

## Similarity Solution of MHD Heat and Mass Transfer over a Vertical Permeable Flat Plate with Variable Concentration, Chemical Reaction and Viscous Dissipation

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**Abstract:** In this paper we study magnetic field, chemical reaction and viscous dissipation on combined heat and mass transfer over a moving permeable vertical flat plate. A variable magnetic field parallel to the  $y$  axis and convective surface boundary condition are applied. The similarity representations of the system of partial differential equations of the problem are obtained via the group method. These similarity equations are then solved numerically by a MATLAB package which uses an implicit finite difference technique. Influences of the various parameters on the velocity, the temperature and the concentration fields across the boundary layer are investigated and numerical results for the velocity, temperature and concentration distributions are shown. The skin-friction factor, the Nusselt number and the Sherwood number are also calculated and displayed in graphs to show the effects of various parameters on them.

**Key words:** Convective Surface • MHD Flow • Similarity Solutions • Chemical Reaction

### INTRODUCTION

Effects of chemical reaction on heat and mass transfer laminar boundary layer flow have been discussed by a number of researchers due to its industrial applications. Heat and mass transfer of a continuously moving isothermal vertical surface with uniform suction and chemical reaction was reported by Muthucumaraswamy [1]. It was concluded that the velocity and concentration increased during the generative reaction and the situation was reversed in the case of the destructive reaction. The same type of problem over a stretching surface in the presence of a constant transverse magnetic field was analyzed by Afify [2]. A similar type of problem on a vertical permeable surface in the presence of a radiation, a first-order homogeneous chemical reaction and the mass flux was examined by Ibrahim *et al.* [3] analytically. Mansour *et al.* [4] have analysed the effects of chemical reaction on MHD free convective heat and mass transfer on a vertical stretching surface in a saturated porous medium. Recently, Joneidi *et al.* [5] have presented homotopy analysis method to investigate the effect of chemical reaction on free convection of a viscous,

incompressible and electrically conducting fluid over a stretching surface in the presence of a constant magnetic field. They have found that velocity, temperature and concentration have a direct relationship with the chemical reaction parameter whilst the magnetic parameter has direct relation with temperature and concentration. Chandrakala [6] has recently presented the finite difference solution of the homogeneous first order chemical reaction on unsteady flow past a vertical plate in the presence of magnetic field. It was observed that the velocity decreases in the presence of chemical reaction or magnetic field. It was also observed that due to the presence of first order chemical reaction, the velocity increases during the generative reaction and decreases in destructive reaction. Also, Muhaimin *et al.* [7] have investigated natural convective flow with temperature-dependent fluid viscosity, chemical reaction and thermal radiation over a vertical stretching surface in the presence of suction by scaling transformations.

Magneto-hydrodynamics (MHD) deals with the flow of electrically conducting fluids in electric and magnetic fields. The study of MHD flow and the effect on mass and heat transfer is of great interest in metallurgy. This is due

to the influence of magnetic fields on the control of boundary-layer flow and on the performance of many systems using fluids which conduct electricity. Researchers have found that the applications of a magnetic field, hydromagnetic techniques can be used for the purification of molten metals from non-metallic inclusions [8]. In recent years, flow of this nature has evoked the interest of many researchers due to its application in many engineering problems. In this respect, Rajeswari *et al.* [9] have focused on the effect of chemical reaction on the free and forced convection boundary layer that flows past vertical porous plate with magnetic field. The effects of variable electric conductivity and temperature dependent viscosity on hydromagnetic heat and mass transfer flow were analyzed using numerical methods for various values of the relevant physical parameters by Rahman and Salaudddin [10]. Ibrahim *et al.* [11] discussed the effect of chemical reaction on free convection for a non-Newtonian power law fluid over a vertical flat plate in porous medium. They have used similarity transformations to reduce the boundary layer equations into similarity equations and solved them by a finite difference method. Muhaimin *et al.* [12] have reported the nonsimilarity solutions of viscous flow due to a shrinking sheet in the presence of suction with variable stream conditions. The heat transfer of an incompressible Ostwald de-Waele power-law fluid past an infinite porous plate, subject to suction at the plate with viscous dissipation and radiation effect was analysed by Cortell [13]. Makinde [14] has studied the MHD boundary layer flow with heat and mass transfer over a moving vertical plate in the presence of magnetic field and a convective heat exchange at the surface with the surroundings.

Methods available in the literature for similarity analysis are broadly classified as (i) Dimensionless Analysis, (ii) Free Parameter, (iii) Separation of variables and (iv) Group Theory [15, 16]. In case of group theory, the similarity solution is the invariant solution of initial and boundary value problems. Group invariant transformations do not alter the structural form of the equations under investigation [8]. This technique has been applied by many researchers to analyze different flow phenomena arising in fluid mechanics, plasma physics, meteorology, chemical engineering and other engineering branches. Abd-el-Malek and Helal [17] have reported the solution of unsteady forced free convective flow under the influence of external velocity and magnetic

field. Recently, Hamad and Pop [18] have analyzed the similarity solutions over flat plate in porous medium saturated by nano fluid (a fluid with nanometre sized particles) with heat generation/absorption. Salem [19] has investigated the effect of temperature-dependent viscosity on free convection flow along a vertical wedge adjacent to a porous medium in the presence of heat generation or absorption using the group method. Reviews for the fundamental theory and applications of group theory to differential equations may be found in the texts by Na [20], Meleshko [21] and Ibragimov and Kovalev [22].

