Generalized Multi-Phase Multivariate Regression Estimator for Partial Information Case Using Multi-Auxiliary Variables

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Abstract: In this paper we propose generalized multi-phase multivariate regression estimator in the presence of multi-auxiliary variables for estimating population mean vector of variables of interest. Some special cases have been deduced from the suggested estimator in the form of remarks. The expressions for mean square errors of proposed estimator and its special cases have also been derived and empirical study has also been carried out.

Key words: Multi-phase sampling, multivariate regression estimator, multi-auxiliary variables

INTRODUCTION

The estimation of the population mean is an unrelenting issue in sampling theory and several efforts have been made to improve the precision of the estimators in the presence of multi-auxiliary variables. A variety of estimators have been proposed following different ideas of ratio, regression and product estimators.


Agarwal et al. [9], moving away from Raj [4], illustrated a new approach to form a multivariate difference estimator which does not require the knowledge of any population parameter. Ahmed [10] put forward chain based general estimators using multivariate auxiliary information under multiphase sampling, while Kadilar and Cingi [11, 12] analyzed combinations of regression type estimators in the case of two auxiliary variables. Pradhan [13] suggested a chain regression estimator for two-phase sampling using three auxiliary variables when the population mean of one auxiliary variable is unknown and other is known. Hanif et al. [14] suggested generalized multivariate ratio estimators in the presence of multi-auxiliary variables for estimating population mean of a study variable for multi-phase sampling. The estimators were proposed for both cases when information on all auxiliary variables is known (full information case) and unknown (no information case).

Ahmad et al. [15] proposed generalized regression-ratio estimators for two phase sampling using multi-auxiliary variables for estimating the population mean of study variable. Ahmad et al. [16] developed generalized regression-in-regression estimators for two-phase sampling using multi-auxiliary variables for estimating the population mean of variable of interest.

Ahmad et al. [17] suggested generalized multivariate multiple ratio estimators for partial information case using multi-auxiliary variables. Ahmad et al. [18] developed multivariate regression estimators for full and no information cases in the presence of multi-auxiliary variables for estimating population mean of a study variable for multi-phase sampling.

In multipurpose surveys, the problem is to estimate population means of several variables simultaneously. Swain [19] Tripathi and Khattree [20] estimated means of several variables of interest, using multi-auxiliary variables for simple random sampling. Further Tripathi [21] extended the results to the case of two phase sampling.

We suggest generalized multivariate regression estimator for estimating a vector of population means of study variables for multi-phase sampling using...
multi-auxiliary variables when information on some multi-auxiliary variables (Partial Information Case) is available for population [22].

Before suggesting the estimators we provide multi-phase sampling scheme and some useful notations and results in the following section.

MULTI-PHASE SAMPLING USING MULTI-AUXILIARY VARIABLES

Consider a population of size $N$ units. Let $Y_1, Y_2, \ldots, Y_p$ be $p$ variables of interest and $X_1, X_2, \ldots, X_q$ be $q$ auxiliary variables. For multi-phase sampling design let $n_h$ and $n_k$ ($n_h \leq n_k$) be sample sizes for $h$th and $k$th phase respectively. $X_{h,i}$ and $X_{k,i}$ denote the $i$th auxiliary variables from $h$th and $k$th phase samples respectively and $y_{h,i}$ denote $i$th study variable from $h$th phase sample. Let $\bar{X}_h$, $C_{x_h}$, $C_{y_h}$, $\rho_{x_hy_h}$, $\rho_{x_hy}$ and $\rho_{x_h}$ denote the population mean, coefficient of variation of $i$th auxiliary variable, coefficient of variation of $i$th variable of interest, correlation coefficient of $i$th variable of interest and $i$th auxiliary variable, correlation coefficient of $i$th and $j$th variable of interest and correlation coefficient of $i$th and $j$th auxiliary variables respectively. Further let

$$\theta_h = \frac{1}{n_h} - \frac{1}{N}$$

and

$$\theta_k = \frac{1}{n_k} - \frac{1}{N}$$

are sampling fractions for $h$th and $k$th phase respectively. Also

$$y_{h,i} = Y_i + e_{y_{h,i}}, x_{h,i} = X_i + e_{x_{h,i}}$$

and

$$x_{k,i} = X_i + e_{x_{k,i}} (i = 1, 2, \ldots, k)$$

where $e_{y_{h,i}}$, $e_{x_{h,i}}$ and $e_{x_{k,i}}$ are sampling errors. We assume that

$$E_h(e_{y_{h,i}}) = E_h(e_{x_{h,i}}) = E_h(e_{x_{k,i}}) = 0$$

where $E_h$ and $E_k$ denote the expectations of errors of $h$th and $k$th phase sampling. Then for simple random sampling without replacement for both first and second phases, we write by using phase wise operation of expectations as:

$$E_h\left(e_{y_{h,i}}^2\right) = \left(1 - \frac{n_h}{N}\right)\sigma_y^2$$

$$E_h\left(e_{y_{h,i}}^2\right) = \left(1 - \frac{n_h}{N}\right)\frac{\sigma_y^2}{n_h} = \theta_h N_1 C_y^2$$

$$E_h\left(e_{y_{h,i}} e_{y_{h,j}}\right) = \left(1 - \frac{n_h}{N}\right)\frac{\sigma_{y_hy_h}}{n_h} = \theta_h N_1 C_y^2 C_{x_h} \rho_{y_hy}$$

$$E_h\left(e_{y_{h,i}} e_{y_{h,j}}\right) = \left(1 - \frac{n_h}{N}\right)\frac{\sigma_{y_ky_k}}{n_h} = \theta_k N_1 C_y^2 C_{y_h} \rho_{x_hy}$$

$$E_h E_k \left[\sum_{i=1}^{n_h} \left(x_{h,i} - x_{h,i}\right)\right] = E_h E_k \left[\sum_{i=1}^{n_h} \left(y_{h,i} - y_{h,i}\right)\right] = 0$$

and

$$E_h E_k \left[\sum_{i=1}^{n_k} \left(z_{k,i} - z_{k,i}\right)\right] = E_h E_k \left[\sum_{i=1}^{n_k} \left(v_{k,i} - v_{k,i}\right)\right] = 0$$

