

Fractional Derivative of a Product of Some Special Functions

Ahmad Ali Al-Jarrah

Department of Mathematics and Statistics, Mu'tah University, P.O. Box: 25, Mu'tah, Al-Karak, Jordan

Abstract: Our aim in the present paper is to study certain fractional calculus operator pertaining to many families of expansion involving two some product of special function with H-function of several complex variables. The main results of our paper are unified in nature and capable of yielding a very large number of corresponding results (new and known) involving simpler special functions and polynomials (of one or more variables) as special cases of our formulae. The present study unifies and extends a number of composition formulae lying scattered in the literature hitherto.

Key words: H- function • G- function • Gaus hypergeometric function

INTRODUCTION

The fractional derivatives of certain product of special functions, which are Gauss hypergeometric function [1], generalized Lauricella functions [2] a general class of multivariable polynomials [3]and the multivariable H-function is established. The generalized Leibnitz rule has been used to derive the main result pertaining to the product of these functions. The multivariable H-function which was introduced and studied systematically in a series of papers by [4, 5] is an extension of multivariable G-function. The multivariable H-function includes Fox's H-function and Meijer's G-function of one and two variables, the generalized Lauricella function of [2] Appell functions, the Whittaker functions etc.. Therefore the results derived in this paper are of general character from which several known and new results can be deduced.

Definition and Notations: Following [6, 7, 8] we define the fractional derivative of a function $f(x)$ of complex order μ (or alternatively, a $-\mu^{\text{th}}$ order fractional integral of $f(x)$) by

$${}_a D_x^\mu \{f(x)\} = \begin{cases} \frac{1}{\Gamma(-\mu)} \int_a^x (x-y)^{-\mu-1} f(y) dy, & \text{Re}(\mu) < 0, \\ \frac{d^m}{dx^m} {}_a D_x^{\mu-m} \{f(x)\}, & 0 \leq \text{Re}(\mu) < m, \end{cases} \quad (1)$$

Where m is a positive integer.

The well-known binomial expansion

$$(t-\xi)^\lambda = (-\xi)^\lambda \sum_{m=0}^{\infty} \binom{\lambda}{m} \left(-\frac{t}{\xi}\right)^m, \quad \left| -\frac{t}{\xi} \right| < 0. \quad (2)$$

The special case of fractional derivative [6] is

$$D_t^\mu \{t^\lambda\} = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)} t^{\lambda-\mu}, \quad \text{Re}(\lambda) > -1. \quad (3)$$

The following known result of [7]

If $\mu \geq 0$, $0 < t < 1$, $\text{Re}(1+p) > 0$, $\text{Re}(q) > -1$, $\mu_i > 0$ and $\Delta_i > 0$, or $\Delta_i = 0$ and $|z_i| < \sigma_i$, $i = 1, 2, \dots, r$ then

$$x^\mu F \begin{bmatrix} z_1 x^{\mu_1} \\ \vdots \\ z_r x^{\mu_r} \end{bmatrix} = \sum_{M=0}^{\infty} \frac{(1+p+q+2M)(-\mu)_M (1+p)_\mu}{M! (1+p+q+M)_{\mu+1}} F_M [z_1, \dots, z_r] \cdot {}_2F_1 \begin{bmatrix} -M, 1+p+q+M ; \\ 1+p ; \end{bmatrix} x, \quad (4)$$

Where

$F_M [z_1, \dots, z_r]$

$$= F \begin{bmatrix} E+2 : U, \dots, U^{(r)} \\ P+2 : V, \dots, V^{(r)} \end{bmatrix} \begin{bmatrix} [(e) : \eta^1, \dots, \eta^{(r)}], [1+p+\mu, \mu_1, \dots, \mu_r], [\mu+1, \mu_1, \dots, \mu_r], \\ [(g) : \xi^1, \dots, \xi^{(r)}], [2+p+q+M+\mu, \mu_1, \dots, \mu_r], [\mu-M+1, \mu_1, \dots, \mu_r], \\ [(w) : x^1, \dots, x^{(r)}], [v^{(r)} : t^{(r)}], z_1, \dots, z_r \end{bmatrix}, \quad (5)$$

Where $M \geq 0$.