The objectives of our present study is to find the similarity solutions of steady MHD laminar flow, heat and mass transfer of Newtonian fluid with homogenous chemical reaction over a moving permeable flat plate with convective surface boundary condition. For our study, the concentration of the reactant is not constant. We apply convective surface boundary condition. The governing nonlinear partial differential equations are transformed into a coupled similarity differential equations using a set of dimensionless variables obtained by the group method and are then solved them numerically using a finite difference scheme. The computation is carried out by MATLAB software. We will plot graphs for the non-dimensional velocity, temperature and concentration profiles for various values of controlling parameters entering into the problem. We have also estimated the wall shear stress; the rate of heat transfer and the rate of mass transfer and exhibit them graphically which are the chief quantities of practical interest.

### Mathematical Simulation

**Problem Analysis:** Consider a continuous moving permeable flat plate with a uniform velocity  $U_0$  shown in figure1. Parallel to the  $y$  direction, a variable magnetic field of strength  $B(x)$  is applied. As in Ishak [23], the bottom surface of the plate is heated by convection from a hot fluid of temperature  $T_f$ . This then provides a heat transfer coefficient  $h_f$ . Consider fluid properties as constant except the density of the buoyancy term. We shall further assume that particle coagulation, the magnetic Reynolds number, the electric field on account of the polarization of charges and Hall effects are negligible. The field variables are the velocity  $\vec{v} = (u, v)$ , the temperature  $T$  and the concentration  $C$ . Under the forgoing assumptions the governing boundary layer equations relevant to our problem are (Incropera *et al.* [24]).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u + g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty), \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2, \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty). \tag{4}$$

The boundary conditions are ((see Aziz [25], Ishak [23]).

$$u = U_0, \quad v = v_w(x), \quad C_w(x) = A x^p + C_\infty, \quad -\kappa \frac{\partial T}{\partial y} = h_f [T_f - T_w] \quad \text{if } y = 0, \tag{5}$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{if } y \rightarrow \infty,$$

The meanings of the symbols are as follows:

$(u, v)$ : the velocity components along  $x$  (along the plate) and  $y$  (perpendicular to the plate) axes,  $v_w(x)$ : the mass transfer velocity with  $v_w(x) > 0$  for injection (blowing),  $v_w(x) < 0$  for suction and  $v_w(x) = 0$  corresponds to an impermeable plate,  $\nu = \mu/\rho$ : the kinematic coefficient of viscosity,  $\mu$ : the coefficient of viscosity,  $\rho$ : density of the fluid,  $\sigma$ : the electric conductivity,  $p$ : real exponent,  $\kappa$ : thermal conductivity,  $\alpha$ : thermal diffusivity,  $D = \kappa / \rho c_p$ : mass diffusivity of species in fluid,  $c_p$ : specific heat at constant pressure,  $\beta_T$ : volumetric thermal coefficient,  $\beta_C$ : volumetric concentration coefficient,  $g$ : acceleration due to gravity,  $k_0$ : the reaction rate,  $k_0 > 0$  for destructive reaction,  $k_0 = 0$  for no reaction,  $k_0 < 0$  for generative reaction,  $T_w, C_w(x)$ : temperature and concentration at the plate while  $T_\infty, C_\infty$  are the temperature and concentration far away the plate respectively.

**Method of Solution:** In order to deal with our problem, we introduce the potential  $\Psi$  defined by  $u = \frac{\partial \Psi}{\partial y}, v = -\frac{\partial \Psi}{\partial x}$ . The

mathematical significance of its use is that the continuity equation (1) is satisfied. The physical motivation for introducing the function is that constant  $\Psi$  lines are stream lines. We also introduce the following dimensionless temperature and concentration functions  $\theta$  and  $\phi$  defined by

$$\theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{6}$$

For the problem being considered, it is appropriate to assume the following form of the magnetic field strength  $B(x)$  can be assumed to be of the form

$$B(x) = B_0 / \sqrt{x} \tag{7}$$

Where  $B_0$  is a constant (see Helmy [26], Aissa and Mohammadein [27], Rahman and Salauddin [10]).

Applying Eqs. (6) and (7) and potential  $\Psi$ , we get from Eqs. (2) to (4)

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B_0^2}{\rho x} \frac{\partial \psi}{\partial y} + g \beta_T \Delta T \theta + g \beta_C \Delta C \phi, \quad (8)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2} + \frac{\mu}{(T_f - T_\infty) \rho c_p} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2, \quad (9)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} (\phi \Delta C) - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} (\phi \Delta C) = D \frac{\partial^2}{\partial y^2} (\phi \Delta C) - k_0 (\phi \Delta C), \quad (10)$$

The boundary conditions (5) becomes

$$\begin{aligned} \frac{\partial \psi}{\partial y} = U_0, \quad \frac{\partial \psi}{\partial x} = -v_w(x), \quad \phi = 1, \quad \frac{\partial \theta}{\partial y} = -\frac{h_f}{\kappa} [1 - \theta(0)] \text{ if } y = 0, \\ \frac{\partial \psi}{\partial y} \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \text{ if } y \rightarrow \infty. \end{aligned} \quad (11)$$

Let us define the similarity variable and stream function (see White [28], Christopher and Wang [29]) in the usual form as follows

$$\eta = c_1 x^r y, \quad \psi = c_2 x^{-s} f(\eta), \quad \theta = \theta(\eta), \quad \phi = \phi(\eta) \quad (12)$$

Here  $c_1 > 0$ ,  $c_2 > 0$  and  $r, s$  are real numbers and will be determined by using the invariant condition of the differential equations.

