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Entropy Generation Analysis of a Variable-Property Fluid Convection in a Helical Tube

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Abstract: In the present study, fully developed laminar temperature-dependent-viscosity flow and heat transfer in a helically coiled tube with uniform heat flux at tube wall is investigated analytically based on minimal entropy generation principle. In this work, temperature dependence of viscosity is taken into account and the results corresponding to the temperature-dependent and constant viscosity cases are compared. Expressions involving relevant variables for entropy generation rate contributed to heat transfer and friction loss and total entropy generation rate are derived. Influence of viscosity on the variation of entropy generation rate with coil passage, inlet temperature and tube curvature has been studied. Investigating the results, it is revealed that constant viscosity assumption may lead to a considerable deviation in entropy generation results from those of the temperature-dependent viscosity case. Also, it is found that there are optimum values for curvature and fluid inlet temperature and these values are certain functions of viscosity.

Key words: Coiled tube · Entropy generation · Variable viscosity · Thermodynamic second law · Irreversibility

INTRODUCTION

Laminar convective heat transfer in helically coiled tubes is commonly encountered in the design of compact heat exchangers, chemical reactors, condensers and many other engineering applications. The secondary flow motion induced by the curvature effect and the resultant centrifugal force makes both heat transfer coefficient and friction factor greater than those in a straight pipe. Also, torsion of helically coiled tubes causes more complication in temperature and velocity fields. Because of practical importance, abundant studies have been done on the heat transfer and pressure drop in these pipes [1-3].

Heat transfer and pressure drop in a heat exchanger are strongly dependent upon the type of fluid flowing through the heat exchanger. Knowing fluid properties and their dependence upon temperature is essential for a heat exchanger analysis. The temperature variation along the flow passage influences fluid properties which will be led to friction factor and convective heat transfer coefficient changes. Viscosity of a fluid is one of the properties which are most sensitive to temperature. In most cases, viscosity becomes the only property which may have considerable effect on heat transfer and temperature variation and thus temperature dependence of other thermo-physical properties is often negligible. Heat flux and temperature differences in many thermo-fluid systems are large and the viscosity of the fluid varies significantly as a function of temperature. For example, when the temperature is increased from 20 to 80°C, the viscosity of engine oil decreases 24 times [4].

Improving the heat transfer performance is a chief task in heat exchanger designing. Due to the fact that the heat transfer enhancement is always accompanied by more cost on friction loss, the optimal trade-off has become the critical challenge for design work. From thermodynamic second law viewpoint, the optimal design can be achieved by minimization of total generated entropy due to heat transfer and friction loss. Bejan [5] has described the systematic methodology of computing entropy generation through heat and fluid flow in heat exchangers. Based on the entropy generation minimization principle, considerable optimal designs of thermal systems have been proposed. For example, Nag and Mukherjee [6] studied the thermodynamic optimization of convective heat transfer through a duct with constant wall temperature. In their study, they investigated the variation of entropy generation with the difference of bulk flow inlet

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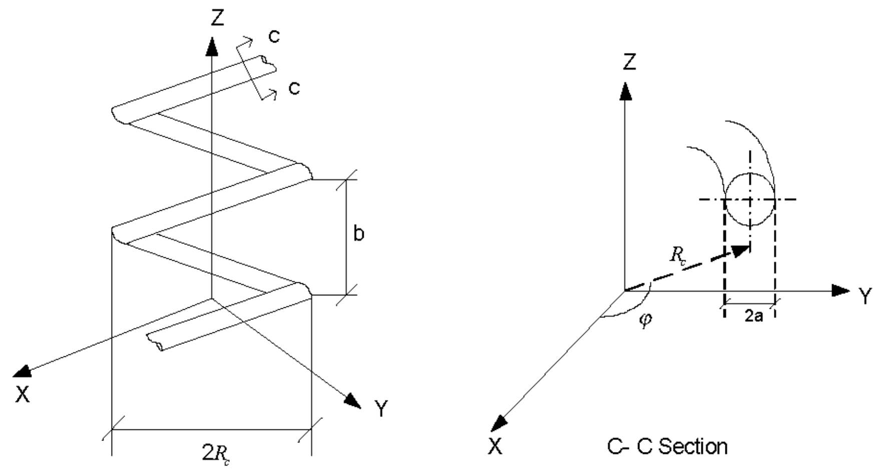