For the sake of brevity, we use the following notation

$$F_{\substack{E:U_1,\dots,U_r \\ P:V_1,\dots,V_r}} \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_r \end{bmatrix} = F \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_r \end{bmatrix} \quad (6)$$

The Gauss hypergeometric function [1] is defined as

$${}_2F_1 \left(\begin{matrix} \rho, \sigma \\ \in \end{matrix}; x \right) = \sum_{\lambda=0}^{\infty} \frac{(\rho)_\lambda (\sigma)_\lambda}{\lambda! (\in)_\lambda} x^\lambda, \quad (7)$$

for \in neither zero nor a negative integer and $\operatorname{Re}(\in - \rho - \sigma) > 0$.

The multivariable H-function is defined by [4, 5], see also [9] in the following manner

$$\begin{aligned} H[z_1, \dots, z_r] &= H_{\substack{0, \lambda+3 : (u_1, v_1), \dots, (u_r, v_r) \\ AC : [B'D']_1, \dots, [B'^r D'^r]}} \begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix} \\ &= \frac{1}{(2\pi i)^r} \int_{L_1} \dots \int_{L_r} \psi(\xi_1, \dots, \xi_r) \Phi_1(\xi_1) \dots \Phi(\xi_r) z_1^{\xi_1} \dots z_r^{\xi_r} d\xi_1 \dots d\xi_r, \end{aligned} \quad (8)$$

Where $i = \sqrt{(-1)}$.

The convergence conditions and other details of the H-function of several complex variables $H[z_1, \dots, z_r]$ are given by [9]. For general class of multivariable polynomials [3]

$$\begin{aligned} S_N^{M_1, \dots, M_s} [w_1, \dots, w_s] &= \sum_{k_1, \dots, k_s=0}^{M_1 k_1 + \dots + M_s k_s \leq N} (-N)_{M_1 k_1 + \dots + M_s k_s} B(N; k_1, \dots, k_s) \frac{(w_1)^{k_1}}{k_1!} \dots \frac{(w_s)^{k_s}}{k_s!}, \end{aligned} \quad (9)$$

Where M_1, \dots, M_s are arbitrary positive integers and the coefficients $B(N; k_1, \dots, k_s)$. $(N; k_i \geq 0, i = 1, \dots, s)$ are arbitrary constants, real or complex.

Main Result: The fractional derivative to be established here is

$$D_t^\mu \left\{ (t-x)^\sigma \eta^\sigma (y-t)^{\sigma+\rho} {}_2F_1 \left(\begin{matrix} u, v \\ w \end{matrix}; R(t-x)^{A_1} (y-t)^{B_1} \right) \right\}$$