Applying Eq. (12), we have from Eqs. (8) to (11), the following relationships among the exponents, for constant conformally invariant (an equation is said to be an invariant under the transformations if it remains unaltered by the transformations, for details about invariant see Hansen, 1964),

$$\begin{aligned} 2r - 2s - 1 = r - s - 1 = 3r - s = p, \\ r - s - 1 = 2r. \end{aligned} \quad (13)$$

Solving the linear system (12), we get

$$r = -1/2, \quad s = -1/2, \quad p = -1. \quad (14)$$

Substitution of Eq. (14) in Eq. (13), leads to

$$\eta = c_1 \frac{y}{x^{1/2}}, \quad \psi = c_2 x^{1/2} f(\eta), \quad \theta = \theta(\eta), \quad \phi = \phi(\eta). \quad (15)$$

Again, substitution of various transformations in Eq. (15) into Eqs. (8) to (10), yields

$$\nu c_1^3 c_2 f''' - \frac{\sigma B_0^2}{\rho} c_1 c_2 f' + g \beta_T \Delta T x \theta + g \beta_C \Delta C x \phi = c_1^2 c_2^2 s f f'', \quad (16)$$

$$\alpha c_1^2 \theta'' + \frac{\mu}{\rho c_p (T_f - T_\infty)} c_1^4 c_2^2 f'^2 - c_1 c_2 s f \theta' = 0, \quad (17)$$

$$c_1 c_2 s f \phi' - c_1 c_2 f' \phi = D c_1^2 \phi'' - k_0 \phi \Delta C x^2. \tag{18}$$

Here prime means the differentiation with respect to similarity independent variable  $\eta$ .

To get the non-dimensional forms of  $\eta$ ,  $\Psi$  and to get rid of the fluid properties appearing in the coefficients of the Eqs.(16) to (18) we choose  $c_1 = \sqrt{\frac{U_0}{\nu}}$ ,  $c_2 = \sqrt{\nu U_0}$  and consequently The new transformations becomes

$$\eta = y \sqrt{\frac{U_0}{\nu x}}, \quad \Psi = \sqrt{U_0 \nu} x^{1/2} f(\eta), \quad \theta = \theta(\eta), \quad \phi = \phi(\eta). \tag{19}$$

Using various transformations in Eq. (19) into Eqs. (8) to (10) we obtain,

$$f''' + \frac{1}{2} f f'' - M f' + Gr \theta + Gc \phi = 0, \tag{20}$$

$$\frac{1}{Pr} \theta'' + \frac{1}{2} f \theta' + Ec f''^2 = 0, \tag{21}$$

$$\frac{1}{Sc} \phi'' + \frac{1}{2} f \phi' - f' \phi - k1 \phi = 0. \tag{22}$$

The parameters involving in the equations (20) to (22) are defined as follows

$$Gr = \frac{g \beta_T (T_f - T_\infty) x}{U_0^2}, \quad Gc = \frac{g \beta_C (C_w - C_\infty) x}{U_0^2}, \quad Sc = \frac{\nu}{D}, \tag{23}$$

$$M = \frac{\sigma B_0^2}{\rho U_0}, \quad Ec = \frac{U_0^2}{c_p \Delta T}, \quad Pr = \frac{\mu c_p}{\kappa}, \quad \text{and } k1 = \frac{k_0 \Delta C x^2}{U_0}$$

Where  $Gr$  is the local Grashof number based on temperature,  $Gc$  is the local Grashof number based on concentration,  $Sc$  is the Schmidt Number,  $Ec$  is the Eckert number,  $M$  is the magnetic parameter and  $Pr$  is the Prandtl number of the fluid,  $k1$  is the local non-dimensional reaction parameter (see Makinde [14]).

The corresponding boundary conditions are

$$f' = 1, \quad f = f_w, \quad \theta'(0) = b(\theta(0) - 1), \quad \phi = 1 \quad \text{at } \eta = 0, \tag{24}$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty,$$

Where  $f_w = \frac{2v_w(x)}{\sqrt{\nu U_0}} x^{1/2}$  is the suction parameter and for constant  $f_w$ ,  $v_w$  must be proportional to  $x^{-1/2}$ . The parameter

$b = \frac{h_f}{\kappa} \sqrt{\frac{\nu x}{U_0}}$  is the local convective heat transfer parameter.