Fig. 1: Schematic view of a typical coiled tube

and wall temperatures. Nag and Kumar [7] studied the second law optimization for convective heat transfer through a duct with constant heat flux. Sahin [8] performed the irreversibility analysis in various duct geometries with constant wall heat flux in laminar flow. Sahin [4] investigated the entropy generation and pumping power in a turbulent flow through a smooth duct subjected to constant wall temperature. He considered temperature dependent viscosity and compared the results of entropy generation from temperature dependent viscosity and constant viscosity cases. Khaliq [9] studied the effect of temperature on the entropy generation due to heat transfer and pressure drop in an isothermal wall duct laminar viscous flow. Ko and Ting [10] analyzed laminar flow heat transfer inside a helically coiled tube considering constant viscosity. Ko [11] found the optimal curvature ratio for fully developed constant property laminar forced convection flow in a helically coiled tube with uniform heat flux. Mass flow rate optimization for steady, laminar, constant property, fully developed, forced convection in a helical coiled tube with fixed size and constant wall heat flux by the thermodynamic second law was done by Ko [12]. Goodarzian and Shobi [13] studied the effect of design parameters on the exergetic opration of gas-turbine power plants.

However, thermodynamic optimization of temperature dependent viscosity flow inside a helically coiled tube has not been found in the open literature. Therefore, the main target of the present article can be summarized as, the investigation of temperature-dependency of fluid properties effect on the entropy generation rates due to heat transfer and friction loss.

Geometry of Helically Coiled Tubes: A typical coiled tube has been shown in Fig. 1. In this figure, a is inner radius of the tube, R_c is curvature radius of the coil and b is the coil pitch. The curvature ratio, δ, is defined as the coil-to-tube radius ratio, a/R_c and the non-dimensional pitch, λ, is defined as $b/2\pi R_c$. The other four important dimensionless parameters namely, Reynolds number (Re), Nusselt number (Nu), Dean number (De) and Helical number (He) are defined as follow.

$$Re = 2\rho Va/\mu,\ Nu = 2\bar{h}a/k,$$
$$De = Re\left(a/R_c\right)^{0.5},\ He = De\Big/\left(1+\lambda^2\right)^{0.5}$$

Where, V and $\bar{h}$ are average velocity and convective heat transfer coefficient respectively.

Viscosity Dependence upon Temperature: It is evident from experiments that the viscosity of liquids varies considerably with temperature. Around room temperature (293K), for instance, 1% change in temperature produces 7% change in the viscosity of water and approximately 26% change in the viscosity of glycerol [14].

In the present study, a highly temperature-dependent-viscosity fluid, glycerol, has been considered for investigation and for evaluation of its viscosity at different temperatures, the empirical correlation given by Sherman [13] is used as

$$\mu(T) = \mu_0\left(\frac{T}{T_0}\right)^n exp\left[B\left(\frac{1}{T}-\frac{1}{T_0}\right)\right] \tag{1}$$

Where, n and B are fluid-dependent constant parameters. For glycerol, n = 52.4 and B = 23100.

Besides equation 1, a constant viscosity model is also included in this work to depict the significance of the variation of viscosity on the results.

