Some New Balanced Designs for Two-component Competition Experiments on Square Lattice

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Abstract: To achieve the goal of maximum test ratio is of vital importance in competition designs. Therefore, in this study, two new methods are introduced for vertical buildability of balanced designs resulting into four new designs. These are in addition to the existing 28 designs producing maximum test ratio and straightforward planting keys in binary form.

Key words: Variety Competition • Square lattice • Balanced Designs • Elementary Array • Complementary Array • Buildability.

INTRODUCTION

Different varieties when sown together may show an increase or decrease in yield depending upon environmental conditions like weather, plant density, mineral resources etc. Hutchings [1] and Crawley and May [2] are of the opinion that in no area of ecology is the role of space more fundamental than in the study of plant communities. Stoll and Weiner [3] stated that basic plant biology suggests that plant-plant interactions are inherently local in nature. Because a plant’s ability to move is restricted (except during dispersal), local conditions are of much greater significance to plants than to animals.

Neighbors influence plants; the type of interaction between plants when grown in mixtures may be termed competition. Altieri [4] define competition in the following terms “Competition occurs when individual plants consume resources which are therefore not available to other individuals. If the lack of resources limits the growth of an individual, then that individual has suffered from competition”.

According to Bulson et al. [5] “components of a mixture use limiting resources more efficiently than pure stands, thus showing resource complementarily”. “Better biological efficiency of mixtures compared with monocultures may result from differences in growing cycle and root architecture” Wilson [6] and Ponce[7], which stimulates the need to construct designs on square lattice in which there are equal numbers of plants of each variety immediately surrounded by exactly 0,1,..., 4 plants of the other variety. Such designs are called balanced designs.

The first real attempt to construct such designs on a square lattice was made by Zafar-Yab [8]. Later Veever and Zafar-Yab [9] published some self-build-able balanced designs. They realized that for building larger designs a balanced array more likely to have self-building property if it is symmetric about its major axis. Therefore, in the search for self-building arrays they confined their attention to symmetric arrays in complementary halves. In pursuit of constructing balanced designs of large size they investigated three-row 32-hills elementary arrays such that the first and the third rows of an array consist of 10 hills each and the middle row of 12 hills. Representing one variety by 0 and the other by 1 their investigation resulted into 4880 unique balanced elementary arrays. Of these balanced arrays only 460 possess the vertically building property. Veever and Zafar-Yab [9] reported 28 design strings each generating a design build-able both horizontally and vertically to any desired size. Langton [10] used these designs and concluded that the mixture performance was better than that of monoculture.

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Table 2.1: The upper half shows the arrays based on the construction Method I and the ones in the lower half on the construction Method II.

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<tr>
<th>N</th>
<th>Elementary Balanced Array</th>
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Re-Examination of Balanced Elementary Arrays:
From Veevers and Zafar-Yab [9] it is not clear how 460 elementary balanced arrays were isolated for consideration. Therefore, the remaining 4420 arrays are reexamined for self-buildability (supposedly) by a different approach than adopted by Veevers and Zafar-Yab [9]. It is found that 4 of these arrays possess the self-building property. Two of these arrays generate basic designs by one method (called Method I) and the other two by 2nd method (called Method II). The arrays so discovered are presented in Table 2.1.

Construction of Balanced Designs: Veevers and Zafar-Yab [9] presented balanced elementary arrays of 32 hills on a square lattice, which are symmetric about their major axis. A design string of ten symbols together with two rules for building a basic design is all that is required. The 28 design strings presented therein constitute (apparently) an exhaustive set which give rise to designs of larger size by extending in both horizontal and vertical directions.

Construction of balanced designs of desired size based on the newly identified balanced elementary arrays and their planting key consists of the following three steps:

Build a Larger Sized Balanced Array as Follows:

- Take three-rows 32-hills balanced elementary array
- Add two rows at a time such that rows 4 and 5 are

Method I: Rotation of rows 2 and 3 respectively of the elementary balanced array about their minor axis.

Method II: Rotational complement of rows 2 and 3 respectively of the elementary balanced array about their minor axis.

- For each of the above methods further extension is obtained by adding successively the minor axis rotation of the preceding 2 rows.

Complete balanced design of desired size is realized by placing side by side the above built larger sized balanced array and its complement.

Planting key is constructed such that it follows rule 1A of Veevers and Zafar-Yab [9] for the construction of larger sized balanced design.

The construction of larger sized balanced array by the above two Methods is explained with the help of Fig 2.1 and Fig 2.2.

The first three rows in Fig 2.1 represent a balanced elementary array, which is symmetric about its major axis. The symbol B stands for the second row and the symbol A both for the first and the third rows. The symbols AR and BR represent respectively the minor axis rotation (mirror image) of rows represented by the symbols A and B. The elementary array and minor axis rotation of its last two rows are stacked as row 4 and row 5 respectively. It can be verified that the array thus constructed is balanced for five rows.

Furthermore, to obtain the balanced array for seven such rows, minor axis rotation of row 4 and row 5 produce row 6 and row 7 respectively. There are two arrays, which possess the above property and are presented in the upper half of Table 2.1. Their planting keys are

1) 0 A 1 A 0 1 A 0 1 A 1 1 A 1 1 1 A 1 1 1 A 1 A 0 1 A 0 A 1 A 0 1 A 0 A 1 A 0 1 A 0

Here 0 and 1 denote the columns for varieties 0 and 1, respectively. A0 and A1 denote the columns on which two varieties are strictly alternating, starting with 0 and 1, respectively. Basic balanced design is obtained from these planting keys after placing side
Fig. 2.1: First elongated rectangle is the balanced elementary array; subsequent successive two rows are minor axis rotation of the preceding two rows.

Fig. 2.2: First elongated rectangle is the elementary array; subsequent successive two rows are complement of minor axis rotation of the preceding two rows.

by side the elementary array and its complement (i.e. by interchanging 0’s and 1’s irrespective of the fact weather these appear alone or with the letter A).

The construction procedure by Method II is explained by Fig 2.2 and proceeds exactly as in Fig 2.1 except that the symbols ARC and BRC stand for minor axis rotational compliment of the respective rows represented by symbols A and B. The planting keys for the designs generated by the Method II are:

1) A 1 1 A 0 1 A 0 A 1 0 A 0 0
2) A 1 1 1 A 1 1 A 1 0 A 1 0 A 1

REFERENCES