The following notations will be used in deriving the mean square errors of proposed estimators:

- $R_{x_{h,j}}$ Determinant of population correlation matrix of variables $x_{h,1}, x_{h,2}, \ldots, x_{h,j}$ and $x_{h,j+1}$
- $R_{x_{h,j}}$ Determinant of $j$th minor of $R_{x_{h,j}}$ corresponding to the $j$th element of $\rho_{x_h}$
- $\rho_{x_{h,j}}$ Denotes the multiple coefficient of determination of $y$ on $x_{h,1}, x_{h,2}, \ldots, x_{h,j}$
- $\rho_{y_{h,j}}$ Denotes the multiple coefficient of determination of $y$ on $x_{h,1}, x_{h,2}, \ldots, x_{h,j+1}$
- $R_{x_{h,j}}$ Determinant of population correlation matrix of variables $x_{h,1}, x_{h,2}, \ldots, x_{h,j}$ and $x_{h,j+1}$
- $R_{y_{h,j}}$ Determinant of population correlation matrix of variables $x_{h,1}, x_{h,2}, \ldots, x_{h,j}$ and $x_{h,j+1}$
- $R_{y_{h,j}}$ Determinant of the correlation matrix of $y, x_{h,1}, x_{h,2}, \ldots, x_{h,j}$ and $x_{h,j+1}$
- $R_{x_{h,j}}$ Determinant of the correlation matrix of $y, x_{h,1}, x_{h,2}, \ldots, x_{h,j}$ and $x_{h,j+1}$

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RESULTS: The following result will help in deriving the mean square errors of suggested estimators [23].

\[ \left| R_{ky} \right| = \left( 1 - \rho^2_{xy} \right) \]

GENERALIZED MULTI-PHASE MULTIVARIATE REGRESSION ESTIMATOR FOR PARTIAL INFORMATION CASE

Let we have \( q \) auxiliary variable \( X_1, X_2, \ldots, X_q \) and population means for first \( r \) auxiliary is not known and for the rest \( q-r-s \) is known. Let \( \bar{y}_{(h)} \) denotes the sample mean of \( h \)th study variable from \( h \)th phase and \( \bar{x}_{(h)} \) and \( \bar{x}_{(3)} \) denotes the sample mean of \( h \)th auxiliary variable from \( h \)th and \( 3 \)th phase respectively. The generalized multi-phase multivariate regression estimator for estimating the mean vector of \( p \) variables of interest in the presence of \( q \) auxiliary variables for partial information case is suggested as:

\[
T_{h+k,p} = \left[ \bar{y}_{(h)} + \sum_{i=1}^{r} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) + \sum_{i=r+1}^{r+s} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) \right]
+ \sum_{i=r+1}^{r+s} \gamma_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
+ \sum_{i=r+s+1}^{r+s+q} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) + \sum_{i=r+s+1}^{r+s+q} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
+ \sum_{i=r+s+1}^{r+s+q} \gamma_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
\]

(3.1)

where \( \alpha, \beta \) and \( \gamma \) are unknown constants, the expressions for these constants can be obtained for which elements of variance covariance matrix of estimator suggested in (3.1) will be minimum. We write (3.1) as:

\[
T_{h+k,p} = \left[ \bar{y}_{(h)} + \sum_{i=1}^{r} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) - \sum_{i=r+1}^{r+s} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) \right]
- \sum_{i=r+1}^{r+s} \gamma_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
- \sum_{i=r+s+1}^{r+s+q} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) - \sum_{i=r+s+1}^{r+s+q} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
- \sum_{i=r+s+1}^{r+s+q} \gamma_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
\]

or

\[
T_{h+k,p} = \left[ \bar{y}_{(h)} + \sum_{i=1}^{r} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) + \sum_{i=r+1}^{r+s} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) \right]
+ \sum_{i=r+1}^{r+s} \gamma_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
+ \sum_{i=r+s+1}^{r+s+q} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) - \sum_{i=r+s+1}^{r+s+q} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
- \sum_{i=r+s+1}^{r+s+q} \gamma_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
\]

or

\[
T_{h+k,p} = \left[ \bar{y}_{(h)} + \sum_{i=1}^{r} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) + \sum_{i=r+1}^{r+s} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) \right]
+ \sum_{i=r+1}^{r+s} \gamma_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
+ \sum_{i=r+s+1}^{r+s+q} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) - \sum_{i=r+s+1}^{r+s+q} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
- \sum_{i=r+s+1}^{r+s+q} \gamma_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
\]

or

\[
T_{h+k,p} = \bar{y}_{(h)} + \sum_{i=1}^{r} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) + \sum_{i=r+1}^{r+s} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) - \sum_{i=r+s+1}^{r+s+q} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) - \sum_{i=r+s+1}^{r+s+q} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
\]

or

\[
T_{h+k,p} = \bar{y}_{(h)} + \sum_{i=1}^{r} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) + \sum_{i=r+1}^{r+s} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) - \sum_{i=r+s+1}^{r+s+q} \alpha_i (\bar{x}_{(h) i} - \bar{x}_{(3) i}) - \sum_{i=r+s+1}^{r+s+q} \beta_i (\bar{x}_{(h) i} - \bar{x}_{(3) i})
\]