Temperature Variation along the Coil: In this paper, we focus on the steady fully-developed laminar convection in the helical coils with constant wall heat flux. An incompressible viscous fluid with mass flow rate $\dot{m}$ and inlet temperature T_i, enters the coil of length L. The density, ρ, thermal conductivity, k and specific heat, C_P of the fluid are assumed to be constant within the range of temperatures considered in this study. q', T and T_W are denoted as heat transfer rate per unit coil length, fluid bulk temperature and wall temperature respectively. Taking the coil passage of length dx as the thermodynamic control volume, first law can be expressed as [4].

$$q' dx = \dot{m} C_P dT \tag{2}$$

Integrating this equation along the coil passage from pipe inlet to an arbitrary cross section x, the variation of fluid bulk temperature is obtained as

$$T = T_i + \frac{q'}{\dot{m} C_P} x \tag{3}$$

Defining dimensionless parameter $x^* = x/L$, equation (3) is rewritten as

$$T = T_i + \frac{q' L}{\dot{m} C_P} x^* \tag{4}$$

From energy balance, one can write [4]

$$T_w - T = \frac{q'}{2\pi a \bar{h}} \tag{5}$$

Invoking the definition of Nusselt number ($Nu = 2\bar{h}a/k$) and equation (5), wall temperature variation along the coil will be obtained as Eq. 6:

$$T_w = T + \frac{q'}{\pi k Nu} \tag{6}$$

Entropy Generation: The second law of thermodynamics for an open system in communication with the atmosphere and n additional heat reservoirs can be written in the following general form [5].

$$\dot{S}'_{gen} = \frac{dS}{dt} - \sum_{i=0}^{n} \frac{\dot{Q}_i}{T_i} + \sum_{out} \dot{m}s - \sum_{in} \dot{m}s \geq 0 \tag{7}$$

Assuming the thermodynamic control volume introduced in previous section, this equation can be simplified for steady state flow as

$$\dot{S}'_{gen} = \dot{m}\frac{ds}{dx} - \frac{q'}{T_w} \tag{8}$$

Where, $\dot{S}'_{gen}$ is entropy generation rate per unit coil length. From thermodynamics we have [15].

$$Tds = dh - vdP \tag{9}$$

Substituting ds from equation (9) into equation (8), $\dot{S}'_{gen}$ can be written as:

$$\dot{S}'_{gen} = \frac{q'(T_w - T)}{TT_w} + \frac{\dot{m}}{\rho T}\left(-\frac{dP}{dx}\right) \tag{10}$$

The pressure drop in Eq. (10) is given by Eq. (11)

$$-\frac{dP}{dx} = f\frac{\rho V^2}{4a} \tag{11}$$

The non-dimensional entropy generation number N can be defined as [5]:

$$N_S = \frac{\dot{S}'_{gen}}{q'/T_i} \tag{12}$$

N_s is composed of heat transfer and pressure drop contributions as follow

$$N_s = (N_s)_T + (N_s)P \tag{13}$$

Where $(N_s)_T$ and $(N_s)_P$ will be obtained invoking equations (10)-(12) as follow

$$(N_S)_T = \frac{T_i}{T(1 + k\pi T.Nu/q')} \tag{14}$$

$$(N_S)_P = \left(\frac{\dot{m}^3}{4\pi^2 \rho^2 a^5 q'}\right)\frac{T_i f}{T} \tag{15}$$

Introducing parameters $\xi = k\pi/q'$ and $\eta = \dot{m}^3/4\pi^2\rho^2 a^5 q'$, equations (14) and (15) can be written

$$(N_S)_T = \frac{T_i}{T(1 + \xi T.Nu)} \tag{16}$$

$$(N_s)_P = \eta f T_l/T \tag{17}$$

Paoletti *et al.* [16] proposed an irreversibility distribution parameter, Bejan number (Be), defined as.

$$Be = (N_s)_T/N_S \tag{18}$$

It is obvious that this parameter ranges from 0 to 1 and when $Be = 0.5$, the entropy generation due to heat transfer and fluid friction are equal.

For analyzing entropy generation, variation of Nu and f with various parameters of flow and geometry of tube shall be known. For this purpose, the proposed correlations by Manlapaz and Churchill [3] are used as follow.

$$f = \frac{16}{Re}\left[\left(1-0.18\Big/\left[1+\left(35/He\right)^2\right]^{1/2}\right)^m + \left(1+\delta/3\right)^2 He/88.33\right]^{1/2} \tag{19}$$

$$Nu = \left[\left(\frac{48}{11}+\frac{51/11}{\left(1+1342/Pr\,He^2\right)^2}\right)^3 + 1.816\left(\frac{He}{1+1.15/Pr}\right)^{3/2}\right]^{1/3} \tag{20}$$

Where, values of m are 2, 1 and 0 for De,<20, 20<De<40 and De>40 respectively.