$$\begin{aligned} & F \begin{bmatrix} \tau_1 \eta(y-t)^\sigma \\ \vdots \\ \tau_r \eta(y-t)^\sigma \end{bmatrix} S_N^{M_1, \dots, M_s} \begin{bmatrix} (t-x)^{a_1} (y-t)^{b_1} \\ \vdots \\ (t-x)^{a_s} (y-t)^{b_s} \end{bmatrix} H \begin{bmatrix} z_1 \{t(t-x)\}^{\sigma_1} \{t(y-t)\}^{\rho_1} \\ \vdots \\ z_r \{t(t-x)\}^{\sigma_r} \{t(y-t)\}^{\rho_r} \end{bmatrix} \Bigg) \\ &= \sum_{K, K_1, M, \alpha, \beta=0}^{\infty} \sum_{k_1, \dots, k_s=0}^{M_1 k_1 + \dots + M_s k_s \leq N} \frac{(-N)_{M_1 k_1 + \dots + M_s k_s}}{k_1! \dots k_s!} B(N; k_1, \dots, k_s) \Delta \\ & H_{\substack{0, \lambda+3 : (u_1, v_1), \dots, (u_r, v_r) \\ A+3, C+3 : [B'D']_1, \dots, [B'^r D'^r]}} \begin{bmatrix} z_1 (-x)^{\sigma_1} y^{\rho_1} t^{\rho_1 + \sigma_1} \\ \vdots \\ z_r (-x)^{\sigma_r} y^{\rho_r} t^{\rho_r + \sigma_r} \end{bmatrix} \\ & [(\phi : \psi, \dots, \psi^{(r)})_1, \dots, (\phi : \psi, \dots, \psi^{(r)})_r] [\alpha - \sigma - A_1 K_1 + \sum_{i=1}^s a_i k_i; \sigma_1, \dots, \sigma_r] [\beta - \rho - B_1 K_1 - \sum_{i=1}^s b_i k_i; \rho_1, \dots, \rho_r] \\ & [(\phi : \psi, \dots, \psi^{(r)})_1, \dots, (\phi : \psi, \dots, \psi^{(r)})_r] [\alpha - \sigma - A_1 K_1 + \sum_{i=1}^s a_i k_i; \sigma_1, \dots, \sigma_r] [\beta - \rho - B_1 K_1 - \sum_{i=1}^s b_i k_i; \rho_1, \dots, \rho_r] \\ & [(\phi : \psi, \dots, \psi^{(r)})_1, \dots, (\phi : \psi, \dots, \psi^{(r)})_r] [\alpha - \sigma - A_1 K_1 + \sum_{i=1}^s a_i k_i; \sigma_1, \dots, \sigma_r] [\beta - \rho - B_1 K_1 - \sum_{i=1}^s b_i k_i; \rho_1, \dots, \rho_r] \\ & [(\phi : \psi, \dots, \psi^{(r)})_1, \dots, (\phi : \psi, \dots, \psi^{(r)})_r] [\alpha - \sigma - A_1 K_1 + \sum_{i=1}^s a_i k_i; \sigma_1, \dots, \sigma_r] [\beta - \rho - B_1 K_1 - \sum_{i=1}^s b_i k_i; \rho_1, \dots, \rho_r] \end{aligned} \quad (10)$$

Where

$$\Delta = (-1)^{\rho+K+B_1 K_1} \frac{(1+p+q+2M)(-\sigma)_M (1+p)_\sigma}{M! (1+p+q+M)_{\sigma+1}}$$

$$F_M[\tau_1, \dots, \tau_r] \frac{(-M)_K (1+p+q+M)_K}{(1+p)_K K! \Gamma(\alpha+1) \Gamma(\beta+1)} \frac{(u)_{K_1} (v)_{K_1} R^{K_1}}{K_1! (w)_{K_1}} \eta^K$$

$$(-x)^{\sigma - \alpha + A_1 K_1} (-y)^{\rho + K + B_1 K_1} t^{\alpha + \beta - \mu}$$

$$\text{and } \operatorname{Re}(\sigma) + \sum_{i=1}^r \sigma_i \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) > -1, \operatorname{Re}(\rho) + \sum_{i=1}^r \rho_i \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) > -1,$$

$\rho_i > 0, \sigma_i > 0 (i = 1, \dots, r)$, M_1, \dots, M_s are arbitrary positive integers and the coefficients $B(N; k_1, \dots, k_s)$, $(N; k_i \geq 0, i = 1, \dots, s)$ are arbitrary constants, real or complex. ω is neither zero nor a negative integer and $\operatorname{Re}(\omega - u - v) > 0$.

Proof: To prove (10), replacing the Gauss hypergeometric function, Lauricella function, general class of polynomials and the multivariable H-function by (7), (5), (9) and (8) respectively and collecting the powers of $(t-x)$ and $(y-t)$. Finally, making use of the result (2) and (3), we arrive at (10).