Here it is observed that parameters  $b$ ,  $Gr$ ,  $Gc$  and  $k1$  are all functions of  $x$ . But, we know, in order to get the similarity solution (see Hansen [15], Dresner [30]) the parameters  $b$ ,  $Gr$ ,  $Gc$ ,  $k1$  must be free from  $x$ , as a results they should be constant and we therefore assume:

$$h_f = c_3 x^{-1/2}, \beta_T = c_4 x^{-1}, \Delta C = c_5 x^{-1}, k_0 = c_6 x^{-1} \tag{25}$$

Where  $c_3, c_4, c_5$  are  $c_6$  constants.

**Comparison with the Literature:** It may be noted that if (i) the plate is impermeable ( $fw = 0$ )(ii) the viscous dissipation is absent ( $Ec = 0$ ), (iii) the chemical reaction is absent ( $kl = 0$ ) and  $\Delta C$  is assumed to be constant, then equations (20) to (22) and the boundary conditions (24) reduces to the equations obtained by Makinde [14]. Again, if the wall is impermeable  $fw = 0$  and if  $Gr = Gc = M = 0$  and without the concentration equation, the equations (20) to (21) reduces to the equations that was derived by Aziz [25]. It is also noticed that without concentration equation, magnetic field and viscous dissipation and for impermeable plate, the equations (20) and (21) reduce to the equations obtained Makinde and Olanrewaju [31].

**Physical Parameters of Interest:** For our problem (I) the local skin friction factor  $C_f$  (ii) the local Nusselt number  $Nu$  and (iii) the local Sherwood number  $Sh$  are the parameters of interest. Physically, skin friction factor represents shearing stress on the surface of the plate due to fluid motion whilst  $Nu$  and  $Sh$  indicates the rate of heat transfer and the rate of mass transfer respectively. The physical quantities can be derived using the definitions given below:

$$C_f = \frac{\mu}{\rho U_0^2} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad Nu = \frac{-x}{T_f - T_\infty} \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad Sh = \frac{-x}{C_w - C_\infty} \left( \frac{\partial C}{\partial y} \right)_{y=0} \tag{26}$$

By substituting from Eqs. (6) and (19) into Eq. (26), we get

$$\begin{aligned} \text{Re}^{\frac{1}{2}} C_f &= f''(0) \\ \text{Re}^{-\frac{1}{2}} Nu &= -\theta'(0) \\ \text{Re}^{-\frac{1}{2}} Sh &= -\phi'(0) \end{aligned} \tag{27}$$

Where  $\text{Re} = \frac{U_0 x}{\nu}$  is the local Reynolds number. Thus from Eq. (27) we see that skin friction factor  $C_f$ , Nusselt number  $Nu$  and Sherwood number  $Sh$  are proportional to the numerical values of  $f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  respectively.

### RESULTS AND DISCUSSION

The set of coupled similarity equations (20) to (22) are of third order in  $f$ , second order in  $\theta$  and  $\phi$  respectively. These equations have been reduced to seven simultaneous first order ordinary differential equations for seven unknowns. We solve them numerically by an implicit finite difference technique with Grashof number based on temperature  $Gr$ , Grashof number based on concentration  $Gc$  Prandtl number  $Pr$ , Schmidt number  $Sc$ , Magnetic parameter  $M$ , the convective heat transfer parameter  $b$ , the suction parameter  $fw$ , Eckert number

$Ec$ , and chemical reaction parameter  $kl$  as prescribed parameter. The numerical computations are implemented using the MATLAB software bv4c. Numerical results are plotted in Figs. 1 to 10 to exhibit the influence of the various parameters on the flow.

Figures 1 and 2 show representative velocity profiles, Figures 3 and 4 present temperature profiles while Figures 5 and 6 represent typical concentration profiles for different values of the convective heat transfer parameter ( $b$ ), suction parameter ( $fw$ ), viscous dissipation parameter ( $Ec$ ) and nondimensional reaction parameter ( $kl$ ) respectively.