(3.2)

Where

\[
\bar{d}_a = \left[ (\bar{x}_{(h)1} - \bar{x}_{(3)1}) (\bar{x}_{(h)2} - \bar{x}_{(3)2}) \ldots (\bar{x}_{(h)q} - \bar{x}_{(3)q}) \right]
= \left[ (\bar{x}_{(h)1} - \bar{x}_{(3)1}) (\bar{x}_{(h)2} - \bar{x}_{(3)2}) \ldots (\bar{x}_{(h)q} - \bar{x}_{(3)q}) \right]_{h+r}
\]

\[
\bar{d}_a = \left[ (\bar{x}_{(h)1} - \bar{x}_{(3)1}) (\bar{x}_{(h)2} - \bar{x}_{(3)2}) \ldots (\bar{x}_{(h)q} - \bar{x}_{(3)q}) \right]
= \left[ (\bar{x}_{(h)1} - \bar{x}_{(3)1}) (\bar{x}_{(h)2} - \bar{x}_{(3)2}) \ldots (\bar{x}_{(h)q} - \bar{x}_{(3)q}) \right]_{h+r}
\]

A = [(\alpha_{ij})_{h+p}] for i = 1, 2, 3, ..., q and j = 1, 2, 3, ..., p
B = [(\beta_{ij})_{h+p}] and C = [(\gamma_{ij})_{h+p}] for i = r+1, r+2, r+3, ..., r+s and j = 1, 2, 3, ..., p. Letting, \( \bar{y} = \bar{y} + \bar{d}_a \),

where

\[
\bar{y} = \left[ \bar{y}_1, \bar{y}_2, \ldots, \bar{y}_p \right]
\]
and
\[
\mathbf{\bar{d}}_y = \begin{bmatrix}
\bar{d}_{y1} \bar{d}_{y2} \cdots \bar{d}_{yp}
\end{bmatrix}
\]

We write (3.2) as:
\[
T_{k_{x+p}} = \mathbf{\bar{d}}_y + \mathbf{\bar{d}}_{x} A - \mathbf{\bar{d}}_{x} B - \mathbf{d}_{x} C.
\]

We use information related to auxiliary variables from first and second phase both then the mean square error of \(T_{k_{x+p}}\) can be written as:
\[
\Sigma_{\bar{y}_{x+p}} = \mathbb{E}[\mathbb{E}_\mathbf{\bar{y}}(T_{k_{x+p}} - \mathbf{\bar{y}})(T_{k_{x+p}} - \mathbf{\bar{y}})]
= \mathbb{E}_\mathbf{\bar{y}}\left[\begin{bmatrix}
\bar{d}_{x}^T A - \bar{d}_{x}^T B - \bar{d}_{x} C
\end{bmatrix}^T
\begin{bmatrix}
\bar{d}_{x}^T A - \bar{d}_{x}^T B - \bar{d}_{x} C
\end{bmatrix}\right],
\]

(3.3)

We can write
\[
\mathbb{E}_\mathbf{\bar{y}}\left[\bar{d}_{y}^T d_x\right] = \theta_{k} \sigma_{x,y} = \theta_{k} [\sigma_{x,y}]_{x+p}, \text{ for i,j}, \sigma_{x,y} = \sigma^2_y.
\]

(3.4)

The optimum values of unknown matrices \(A, B\) and \(C\) can be written as:
\[
A_{x+p} = W_{x+p}^{-1} \left(\Sigma_{x+p} - \Sigma_{x+p}^{-1} \Sigma_{x+p} \sigma_{x+p}^2 \Sigma_{x+p} \right)
\]

(3.5)

\[
B_{x+p} = \left(\Sigma_{x+p}^{-1} \Sigma_{x+p} \sigma_{x+p}^2 \Sigma_{x+p}^{-1} \right)
- \left(\Sigma_{x+p}^{-1} \Sigma_{x+p} \sigma_{x+p}^2 \Sigma_{x+p}^{-1} \right)
\]

(3.6)

and
\[
C_{x+p} = \left(\Sigma_{x+p}^{-1} \Sigma_{x+p} \sigma_{x+p}^2 \Sigma_{x+p}^{-1} \right)
- \left(\Sigma_{x+p}^{-1} \Sigma_{x+p} \sigma_{x+p}^2 \Sigma_{x+p}^{-1} \right)
\]

(3.7)

Using the above values of unknown matrices in the expression of mean square error given in (3.4), we write:
\[
\Sigma_{\bar{y}_{x+p}} = \theta_{k} \left(\Sigma_{x+p} - \Sigma_{x,p} \Sigma_{x+p}^{-1} \Sigma_{x,p} \right)
- (\theta_{k} - \theta_{k} ) \left(\Sigma_{x+p} - \Sigma_{x,p} \Sigma_{x+p}^{-1} \Sigma_{x,p} \right)
\]

(3.8)

\[
W_{x+p}^{-1} = \left(\Sigma_{x+p} - \Sigma_{x,p} \Sigma_{x+p}^{-1} \Sigma_{x,p} \right)^{-1}
\]

where
\[
W_{x+p}^{-1} = \left(\Sigma_{x+p} - \Sigma_{x,p} \Sigma_{x+p}^{-1} \Sigma_{x,p} \right)^{-1}
\]

(3.9)

provided \(\Sigma_{x+p}^{-1}, W_{x+p}^{-1}\), and \(\Sigma_{x,p}^{-1}\) exist.