Interesting Special Cases:

For $\lambda = A = C = 0$, the multivariable H-function split into product of Fox's H-function and consequently there holds the following result

$$\begin{aligned}
 & D_t^{\mu} \left\{ (t-x)^{\sigma} \eta^{\sigma} (y-t)^{\sigma+\rho} {}_2F_1 \left(\begin{matrix} u, v \\ w \end{matrix}; R (t-x)^{A_1} (y-t)^{B_1} \right) \right. \\
 & F \left[\begin{matrix} \tau_1 \{\eta(y-t)\}^{\sigma_1} \\ \vdots \\ \tau_r \{\eta(y-t)\}^{\sigma_r} \end{matrix} \right] S_N^{M_1, \dots, M_s} \left[\begin{matrix} (t-x)^{a_1} (y-t)^{b_1} \\ \vdots \\ (t-x)^{a_s} (y-t)^{b_s} \end{matrix} \right] \\
 & \left. \prod_{i=1}^r H_{B^{(i)}, D^{(i)}}^{u^{(i)}, v^{(i)}} \left[z_i \{t(t-x)\}^{\sigma_i} \{t(y-t)\}^{\rho_i} \left| \begin{matrix} (b^{(i)}, (\phi^{(i)}) \\ (d^{(i)}, \delta^{(i)}) \end{matrix} \right. \right] \right] \\
 & = \sum_{K, K_1, M, \alpha, \beta=0}^{\infty} \sum_{k_1, \dots, k_s=0}^{M_k k_1 + \dots + M_s k_s \leq N} \frac{(-N)_{M_k k_1 + \dots + M_s k_s}}{k_1! \dots k_s!} B(N; k_1, \dots, k_s) \Delta \\
 & H_{3,3: [B', D'] \dots; [B^{(r)}, D^{(r)}]}^{0,3: (u', v') \dots; (u^{(r)}, v^{(r)})} \left[\begin{matrix} z_1 (-x)^{\sigma_1} y^{\rho_1 t} t^{\alpha+1} \\ \vdots \\ z_r (-x)^{\sigma_r} y^{\rho_r t} t^{\alpha+r} \end{matrix} \right] \\
 & [-\sigma - A_1 K_1 + \sum_{i=1}^s a_i k_i; \sigma_1, \dots, \sigma_r], [-\rho - K - B_1 K_1 + \sum_{i=1}^s b_i k_i; \rho_1, \dots, \rho_r], [1 - \alpha - \beta; \sigma_1 + \rho_1, \dots, \sigma_r + \rho_r] \\
 & [\alpha - \sigma - A_1 K_1 - \sum_{i=1}^s a_i k_i; \sigma_1, \dots, \sigma_r], [\beta - \rho - K - B_1 K_1 - \sum_{i=1}^s b_i k_i; \rho_1, \dots, \rho_r], \\
 & [\phi: \phi], \dots, [(\phi^{(r)}): \phi^{(r)}], \\
 & [\mu - \alpha - \beta; \sigma_1 + \rho_1, \dots, \sigma_r + \rho_r], [(\delta): \delta], \dots, [(\delta^{(r)}): \delta^{(r)}], \\
 & (11)
 \end{aligned}$$

valid under the conditions surrounding (10).

When ϕ_i (i = 1, ..., r) in (10), we find the following

$$\begin{aligned}
 & D_t^{\mu} \left\{ (t-x)^{\sigma} \eta^{\sigma} (y-t)^{\sigma+\rho} {}_2F_1 \left(\begin{matrix} u, v \\ w \end{matrix}; R (t-x)^{A_1} (y-t)^{B_1} \right) \right. \\
 & F \left[\begin{matrix} \tau_1 \{\eta(y-t)\}^{\sigma_1} \\ \vdots \\ \tau_r \{\eta(y-t)\}^{\sigma_r} \end{matrix} \right] S_N^{M_1, \dots, M_s} \left[\begin{matrix} (t-x)^{a_1} (y-t)^{b_1} \\ \vdots \\ (t-x)^{a_s} (y-t)^{b_s} \end{matrix} \right] \\
 & \left. \cdot \prod_{i=1}^r G_{B^{(i)}, D^{(i)}}^{u^{(i)}, v^{(i)}} \left[z_i \{t(t-x)\}^{\sigma_i} \{t(y-t)\}^{\rho_i} \left| \begin{matrix} (b^{(i)}) \\ (d^{(i)}) \end{matrix} \right. \right] \right]
 \end{aligned}$$