RESULTS AND DISCUSSION

Although the primary concern in heat exchanger design is the amount of heat transfer, however the entropy generation may be large which would lead to low second law efficiency. Hence, entropy generation rate per unit amount of heat transfer rate could be a suitable means to evaluate a heat exchanger from second law point of view.

To investigate the effect of viscosity variation on the entropy generation in a coiled tube, a highly temperature-dependent-viscosity fluid, glycerol, has been chosen. Two models have been focused in this study: constant viscosity and temperature-dependent-viscosity models. In the former case, all the properties of fluid are evaluated at the inlet bulk temperature. All the results have been provided and compared for both models to illustrate the influence of viscosity variation on the entropy generation.

It should be noted that flow regime was always assured to be in laminar region by checking the Reynolds number be less than the critical value given by Srinivasan [17], as follows.

$$Re_{cr} = 2100(1+12\delta^{1/2}) \tag{21}$$

As entropy generation rate is a function of several variables, a baseline case has been considered as $\xi = 4.15\times10^{-4}$, $\eta = 0.783$ and $\lambda = 0.05$.

Since friction factor and Nusselt number are closely related with entropy generation rate in flow fields, their variation along the coil passage are presented in Figs.2 and 3. As it can be seen, assuming constant viscosity will result in a lower Nusselt number and a higher friction factor. The former can be explained as increasing the fluid temperature causes viscosity to decrease which in turn ascends the Reynolds number and consequently Nusselt number; and the latter is obvious as a result of lower viscosity.

In order to understand the detail contribution of entropy generation rate due to heat transfer and frictional irreversibility, investigation of the effects of viscosity variation on $(N_S)_T$ and $(N_S)_P$ have been performed for baseline case. Figs.4 and 5 show the variation of $(N_S)_T$ and $(N_S)_P$ along the coil passage for constant and variable viscosity models. From these figures, it is evident that assuming constant viscosity will overestimate the values of $(N_S)_T$ and $(N_S)_P$, also getting closer to the end of pipe, the value of this deviation increases. It is also found that, going along the coil passage, both $(N_S)_T$ and $(N_S)_P$ will reduce as a result of higher temperature.

A similar trend can be seen in Fig.6 which illustrates the variation of total entropy generation number N_s along the coil passage. The variation of N_s is similar to $(N_S)_T$ and $(N_S)_P$ for both constant and variable viscosity models.

A distribution parameter which is an appropriate measure in second law analysis is Bejan number, Be, which was defined in Eq. (18). Fig.7 illustrates the variation of this parameter along the coiled tube for temperature dependent viscosity model. Besides, to validate the present study, same graph is presented for constant viscosity model from [12] in this figure. These graphs can be interpreted as, the entropy generation rate contribution due to heat transfer dominates the total entropy generation rate, however going along the coil pass this contribution weakens. Also, constant viscosity assumption will underestimate this contribution.

Figure 8 depicts the variation of $(N_S)_T$ vs x^* for three cases (δ = 0.1, 0.6, 1.1) to illustrate the effect of curvature on $(N_S)_T$. It is revealed from this figure that, increasing δ will reduce $(N_S)_T$ for both constant and variable viscosity models. However, this influence becomes smaller in the larger curvatures. The effect of curvature on the entropy generation rate due to friction loss is different, Fig. 9.