The variance covariance matrix in the form of variance of \(y_1\), covariances and correlation coefficients of \(x_i\) and \(y_1\) can be written as:
\[
\Sigma_{\bar{y}_{x+p}} = \begin{bmatrix}
\sigma_{x,y}^2 & \theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y} \\
\theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y} \\
\theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y}
\end{bmatrix},
\]

(3.10)

\[
(i,j=1,2,\ldots,p)
\]

for
\[
\rho_{x,y} = \rho_{x,y}.
\]

(3.9)

In determinants of correlation matrices for \(R_x \neq 0\) and \(R_{x+p} \neq 0\), (3.9) can be written as:
\[
\Sigma_{\bar{y}_{x+p}} = \begin{bmatrix}
\sigma_{x,y}^2 & \theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y} \\
\theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y} \\
\theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y} & \theta_{k} \sigma_{x,y} \rho_{x,y}
\end{bmatrix}
\]

(3.11)

\[
(i,j=1,2,\ldots,p)
\]

\[
\rho_{x,y} = \rho_{x,y}.
\]
for
\[ i = j, \sigma_i \sigma_j = \sigma_i^2, |R_{i,j}^2| = |R_{i,j}^2| \]
and
\[ |R_{i,j}^2| = |R_{j,i}^2| \]

**Remark-1:** To develop generalized multivariate regression estimator for two-phase sampling using multi-auxiliary variables for Partial Information Case, replace \( h \) by 1 and \( k \) by 2 in (3.1), we get the following estimator,

\[
T_{i,j}^{(h)} = \left[ \bar{x}_{i} \left( x_{i1} - \bar{x}_{i1} \right) + \sum_{m=1}^{r} \alpha_{m} \left( x_{i2} - \bar{x}_{i2} \right) \right] + \sum_{m=1}^{r} \beta_{m} \left( \bar{x}_{i1} - \bar{x}_{i1} \right) + \sum_{i=1}^{r} \gamma_{i} \left( x_{i2} - \bar{x}_{i2} \right) + \sum_{j=1}^{r} \gamma_{j} \left( x_{i1} - \bar{x}_{i1} \right) \]

(3.11)

The expressions of unknown matrices for which the mean square error of above estimator will be minimum and are same as given in (3.5), (3.6) and (3.7). The expression for variance covariance matrix can be directly written from (3.8) just replacing \( h \) by 1 and \( k \) by 2 as:

\[
\Sigma_{i,j} = \Theta_{i} \left( \Sigma_{i}, \Sigma_{j} + \sum_{m=1}^{r} \alpha_{m}^{2} \Sigma_{m} \right) - \Theta_{j} \left( \Sigma_{j}, \Sigma_{i} + \sum_{m=1}^{r} \alpha_{m}^{2} \Sigma_{m} \right) - \Theta_{i} \Theta_{j} \left( \Sigma_{i}, \Sigma_{j} + \sum_{m=1}^{r} \alpha_{m}^{2} \Sigma_{m} \right) + \Theta_{j} \Theta_{i} \left( \Sigma_{j}, \Sigma_{i} + \sum_{m=1}^{r} \alpha_{m}^{2} \Sigma_{m} \right)
\]

(3.12)

The variance covariance matrix in the form of variance of \( y_{1} \), covariances and correlation coefficients of \( x_{i} \) and \( y_{1} \) is written as:

\[
\Sigma_{i,j} = \left[ \sigma_{y_{1}} \sigma_{y_{1}} \right] + \left[ \Theta_{i} \left( \rho_{x_{i},y_{1}}, \rho_{y_{1},y_{1}} \right) - \Theta_{j} \left( \rho_{x_{j},y_{1}}, \rho_{y_{1},y_{1}} \right) \right]
\]

(3.13)

for
\[ i = j, \sigma_{y_{1}} = \sigma_{y_{1}}^2, |R_{i,j}^2| = |R_{j,i}^2| \]

and
\[ |R_{i,j}^2| = |R_{j,i}^2| \]

In determinants of correlation matrices for \( |R_{i,j}^2| \neq 0 \) and \( |R_{i,j}^2| \neq 0 \), (3.13) can be written as:

\[
\Sigma_{(i,j)} = \left[ \sigma_{y_{1}} \sigma_{y_{1}} \right] \left( \Theta_{i} \left( \rho_{x_{i},y_{1}}, \rho_{y_{1},y_{1}} \right) + \Theta_{j} \left( \rho_{x_{j},y_{1}}, \rho_{y_{1},y_{1}} \right) \right)
\]

(3.14)

for
\[ i = j, \sigma_{y_{1}} = \sigma_{y_{1}}^2, |R_{i,j}^2| = |R_{j,i}^2| \]

and
\[ |R_{i,j}^2| = |R_{j,i}^2| \]

**Remark-2:** We can develop a univariate generalized regression estimator for multiphase sampling using multi auxiliary variable for Partial Information Case if we put \( p = 1 \) in (3.1) as:

\[
T_{i,j}^{(h)} = \bar{x}_{i} \left( x_{i1} - \bar{x}_{i1} \right) + \sum_{m=1}^{r} \alpha_{m} \left( x_{i2} - \bar{x}_{i2} \right) + \sum_{m=1}^{r} \beta_{m} \left( \bar{x}_{i1} - \bar{x}_{i1} \right) + \sum_{j=1}^{r} \gamma_{j} \left( x_{i2} - \bar{x}_{i2} \right) + \sum_{j=1}^{r} \gamma_{j} \left( x_{i1} - \bar{x}_{i1} \right)
\]