$$\begin{aligned}
 & = \sum_{K, K_1, M, \alpha, \beta=0}^{\infty} \sum_{k_1, \dots, k_s=0}^{M_k k_1 + \dots + M_s k_s \leq N} \frac{(-N)_{M_k k_1 + \dots + M_s k_s}}{k_1! \dots k_s!} B(N; k_1, \dots, k_s) \Delta \\
 & H_{3,3: [B', D'] \dots; [B^{(r)}, D^{(r)}]}^{0,3: (u', v') \dots; (u^{(r)}, v^{(r)})} \left[\begin{matrix} z_1 (-x)^{\sigma_1} y^{\rho_1 t} t^{\alpha+1} \\ \vdots \\ z_r (-x)^{\sigma_r} y^{\rho_r t} t^{\alpha+r} \end{matrix} \right] \\
 & [-\sigma - A_1 K_1 + \sum_{i=1}^s a_i k_i; \sigma_1, \dots, \sigma_r], [-\rho - K - B_1 K_1 + \sum_{i=1}^s b_i k_i; \rho_1, \dots, \rho_r], [1 - \alpha - \beta; \sigma_1 + \rho_1, \dots, \sigma_r + \rho_r] \\
 & [\alpha - \sigma - A_1 K_1 - \sum_{i=1}^s a_i k_i; \sigma_1, \dots, \sigma_r], [\beta - \rho - K - B_1 K_1 - \sum_{i=1}^s b_i k_i; \rho_1, \dots, \rho_r], \\
 & [\phi: \phi], \dots, [(\phi^{(r)}): \phi^{(r)}], \\
 & [\mu - \alpha - \beta; \sigma_1 + \rho_1, \dots, \sigma_r + \rho_r], [(\delta): \delta], \dots, [(\delta^{(r)}): \delta^{(r)}], \\
 & (12)
 \end{aligned}$$

Which holds true under the same conditions as given in (10).

Taking $B(N; k_1, \dots, k_s) = I(\theta)$ in (9) where

$$I(\theta) = \frac{\prod_{j=1}^E (e_j)_{k_1 \eta_j^{(1)} + \dots + k_s \eta_j^{(s)}} \prod_{j=1}^{U'} (u_j')_{k_1 x_j^{(1)} \dots k_s x_j^{(s)}}}{\prod_{j=1}^P (g_j)_{k_1 \xi_j^{(1)} + \dots + k_s \xi_j^{(s)}} \prod_{j=1}^{V'} (v_j')_{k_1 t_j^{(1)} \dots k_s t_j^{(s)}}} \prod_{j=1}^{U^{(s)}} (u_j^{(s)})_{k_s x_j^{(s)}} \quad (13)$$

$S_N^{M_1, \dots, M_s} [\omega_1, \dots, \omega_s]$ reduces to the generalized Lauricella function of [2],

$$F_{P: V' \dots; V^{(s)}}^{1+E: U' \dots; U^{(s)}} \left[\begin{matrix} [-N; M_1, \dots, M_s], [(\omega); \eta^{(1)}, \dots, \eta^{(s)}], [(\omega) x^{(1)}, \dots, [(\omega^{(s)}) x^{(s)}]; \\ [(\omega); \xi^{(1)}, \dots, \xi^{(s)}], [(\nu); t^{(1)}, \dots, [(\nu^{(s)}) t^{(s)}]; \end{matrix} \right] \omega_1, \dots, \omega_s \quad (14)$$

and our main result (10) reduces to the following formula

$$D_t^{\mu} \left\{ (t-x)^{\sigma} \eta^{\sigma} (y-t)^{\sigma+\rho} {}_2F_1 \left(\begin{matrix} u, v \\ w \end{matrix}; R (t-x)^{A_1} (y-t)^{B_1} \right) F \left[\begin{matrix} \tau_1 \{\eta(y-t)\}^{\sigma_1} \\ \vdots \\ \tau_r \{\eta(y-t)\}^{\sigma_r} \end{matrix} \right] \right\}$$