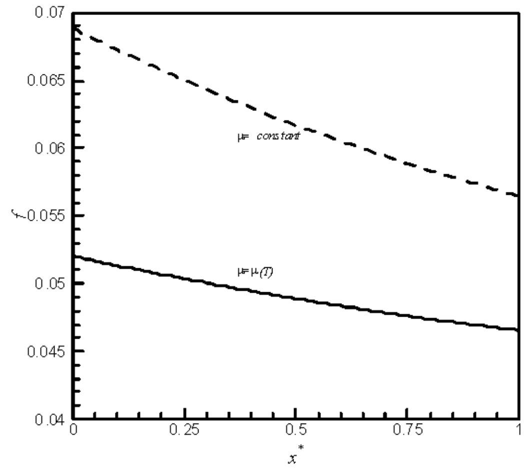

Fig. 1: Variation of friction factor along the coil passage for constant and variable viscosity models ($\delta = 0.1$, $T_i = 373K$)

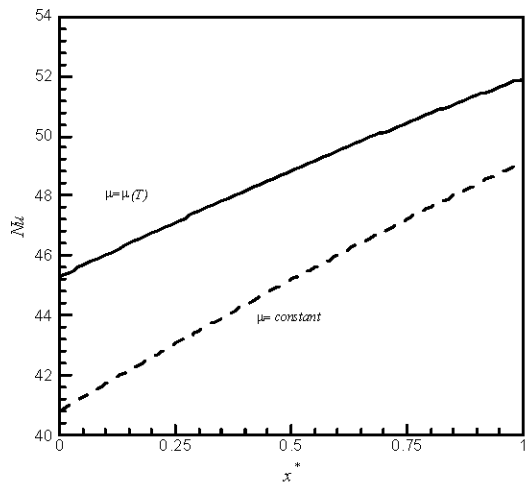

Fig. 2: Variation of Nusselt number along the coil passage for constant and variable viscosity models ($\delta = 0.1$, $T_i = 373K$)

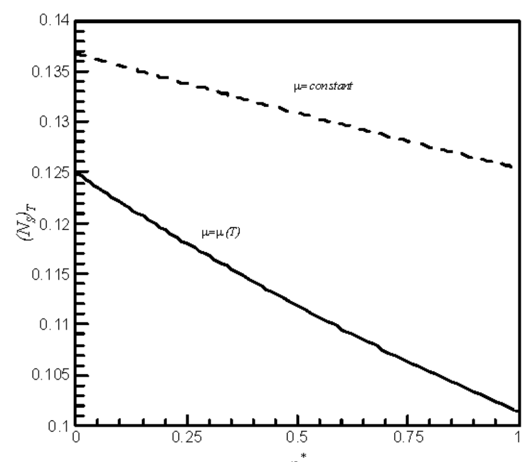

Fig. 3: Variation of heat transfer entropy generation number along the coil passage for constant and variable viscosity models ($\delta = 0.1$, $T_i = 373K$)

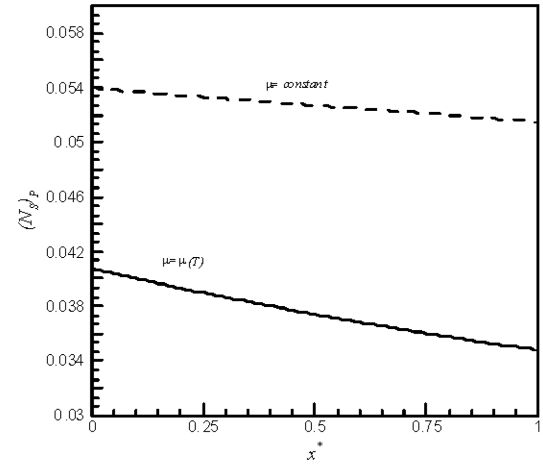

Fig. 4: Variation of friction loss entropy generation number along the coil passage for constant and variable viscosity models ($\delta = 0.1$, $T_i = 373K$)