(3.15)

The expression for vectors of unknown constants for which the mean square error will be minimum can be written from (3.5), (3.6) and (3.7) as

\[
A_{i,j} = W_{i,j} \left( \Sigma_{x_{i},x_{j}} - \Sigma_{x_{i},y_{1}} \Sigma_{y_{1},y_{1}}^{-1} \Sigma_{y_{1},x_{j}} \right)
\]

(3.16)

\[
B_{i,j} = \left( \Sigma_{x_{i}} - \Sigma_{x_{i},y_{1}} \Sigma_{y_{1},y_{1}}^{-1} \Sigma_{y_{1},x_{j}} \right)
\]

(3.17)

and

\[
C_{i,j} = \left( \Sigma_{x_{i}} - \Sigma_{x_{i},y_{1}} \Sigma_{y_{1},y_{1}}^{-1} \Sigma_{y_{1},x_{j}} \right) + \left( \Sigma_{x_{j}} - \Sigma_{x_{j},y_{1}} \Sigma_{y_{1},y_{1}}^{-1} \Sigma_{y_{1},x_{j}} \right)
\]

(3.18)

The above expressions for unknown matrices can be written in determinants form as:

\[
\alpha_{i} = \left( -1 \right)^{i+j} \frac{C_{i,j}}{C_{j,i}} \frac{|R_{i,j}^2|_{i,j}}{C_{j,i}} \left( -1 \right)^{i+j} \frac{C_{i,j}}{C_{j,i}} \frac{|R_{i,j}^2|_{i,j}}{C_{j,i}} \left( i = 1, 2, \ldots, r \right)
\]

(3.19)
\[ \beta_i = (-1)^{i+1} C_{x_i} \begin{bmatrix} R_{y_i, x_{i1}} & R_{y_i, x_{i2}} \\ R_{x_{i1}, x_{i2}} & R_{x_{i1}, x_{i1}} \end{bmatrix} \]
\[ \gamma_i = (-1)^{i+1} C_{x_i} \begin{bmatrix} R_{y_i, x_{i1}} & R_{y_i, x_{i2}} \\ R_{x_{i1}, x_{i2}} & R_{x_{i1}, x_{i1}} \end{bmatrix} \]
\[ (i = r + 1, r + 2, \ldots, r + s) \]

The expression for mean square error can be directly written from (3.8) as:
\[
\text{MSE}(T_{1k}) = \sum_{i=1}^{r} \left( \sigma_i^2 - \sum_{q=1}^{r} \beta_i (1 - \rho_{y_i x_{i1}}^2) + \theta_i (1 - \rho_{y_i x_{i2}}^2) \right) \]
\[
+ \left( \theta_i - \theta_0 \right) \left( \sum_{q=1}^{r} \gamma_i - \sum_{q=1}^{r} \sum_{m=1}^{r} \beta_i \gamma_m \right) \]
\[ \text{MSE}(T_{2k}) = \left( \sum_{i=1}^{r} \sum_{q=1}^{r} \sigma_i^2 (1 - \rho_{y_i x_{i1}}^2) + \theta_i (1 - \rho_{y_i x_{i2}}^2) \right) \]
\[ (3.26) \]

Remark-3: To develop a generalized univariate regression estimator for two-phase sampling using multi-auxiliary variables for Partial Information Case we put \( h = 1 \) and \( k = 2 \) in (3.15). The required estimator becomes:
\[ T_{1k+i} = \bar{y}_{(n)} + \sum_{i=1}^{r} \left( \bar{x}_{(i)} - \bar{x}_{(n)} \right) + \beta_i \left( \bar{x}_{(i)} - \bar{x}_{(n)} \right) \]
\[ + \sum_{i=1}^{r} \gamma_i \left( \bar{x}_{(i)} - \bar{x}_{(n)} \right) \]
\[ (3.27) \]

The expression for vectors of unknown constants for which the mean square error will be minimum are same as given in (3.16), (3.17) and (3.18) and these expressions are also given in determinants of correlation matrices in (3.19), (3.20) and (3.21). The expression for mean square error can be written from (3.22) just by replacing \( h = 1 \) and \( k = 2 \) as:
\[
\text{MSE}(T_{12}) = \sum_{i=1}^{r} \left( \sigma_i^2 - \sum_{q=1}^{r} \beta_i (1 - \rho_{y_i x_{i1}}^2) + \theta_i (1 - \rho_{y_i x_{i2}}^2) \right) \]
\[
+ \left( \theta_i - \theta_0 \right) \left( \sum_{q=1}^{r} \gamma_i - \sum_{q=1}^{r} \sum_{m=1}^{r} \beta_i \gamma_m \right) \]
\[ (3.28) \]

Remark-4: Generalized multivariate regression estimator as suggested by Hanif et al. (2009) for multi-phase sampling using multi-auxiliary variables when information on all auxiliary variables is not available for population (No Information Case) can be developed by putting \( \beta_i \)'s and \( \gamma_i \)'s equals to zero in (3.1) as:
\[ T_{11} = \bar{y}_{(n)} + \sum_{i=1}^{r} \sum_{q=1}^{r} \left( \bar{x}_{(i)} - \bar{x}_{(n)} \right) + \sum_{i=1}^{r} \gamma_i \left( \bar{x}_{(i)} - \bar{x}_{(n)} \right) + \ldots \]
\[ (3.29) \]

The expression of unknown matrix for which the mean square error will be minimum can be directly obtained by considering only those matrices from (3.5), (3.6) and (3.7) those includes only order \( p \times r \) and \( r \times r \) than we get the required matrix that is \( \Sigma_{\tau_{x_i y_i}} \Sigma_{\tau_{y_i x_i}} \). The variance covariance matrix can be obtained from (3.8) just by considering those matrices having order \( p \times r \) and \( r \times r \). The variance covariance matrix is:
\[ \Sigma_{\tau_{x_i y_i}} = \sum_{i=1}^{r} \sum_{j=1}^{r} \sigma_i^2 (1 - \rho_{y_i x_{i1}}^2) + \theta_i (1 - \rho_{y_i x_{i2}}^2) \]
\[ (3.30) \]

All special cases of estimator given in (3.27), in the case on no information and full information, have been discussed by Hanif et al. (2009).