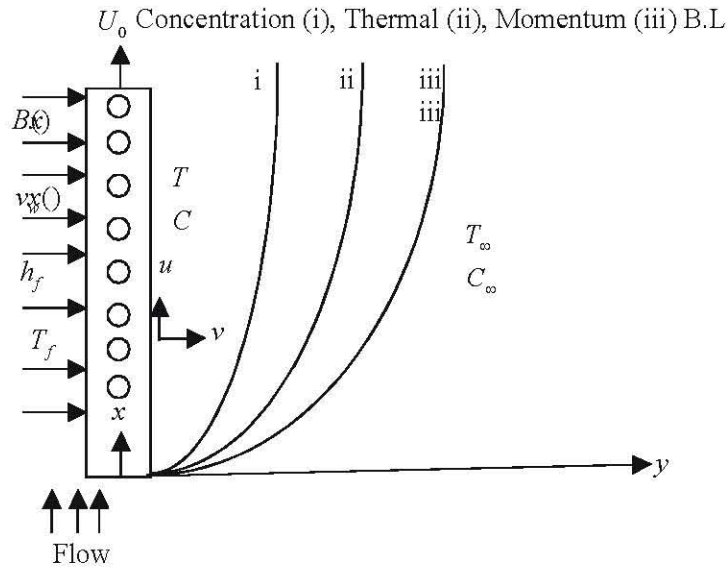


Fig. 1: Physical configuration and coordinate system the problem

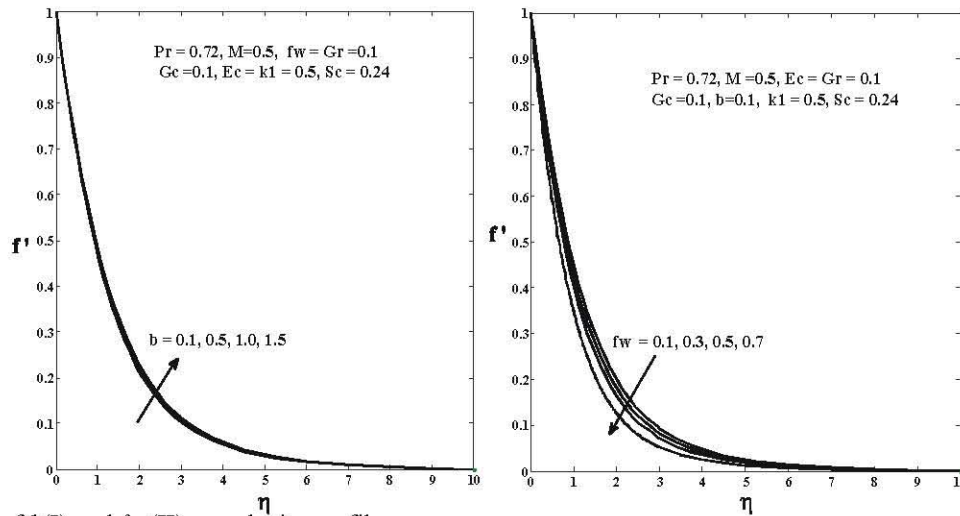


Fig. 2: Effects of  $b(I)$  and  $fw(II)$  on velocity profiles.

Figures 1(I), 3 (I) and 5(I) reveal the effect of the convective heat transfer parameter  $b$  on the velocity, temperature and concentration profiles respectively. From the figures, it can be seen that the fluid velocity, temperature and concentration species and their corresponding boundary layers thickness increases with increasing  $b$ .

The effects of the suction parameter  $fw$  on the velocity, temperature and concentration profiles are shown in Figures 1 (II), 3(II) and 5(II) respectively. Imposing fluid suction  $fw > 0$  tends to decrease the fluid velocity, temperature and concentration as well

as their boundary layers thicknesses. These trends are clear from Figures 1(II), 3(II) and 5(II) respectively.

Figures 2(I), 4(I) and 6(I) demonstrate the velocity distribution, the temperature distribution and the concentration distribution for different values of Eckert number  $Ec$ . It is observed that increasing values of  $Ec$  increase the velocity, temperature and concentration distribution in flow region. This is due to the heat energy stored in the liquid because of the frictional heating [10]. We also note that increasing the values of  $Ec$  increases the momentum, thermal and concentration boundary layers thickness.