$$\begin{aligned}
& \text{F}^{\text{I+E}, U^*, \dots, U^{(S)}}_{P^*: V^*, \dots, V^{(S)}} \left[\begin{array}{l} [-N; M_1, \dots, M_s], [(\epsilon), \eta^*, \dots, \eta^{(S)}] \\ [(w), x^*, \dots, (w^{(S)}), x^{(S)}], \\ [(g), t^*, \dots, t^{(S)}], [(v), t^*], \dots, [(v^{(S)}), t^{(S)}] \end{array} \right], \quad \frac{(t-x)^{a_1}}{\vdots} \frac{(y-t)^{b_1}}{\vdots} \\
& .H \left[\begin{array}{l} z_1 \{t(t-x)\}^{\sigma_1} \{t(y-t)\}^{\rho_1} \\ \vdots \\ z_r \{t(t-x)\}^{\sigma_r} \{t(y-t)\}^{\rho_r} \end{array} \right] \\
& = \sum_{K, K_1, M, \alpha, \beta=0}^{\infty} \sum_{k_1, \dots, k_s=0}^{M_1 k_1 + \dots + M_s k_s \leq N} \frac{(-N)_{M_1 k_1 + \dots + M_s k_s}}{k_1! \dots k_s!} I(\theta) \Delta \\
& H^{0, +\lambda+3 : (u^*, v^*) ; \dots ; (u^{(r)}, v^{(r)})}_{A+3, C+3 : [B^* D] ; \dots ; [B^{(r)}, D^{(r)}]} \left[\begin{array}{l} z_1 (-x)^{\sigma_1} y^{\rho_1} t^{\rho_1 + \sigma_1} \\ \vdots \\ z_r (-x)^{\sigma_r} y^{\rho_r} t^{\rho_r + \sigma_r} \end{array} \right] \\
& [-\sigma - A_1 K_1 + \sum_{i=1}^s a_i k_i \cdot \sigma_1, \dots, \sigma_r], [-\rho - K - B_1 K_1 + \sum_{i=1}^s b_i k_i \cdot \rho_1, \dots, \rho_r], [1 - \alpha - \beta \cdot \sigma_1 + \rho_1, \dots, \sigma_r + \rho_r] \\
& [(c) : \psi, \dots, \psi^{(r)}], [\alpha - \sigma - A_1 K_1 - \sum_{i=1}^s a_i k_i \cdot \sigma_1, \dots, \sigma_r], [\beta - \rho - K - B_1 K_1 - \sum_{i=1}^s b_i k_i \cdot \rho_1, \dots, \rho_r], \\
& [(\dots) : \phi, \dots, \phi^{(r)}], [(\phi) : \phi^*], \dots, [(\phi^{(r)}) : \phi^{(r)}], \\
& [\mu - \alpha - \beta \cdot \sigma_1 + \rho_1, \dots, \sigma_r + \rho_r], [(\delta) : \delta^*], \dots, [(\delta^{(r)}) : \delta^{(r)}]
\end{aligned} \tag{15}$$

Which holds true under the same conditions as given in (10).

Letting $R \rightarrow 0$ in (10), we find a known result recently obtained by [8].

Taking $M_i = 0$, ($i = 1, 2, \dots, s$), $N \rightarrow 0$ and $R \rightarrow 0$, the result in (10) reduces to a known result given in [10].

For $R \rightarrow 0$, the results given in (11), (12) and (13) reduce to known results given by [8].

For $R \rightarrow 0$, the results given in (11), (12) and (13) reduce to known results given by [8].

The results established here are quite general in nature and a large number of (known or new) results can be obtained by specializing the parameters suitably of the various functions involved in them.

Fractional derivative formula is used to solve many differential equations in mathematics, physics and engineering science.

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