**EMPIRICAL STUDY**

Obviously estimator for which the information on all auxiliary variables is available for population (full information case) will be more efficient than that for which the information on some auxiliary variables is available for population (partial information case). The estimator for partial information case will be efficient than the estimator for which the information on all auxiliary variables is not available for population (no information case). In the case of multivariate, the estimator will be less efficient by increasing the phases but cost effective. To illustrate the above statements, empirical study has been carried.

For the justification of above statements, the empirical study is carried out by using determinant of
Appendix A

Table A-1.1: Detail of populations

<table>
<thead>
<tr>
<th>Sr. #</th>
<th>Source of populations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Population census report of Jhang district (1998), Pakistan</td>
</tr>
<tr>
<td>2</td>
<td>Population census report of Gujrat district (1998), Pakistan</td>
</tr>
<tr>
<td>3</td>
<td>Population census report of Kasur (1998), Pakistan</td>
</tr>
<tr>
<td>4</td>
<td>Population census report of Sialkot district (1998), Pakistan</td>
</tr>
</tbody>
</table>

Table A-1.2: Description of variables (Each variable is taken from rural locality)

<table>
<thead>
<tr>
<th>Description of variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_1 ) Population of currently married</td>
</tr>
<tr>
<td>( Y_2 ) Total household</td>
</tr>
<tr>
<td>( X_1 ) Population of both sexes</td>
</tr>
<tr>
<td>( X_2 ) Population of primary but below metric</td>
</tr>
<tr>
<td>( X_3 ) Population of metric and above</td>
</tr>
<tr>
<td>( X_4 ) Population of 18 years old and above</td>
</tr>
<tr>
<td>( X_5 ) Population of women 15-49 years old</td>
</tr>
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</table>

Table A-1.3: Parameters of populations for calculating the Matrices of MSE's of multivariate estimators and MSE's of univariate estimators

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<thead>
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<th>District</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( n_3 )</th>
<th>( n_4 )</th>
<th>( n_5 )</th>
<th>( n_6 )</th>
<th>( \sigma_n )</th>
<th>( \sigma_{n_1} )</th>
<th>( \sigma_{n_2} )</th>
<th>( \sigma_{n_3} )</th>
<th>( \rho_{n_1 n_2} )</th>
<th>( \rho_{n_1 n_3} )</th>
<th>( \rho_{n_1 n_4} )</th>
<th>( \rho_{n_1 n_5} )</th>
<th>( \rho_{n_1 n_6} )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>92</td>
<td>46</td>
<td>23</td>
<td>12</td>
<td>29.705</td>
<td>860.11</td>
<td>897.71</td>
<td>0.270</td>
<td>0.595</td>
<td>0.512</td>
<td>0.182</td>
<td>0.448</td>
<td>0.847</td>
</tr>
<tr>
<td>Gujrat</td>
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<td>102</td>
<td>51</td>
<td>26</td>
<td>13</td>
<td>6</td>
<td>57.535</td>
<td>1101.280</td>
<td>1102.540</td>
<td>0.145</td>
<td>0.484</td>
<td>0.847</td>
<td>0.055</td>
<td>0.010</td>
<td>0.055</td>
</tr>
<tr>
<td>Kasur</td>
<td>181</td>
<td>91</td>
<td>45</td>
<td>23</td>
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<td>6</td>
<td>31.890</td>
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<td>1449.020</td>
<td>0.747</td>
<td>0.551</td>
<td>0.530</td>
<td>0.295</td>
<td>0.789</td>
<td>0.799</td>
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<tr>
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<td>269</td>
<td>135</td>
<td>67</td>
<td>34</td>
<td>17</td>
<td>8</td>
<td>52.061</td>
<td>1058.740</td>
<td>998.220</td>
<td>0.147</td>
<td>0.647</td>
<td>0.646</td>
<td>0.324</td>
<td>0.931</td>
<td>0.931</td>
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<table>
<thead>
<tr>
<th>District</th>
<th>( \rho_{n_1 n_2} )</th>
<th>( \rho_{n_1 n_3} )</th>
<th>( \rho_{n_1 n_4} )</th>
<th>( \rho_{n_1 n_5} )</th>
<th>( \rho_{n_1 n_6} )</th>
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<th>( \rho_{n_3 n_4} )</th>
<th>( \rho_{n_3 n_5} )</th>
<th>( \rho_{n_3 n_6} )</th>
<th>( \rho_{n_4 n_5} )</th>
<th>( \rho_{n_4 n_6} )</th>
<th>( \rho_{n_5 n_6} )</th>
<th>( \rho_{n_6 n_6} )</th>
</tr>
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<td>Jhang</td>
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<td>0.376</td>
<td>0.460</td>
<td>0.486</td>
<td>0.185</td>
<td>0.129</td>
<td>0.428</td>
<td>0.512</td>
<td>0.659</td>
<td>0.484</td>
<td>0.425</td>
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<td></td>
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<tr>
<td>Gujrat</td>
<td>0.056</td>
<td>0.988</td>
<td>0.092</td>
<td>0.334</td>
<td>0.543</td>
<td>0.069</td>
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<td>0.955</td>
<td>0.764</td>
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<td>0.996</td>
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<tr>
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<td>0.989</td>
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<td>0.988</td>
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<td>0.989</td>
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<tr>
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<td>0.316</td>
<td>0.997</td>
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</table>

