k_2 e^{-.2u} x_2 + d_5^- - d_5^+ = t_u g, \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, d_1^- \geq 0, d_2^- \geq 0, d_3^- \geq 0, \\ & d_4^- \geq 0, d_5^- \geq 0, d_1^+ \geq 0, d_2^+ \geq 0, d_3^+ \geq 0, d_4^+ \geq 0, d_5^+ \geq 0. \end{aligned}$$

Numerical Results: Here an examples is provided for illustration:

Example: We suppose that:

$$y_1 = 78000, y_2 = 600, y'_2 = 550, y_3 = 10000, m_1 = 2, m_2 = 4, \\ n_1 = 20, n_2 = 35, k_1 = 100, k_2 = 165, t_1 = \%89, g = 100000.$$

Therefore, Our Model Will Be as Follows:

$$\min p_1(d_5^-), p_2(d_1^- + d_1^+), p_3(d_3^- + d_3^+), p_4(d_4^- + d_4^+), p_5(d_2^- + d_2^+) \\ 2x_1 + 4x_2 + d_1^- - d_1^+ = 78000, \\ 20x_1 + 35x_2 + d_2^- - d_2^+ = 10000, \\ x_3 + d_3^- - d_3^+ = 600, \\ x_4 + d_4^- - d_4^+ = 550, \\ 30x_1 + 50x_2 + d_5^- - d_5^+ = 90000, \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, d_1^- \geq 0, d_2^- \geq 0, d_3^- \geq 0, \\ d_4^- \geq 0, d_5^- \geq 0, d_1^+ \geq 0, d_2^+ \geq 0, d_3^+ \geq 0, d_4^+ \geq 0, d_5^+ \geq 0.$$

Now We Solve the Problem by Means of Genetic Algorithm:

$$x_1 = 1000, x_2 = 1200, x_3 = 1000, x_4 = 1200, d_1^- = 1000, d_2^- = 0, d_3^- = 0, \\ d_4^- = 0, d_5^- = 10000, d_1^+ = 0, d_2^+ = 52000, d_3^+ = 400, d_4^+ = 650, d_5^+ = 0.$$

CONCLUSION

In most countries the main reason behind the high casualties after earthquakes goes back to the lack of appropriate organization and optimal allocation of existing rescue workers in the area. This paper tried to provide a model based on goal programming for the optimal allocation of rescue workers and medical staff. we solve this model by means of Genetic Algorithm.

REFERENCES

1. Metzger, M.D., 2005. Formulating earthquake response models in Iran. Master Thesis, Massachusetts Institute of Technology, Department of Electrical Engineering and Computer Sci.,
2. Metzger, M.D., 2003. The Two City Optimal Allocation of Earthquake Rescue Workers. Bachelor Thesis, Massachusetts Institute of Technology, Department of Electrical Engineering and Computer Sci.,
3. Prebys, E.K., 1997. The Genetic Algorithm in Computer Sci.,
4. Marczyk, A., 2004. Genetic algorithms and evolutionary computation. The Talk Origins Archive.
5. Mitchell, M., 1997. An Introduction to Genetic Algorithms. MIT Press, Cambridge, Massachusetts.
6. Charles, A. and W.W. Cooper, 1961. *Management Models and Industrial Applications of Linear Programming*. New York: John Wiley and Sons.
7. Weithman, A.S. and R.J. Ebert, 1981. Goal programming to assist in decision making. Fisheries, 6(1): 5-8.