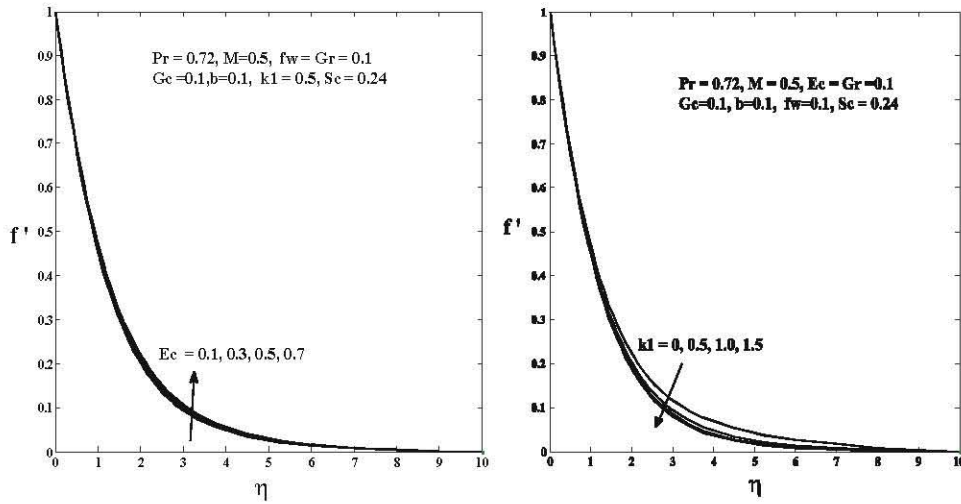


Fig. 3: Effects of  $Ec$  (I) and  $k1$  (II) on velocity profiles

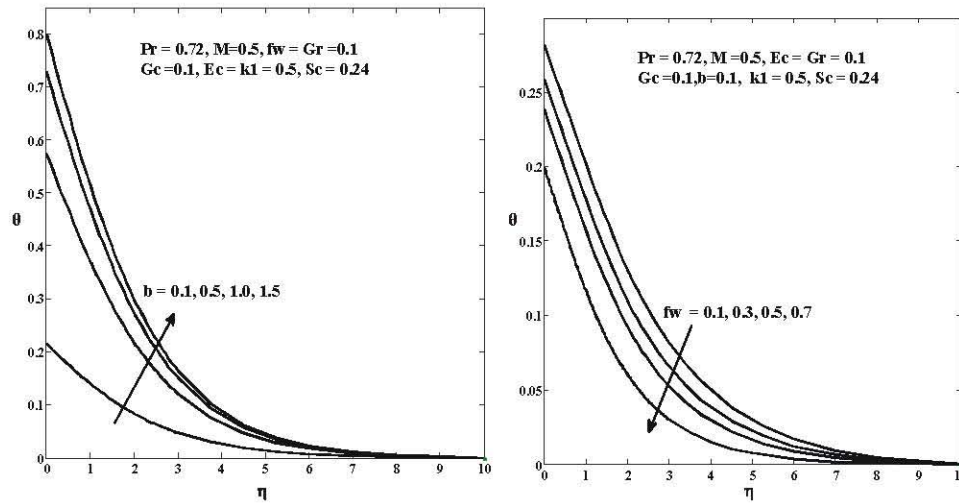


Fig. 4: Effects of  $b$  (I) and  $fw$  (II) on temperature profiles.

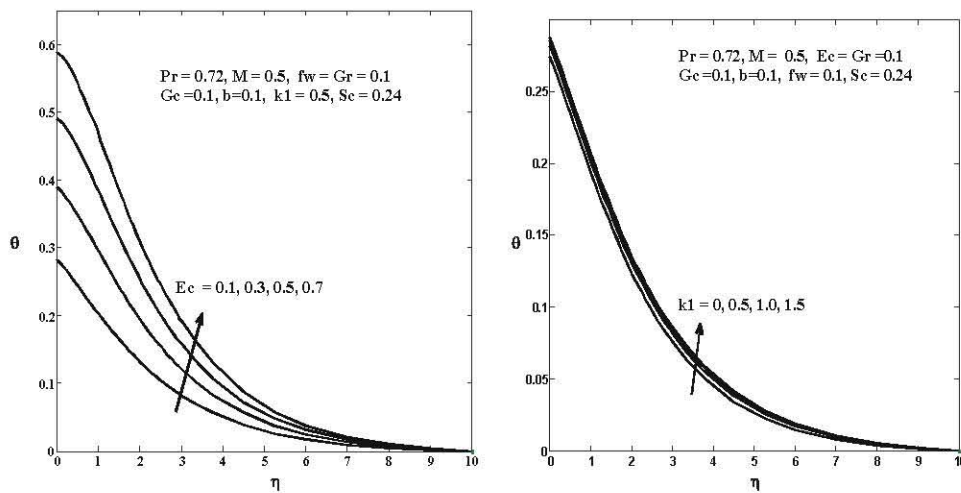


Fig. 5: Effects of  $Ec$  (I) and  $k1$  (II) on temperature profiles.