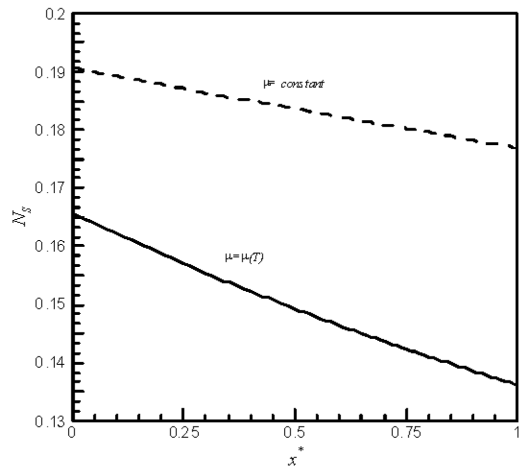

Fig. 5: Variation of total entropy generation number along the coil passage for constant and variable viscosity models ($\delta = 0.1$, $T_i = 373K$)

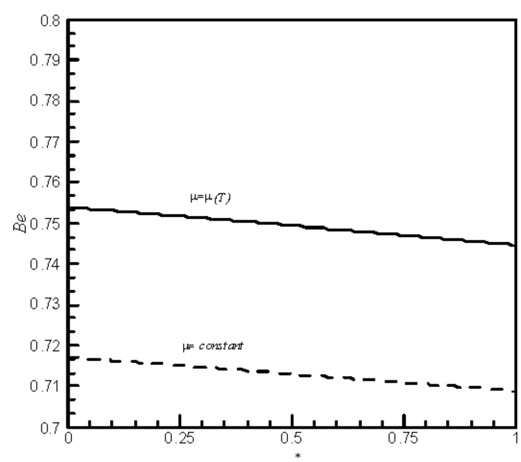

Fig. 6: Variation of Bejan number along the coil passage for variable viscosity model (present study) & constant viscosity model from [12] ($\delta = 0.1$, $T_i = 373K$)

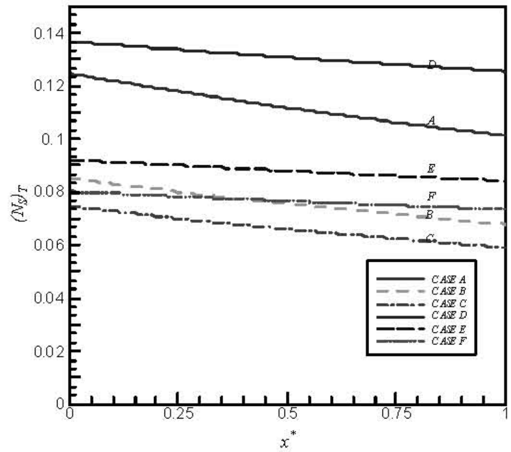

Fig. 7: Variation of heat transfer entropy generation number along the coil passage for variable viscosity model, cases A ($\delta = 0.1$), B ($\delta = 0.6$), & C ($\delta = 1.1$); and constant viscosity model, cases D ($\delta = 0.1$), E ($\delta = 0.6$), & F ($\delta = 1.1$)

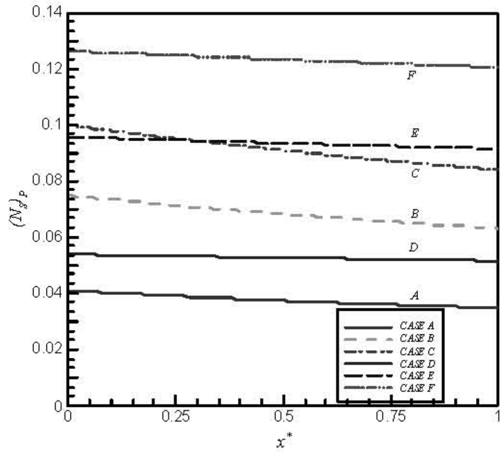

Fig. 8: Variation of friction loss entropy generation number along the coil passage for variable viscosity model, cases A ($\delta = 0.1$), B ($\delta = 0.6$), & C ($\delta = 1.1$); and constant viscosity model, cases D ($\delta = 0.1$), E ($\delta = 0.6$), & F ($\delta = 1.1$)