Table A-1.4.1 Determinants of matrices of MSE's of multivariate Regression estimators for pair-wise phases (No Information Case)

<table>
<thead>
<tr>
<th>District</th>
<th>( T_{11} ) (h=1,k=2)</th>
<th>( T_{12} ) (h=1,k=3)</th>
<th>( T_{14} ) (h=1,k=4)</th>
<th>( T_{15} ) (h=1,k=5)</th>
<th>( T_{23} ) (h=2,k=3)</th>
<th>( T_{24} ) (h=2,k=4)</th>
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<tbody>
<tr>
<td>Jhang</td>
<td>212279.97</td>
<td>1223241.603</td>
<td>7221747.657</td>
<td>46128460.86</td>
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<td>95363.18</td>
<td>312081.54</td>
<td>1102996.85</td>
<td>4211396.91</td>
<td>3327929.29</td>
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<tr>
<td>Kasur</td>
<td>203091.03</td>
<td>901801.71</td>
<td>3838230.04</td>
<td>16192403.14</td>
<td>8973735.68</td>
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</tr>
<tr>
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<td>9555.27</td>
<td>4146.31</td>
<td>17380.26</td>
<td>723929.58</td>
<td>482925.70</td>
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</tr>
<tr>
<td>District</td>
<td>( T_{25} ) (h=2,k=5)</td>
<td>( T_{36} ) (h=3,k=4)</td>
<td>( T_{35} ) (h=3,k=5)</td>
<td>( T_{45} ) (h=4,k=5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jhang</td>
<td>79961159.68</td>
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<td>158095387.70</td>
<td>2.60491E+11</td>
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<td></td>
</tr>
<tr>
<td>Gujrat</td>
<td>11251537.17</td>
<td>10702183.50</td>
<td>30944428.16</td>
<td>93069748.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kasur</td>
<td>3685031.49</td>
<td>19922737.86</td>
<td>79389873.19</td>
<td>170075469.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sialkot</td>
<td>1897366.87</td>
<td>1256890.23</td>
<td>4554991.24</td>
<td>11150157.88</td>
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<td></td>
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</tbody>
</table>
Table A-1.4.2: Determinants of variance covariance matrices of multivariate Regression estimators for pair-wise phases (Partial Information Case)

<table>
<thead>
<tr>
<th>District</th>
<th>$T_{12}$ (h=1,k=2)</th>
<th>$T_{13}$ (h=1,k=3)</th>
<th>$T_{14}$ (h=1,k=4)</th>
<th>$T_{15}$ (h=1,k=5)</th>
<th>$T_{21}$ (h=2,k=3)</th>
<th>$T_{24}$ (h=2,k=4)</th>
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</thead>
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<tr>
<td>Jhang</td>
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<td>890963.2</td>
<td>3826643.1</td>
<td>40379695.2</td>
<td>2250913.1</td>
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</tr>
<tr>
<td>Gujrat</td>
<td>1683.1</td>
<td>10844.4</td>
<td>73847.7</td>
<td>534079.5</td>
<td>18218.9</td>
<td>103581.9</td>
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<tr>
<td>Kasur</td>
<td>247034.4</td>
<td>1500728.6</td>
<td>9605459.6</td>
<td>66188757.3</td>
<td>2573608.2</td>
<td>14114833.3</td>
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<tr>
<td>Sialkot</td>
<td>322.1</td>
<td>2207.0</td>
<td>15843.0</td>
<td>118825.5</td>
<td>3970.3</td>
<td>22574.0</td>
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</table>

<table>
<thead>
<tr>
<th>District</th>
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<th>$T_{14}$ (h=3,k=4)</th>
<th>$T_{15}$ (h=3,k=5)</th>
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</thead>
<tbody>
<tr>
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<td>108298411.4</td>
<td>222062545.3</td>
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<tr>
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Table A-1.4.3: Determinants of matrices of MSE’s of multivariate Regression estimators for each phase (Full Information Case)

<table>
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<tr>
<th>District</th>
<th>$T_1$ (k=1)</th>
<th>$T_2$ (k=2)</th>
<th>$T_3$ (k=3)</th>
<th>$T_4$ (k=4)</th>
<th>$T_5$ (k=5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jhang</td>
<td>1923.378901</td>
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<tr>
<td>Gujrat</td>
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</tr>
<tr>
<td>Kasur</td>
<td>103.7803005</td>
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<td>946342.2579</td>
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<tr>
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</tbody>
</table>

Table A-1.5.1: MSE’s of univariate Regression estimators for pair-wise phases (No Information Case)