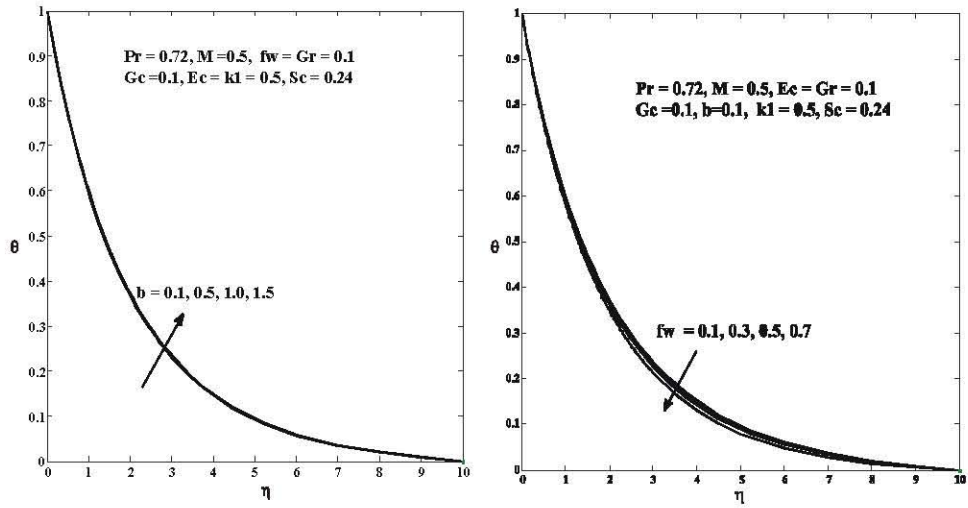


Fig. 6: Effects of  $b$ (I) and  $fw$ (II) on concentration profiles

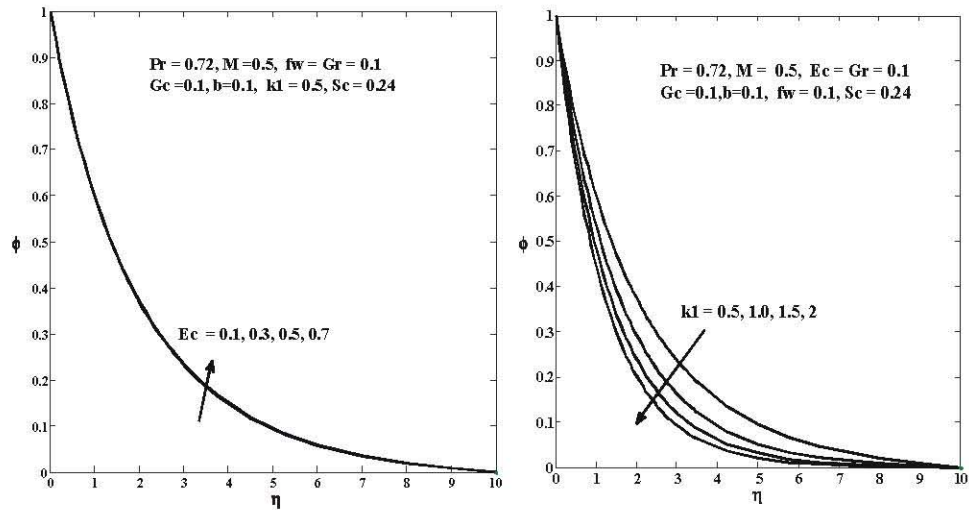


Fig. 7: Effects of  $Ec$ (I) and  $k1$ (II) on concentration profiles.

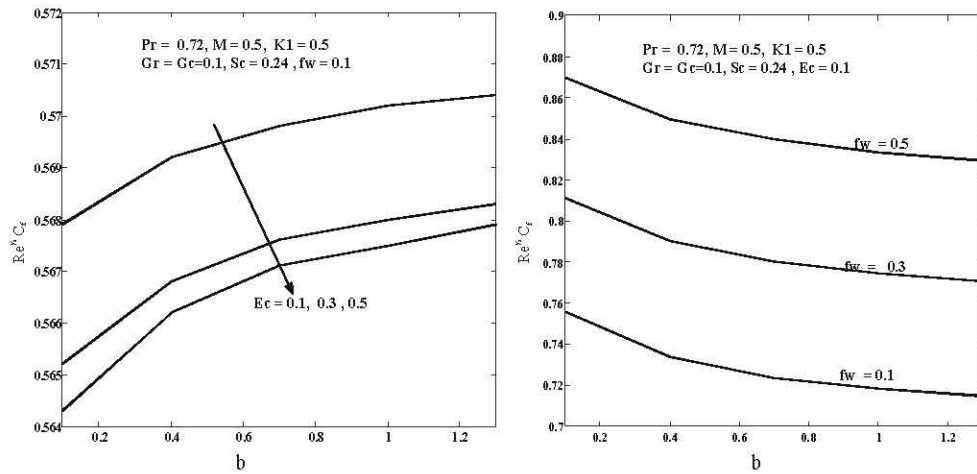


Fig. 8: Effects of  $Ec$  (I) and  $fw$  (II) on skin friction factor.

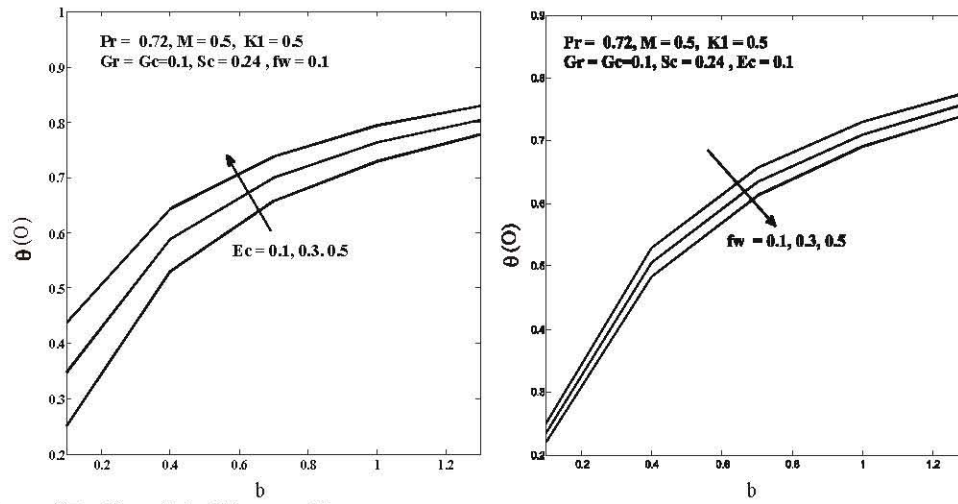


Fig. 9: Effects of  $Ec$  (I) and  $fw$  (II) on wall temperature.