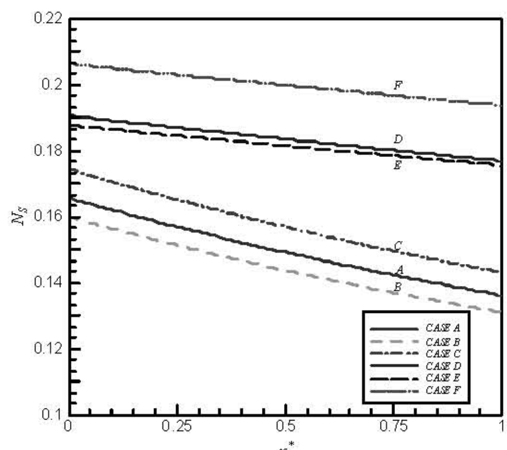

Fig. 9: Variation of total entropy generation number along the coil passage for variable viscosity model, cases A ($\delta = 0.1$), B ($\delta = 0.6$), & C ($\delta = 1.1$); and constant viscosity model, cases D ($\delta = 0.1$), E ($\delta = 0.6$), & F ($\delta = 1.1$)

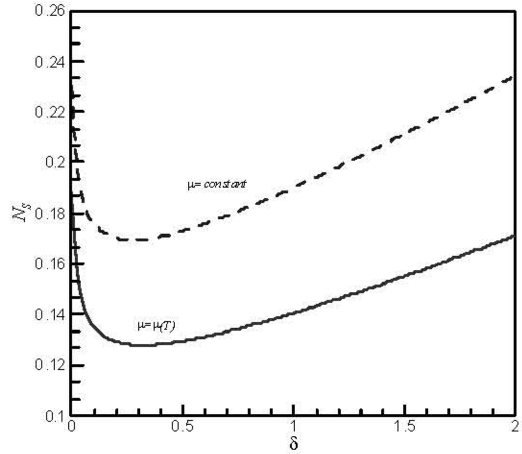

Fig. 10: Variation of total entropy generation number with curvature ratio for constant and variable viscosity models ($x^* = 1$, $T_i = 373K$)

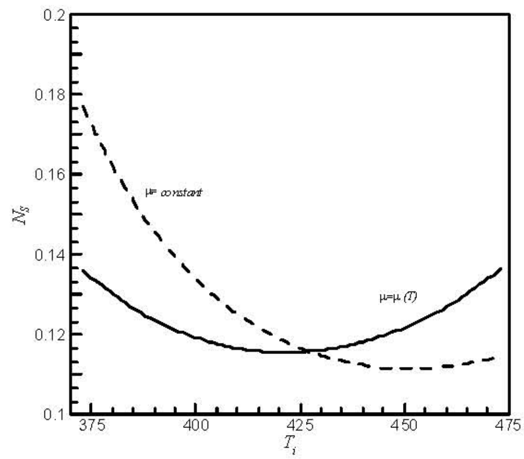

Fig. 11: Variation of total entropy generation number with fluid inlet temperature for constant and variable viscosity models ($\delta = 0.1$, $x^* = 1$)

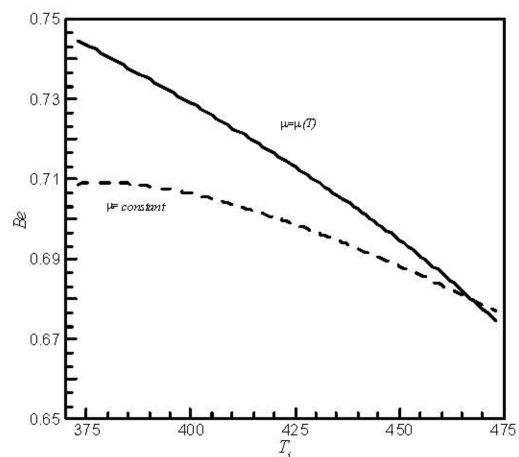

Fig. 12: Variation of Bejan number with fluid inlet temperature for variable viscosity model (present study) & constant viscosity model from Ko (2006b) ($\delta = 0.1$, $x^* = 1$)