<table>
<thead>
<tr>
<th>District</th>
<th>$T_{12}$ (h=1,k=2)</th>
<th>$T_{13}$ (h=1,k=3)</th>
<th>$T_{14}$ (h=1,k=4)</th>
<th>$T_{15}$ (h=1,k=5)</th>
<th>$T_{23}$ (h=2,k=3)</th>
<th>$T_{24}$ (h=2,k=4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jhang</td>
<td>212279.9</td>
<td>1223241.6</td>
<td>7221747.6</td>
<td>46128466.8</td>
<td>3572866.6</td>
<td>1601134.8</td>
</tr>
<tr>
<td>Gujrat</td>
<td>95363.1</td>
<td>312081.5</td>
<td>1160996.8</td>
<td>4211396.9</td>
<td>1123210.6</td>
<td>3372979.2</td>
</tr>
<tr>
<td>Kasur</td>
<td>290391.0</td>
<td>901801.7</td>
<td>3838230.0</td>
<td>16192403.1</td>
<td>2176260.2</td>
<td>8973735.6</td>
</tr>
<tr>
<td>Sialkot</td>
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<td>41464.3</td>
<td>173806.26</td>
<td>723929.5</td>
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<table>
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<th>$T_{25}$ (h=3,k=5)</th>
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</thead>
<tbody>
<tr>
<td>Jhang</td>
<td>7996.1159.68</td>
<td>38868782.02</td>
<td>158093587.7</td>
<td>2.60491E+11</td>
</tr>
<tr>
<td>Gujrat</td>
<td>11251537.17</td>
<td>10702183.50</td>
<td>30944428.16</td>
<td>93069748.63</td>
</tr>
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<td>Kasur</td>
<td>36805031.49</td>
<td>19922737.86</td>
<td>79389873.19</td>
<td>170075469.89</td>
</tr>
<tr>
<td>Sialkot</td>
<td>1897366.87</td>
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<td>4554991.24</td>
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Table A-1.5.2: MSE’s of univariate Regression estimators for pair-wise phases (Partial Information Case)

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<td>4445.753</td>
<td>624.3232</td>
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<tr>
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<td>2376.499</td>
<td>5040.433</td>
<td>771.9548</td>
<td>2103.922</td>
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<table>
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<th>$T_{37}$ (h=3,k=5)</th>
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<td>5389.837</td>
<td>8648.382</td>
<td>4079.586</td>
</tr>
<tr>
<td>Gujrat</td>
<td>1254.024</td>
<td>3617.613</td>
<td>2513.426</td>
<td>1254.024</td>
</tr>
<tr>
<td>Kasur</td>
<td>1558.767</td>
<td>4222.701</td>
<td>3132.391</td>
<td>1558.767</td>
</tr>
<tr>
<td>Sialkot</td>
<td>955.9386</td>
<td>2748.4</td>
<td>1917.382</td>
<td>955.9386</td>
</tr>
</tbody>
</table>

Table A-1.5.3: MSE’s of univariate Regression estimators for each phase (Full Information Case)

<table>
<thead>
<tr>
<th>District</th>
<th>$T_1$ (k=1)</th>
<th>$T_2$ (k=2)</th>
<th>$T_3$ (k=3)</th>
<th>$T_4$ (k=4)</th>
<th>$T_5$ (k=5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jhang</td>
<td>8189064</td>
<td>2456719</td>
<td>5732344</td>
<td>1228.36</td>
<td>2538.61</td>
</tr>
<tr>
<td>Gujrat</td>
<td>2135579</td>
<td>6090655</td>
<td>1099004</td>
<td>2281.698</td>
<td>4645.286</td>
</tr>
<tr>
<td>Kasur</td>
<td>2511011</td>
<td>5840929</td>
<td>1250076</td>
<td>2582.043</td>
<td>5245.977</td>
</tr>
<tr>
<td>Sialkot</td>
<td>142167</td>
<td>3662247</td>
<td>814.34</td>
<td>1710.571</td>
<td>3503.032</td>
</tr>
</tbody>
</table>
variance covariance matrices/MSE’s of newly developed estimators for partial information case and those developed by Hanif et al. (2009) for no and full information cases. We consider four natural populations. The detail of populations and variables description is given in Table A1.1 and Table A1.2 respectively of Appendix A. We consider three variables of interests denoted by Y’s and five auxiliary variables denoted by X’s for computing the determinants of matrices of MSE’s of multivariate Regression estimators and for univariate we consider Y2 as study variable and the same five auxiliary variables as considered in multivariate case. The necessary parameters of populations for computing MSE’s are given in A-1.3. We calculate pair-wise determinant of variance covariance matrices/MSE’s for no information case and for full information case we calculate variance covariance matrices /MSE’s for each phase for first five phases. The determinant of variance covariance matrices of multivariate regression estimators for multiphase sampling using pair-wise phases for no information case are given in Table A-1.4.1, for partial information case, Table A-1.4.2 and using each phase for full information case in Table A-1.4.3. The mean square errors of univariate estimators for multiphase sampling using pair-wise phases for no information case are given in Table A1.5.1 and for partial information case in Table A1.5.3 and for full information case using each phase in A-1.5.3.

From Table A1.4.1, Table A1.4.2 and Table A-1.4.3, we can say that the multivariate Regression estimators for full information case are more efficient than partial information case and estimators for partial information case are more efficient than no information case for each phase e.g. T2 is more efficient than T12, T3 is more efficient than T13 & T23 etc. and the same is true for univariate regression estimators (see Table A-1.5.1, Table A-1.5.2 and Table A-1.5.3). Furthermore we can say for no information case and partial information case from Table A-1.4.1 and Table A-1.4.2 that as we increase phase the efficiency decreases e.g. T12, is more efficient than all others, T3 is more efficient than all others except T12, T34 is more efficient than T35, T45 but less efficient than all others and so on, similarly the same argument can be made for univariate case given in Table A1.5.1. Also for full information case the estimators become less efficient as we increase phases because the sample size decreases by increasing phases, it can be seen from Table A1.4.2 and A1.5.2 for multivariate and univariate estimators respectively.

REFERENCES