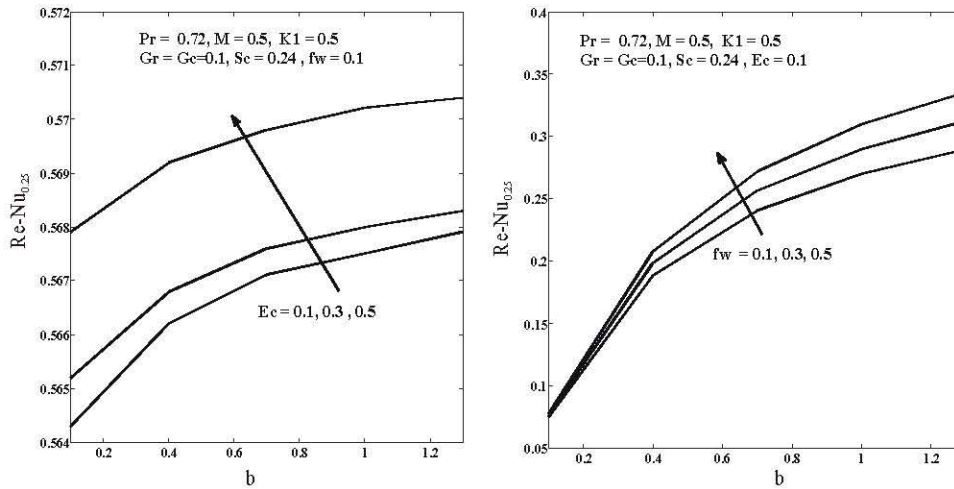


Fig. 10: Effects of  $Ec$  (I) and  $fw$  (II) on the rate of heat transfer.

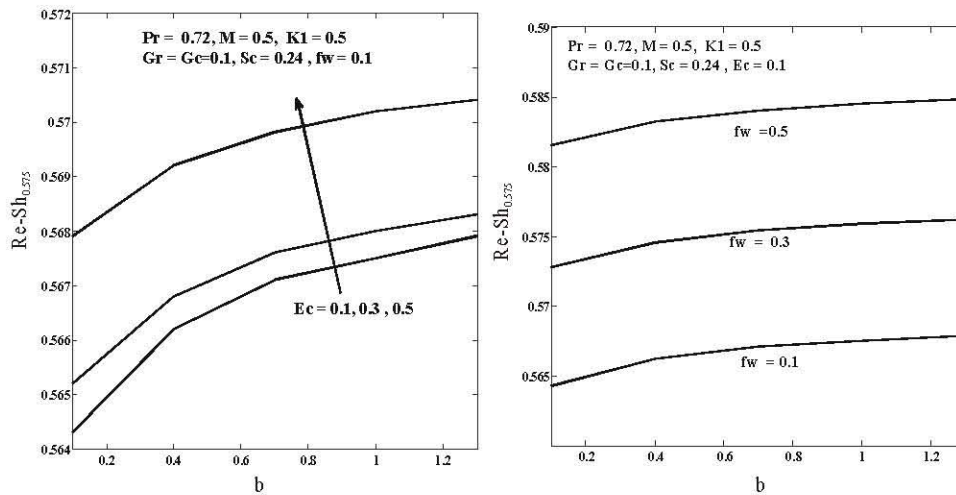


Fig. 11: Effects of  $Ec$  (I) and  $fw$  (II) on the rate of mass transfer.

Figures 2(II), 4(II) and 6(II) show the effect of the chemical reaction parameter  $k_1$  on the velocity, temperature and concentration profiles respectively. The fluid velocity and concentration species and their respective boundary layers decrease with increasing  $k_1$  while the temperature field and the thermal boundary layer increase.

Figures 7(I), 8(I), 9(I) and 10(I) are plotted to exhibit the influence of the Eckert number  $Ec$  on skin friction factor  $C_f$ , the wall temperature  $\theta(0)$ , the rate of heat transfer  $Nu$  and the rate of mass transfer  $Sh$  respectively. It can be remarked that the skin-friction factor decreases while the wall temperature, the Nusselt number and the Sherwood number increases with the increase of the Eckert number.

Figure 7(II) represents skin friction factor  $C_f$  against the convective parameter  $b$  for different values of suction parameter  $f_w$ . The skin friction increases with an increase in  $f_w$  Fig. 8(II) displays the wall temperature  $\theta(0)$  against the parameter  $b$  for various values of  $f_w$ . From the Figure, it can be observed that the wall temperature decreases with the increasing values of  $f_w$ . Figure 9(II) illustrates the effects on suction  $f_w$  on the rate of heat transfer. The rate of heat transfer, (the Nusselt number) increases with an increase in  $f_w$ . A similar phenomena is observed in Fig. 10(II) for rate of mass transfer (the Sherwood number).

### CONCLUSIONS

The steady two dimensional MHD flow and heat as well as mass transfer of a Newtonian fluid past a permeable flat vertical plate subject to convective boundary condition and homogeneous chemical reaction has been examined using the method of group theory. Our analysis revealed that similarity solutions exist if expansion of thermal coefficient, concentration changes and reaction rate are proportional to inverse of the axial distance. The numerical solutions have been reported for various controlling parameters. The following conclusion can be made:

- The thickness of the velocity and concentration boundary layer decreases due to an increase in reaction parameter  $k_1$  while temperature profiles increase.
- The fluid velocity, temperature and concentration species and their corresponding boundary layers increases with increasing values of the convective heat transfer parameter  $b$ .

- The increasing value of the Eckert number (dissipation parameter)  $Ec$  raises the velocity, temperature and concentration distribution in flow region.
- The skin-friction factor  $C_f$ , the Nusselt number  $Nu$  and the Sherwood number increases due to increase of the suction parameter  $f_w$ . However temperature at the wall  $\theta(0)$  decreases.
- The skin-friction factor  $C_f$  increases while the Nusselt number  $Nu$ , the wall temperature  $\theta(0)$  and the Sherwood number  $Sh$  increases with the increasing values of  $Ec$ .

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