Higher curvatures result in more values of $(N_S)_P$, though this effect is smaller in the larger curvature similar to Fig. 8. Fig.10 represents the variation of total entropy generation number with x^* for three aforementioned curvature values. The remarkable point is that increasing δ from 0.1 to 0.6 will decrease N_S, but growing δ from 0.6 up to 1.1 will escalate N_S to a value beyond the one corresponding to primary case of δ = 0.1 which implies that an optimum value for curvature exists. This phenomenon can be explained as the increase of δ causes both f and Nu to increase. The increase of f raises fluid friction and irreversibility, whereas the increase of Nu enhances the heat transfer performance and causes heat transfer irreversibility to reduce. From Fig.10, it is found that these variations have same trend for both constant and variable viscosity models, however the magnitude of N_S in various models are different. It is also noted that inlet bulk temperature of fluid in Figures 8-10 is considered as 373K.

For more investigation on the effect of viscosity on δ_{opt}, variation of N_S with δ has been presented in Fig. 11. From this figure, it is evident that the optimum curvature value is a certain function of viscosity.

For study the influence of inlet temperature of fluid on entropy generation, Fig. 12 has been furnished from which it is lucid that there is an optimum inlet temperature which minimize total entropy generation rate and also this value is a function of viscosity, i.e. considering constant viscosity will cause a large deviation in optimum inlet temperature.

Figure 13 depicts the variation of Bejan number with inlet temperature of fluid for both temperature dependent viscosity model and constant viscosity model, while the former is from the present study and the latter is from [12]. It is seen that the entropy generation rate contribution due to heat transfer becomes smaller as inlet temperature increases. Also, constant viscosity assumption will underestimate this value.

CONCLUSIONS

An analytical study has been carried out to investigate the influence of viscosity variation on the entropy generation rate for a laminar viscous flow in a helically coiled tube subjected to constant wall heat flux. A highly temperature-dependent viscosity fluid, glycerol, has been selected for investigation. Influence of viscosity on the variation of entropy generation rate with coil passage, inlet temperature and tube curvature has been studied. It was found that the constant viscosity assumption will overestimate the entropy generation rate due to both heat transfer and friction loss. It was also revealed that there are optimum values for curvature and fluid inlet temperature and these values are certain functions of viscosity.

Nomenclature

a	Inner radius of tube, m
b	Coil pitch, m
B	Kluid dependent parameter
Be	Bejan number, $=(N_S)_T/N_S$
C_P	Specific heat capacity, J/kg-K
De	Dean number, = $\mathrm{Re}(a/R_c)^{0.5}$
f	Friction factor
h	Specific enthalpy, J/kg
$\bar{h}$	Averaged convective heat transfer, W/m^2-K
He	Helical number, = $Dn/(1=\lambda^2)^{0.5}$
k	Thermal conductivity, W/m^2-K
L	Tube length, m
m	Exponent in equation (19)
$\dot{m}$	Mass flow rate, kg/s
n	Fluid dependent parameter
N_S	Entropy generation number
$(N_S)_P$	Entropy generation number due to friction loss
$(N_S)_T$	Entropy generation number due to heat transfer
Nu	Nusselt number, = $2\bar{h}a/k$
P	Pressure, Pa
Pr	Prandtl number, = $\mu C_P/k$
q'	Heat transfer per unit coil length, W/m
R_c	Curvature radius, m
Re	Reynolds number, = $2\rho Va/\mu$
s	specific entropy, J/kg-K
$\dot{S}'_{gen}$	entropy generation rate per unit length, W/m-K
T	Fluid bulk temperature, K
V	Fluid average velocity, m/s
x	Spatial position, m
x^*	Non-dimensional spatial position, = x/L
Greek letters	
δ	Curvature ratio, = a/R_c
η	Dimensionless parameter, = $\dot{m}^3/4\pi^2\rho^2a^5q'$
λ	Non-dimensional pitch, = $b/2\pi R_c$
μ	Viscosity, kg/m-s
ρ	Density, kg/m^3
ξ	dimensionless parameter, = $k\pi/q'$
Subscripts	
0	Reference condition
i	Inlet condition
cr	Critical value
opt.	optimum value
w	Wall condition

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