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Orthogonal Wavelet Transforms in Forecasting Volatility: An Expermintal Results

[1]S. Al Wadi, [1]Mohd Tahir Ismail, [2]M.H. Alkhahazaleh and [3]Samsul Ariffin Addul Karim

[1]School of Mathematical Science, Universiti Sains Malaysia, 11800 Minden, Penang, Malaysia
[2]Faculty of Science and Technology,
University Kebangsaan Malaysia 43600 UKM Bangi, Selangor D.E. Malaysia
[3]Universiti Teknologi Petronas, 31750 Bandar Seri Iskandar, Tronoh, Perak, Malaysia

Abstract: Wavelet transforms have gotten very high attention in many fields such as physics, engineering, signal processing, applied mathematics, statistics and forecasting. In this paper, we present the advantage of wavelet transforms in forecasting volatility data. Amman stock market (Jordan) was selected as a tool to show the ability of wavelet transforms in forecasting experimentally. This article suggests a novel technique for forecasting the financial time series data based on Wavelet transforms and ARIMA model. The volatility data are decomposed via Haar Wavelet transforms and Daubechies wavelet transform, the future observations of this series are forecasted then the forecasting results are compared using a suitable statistical criteria. Volatility data from 1993 until 2009 are used in this study.

Key words: Wavelet transform · ARIMA model · Volatility data · Forecasting

INTRODUCTION

Stocks markets forecasting is required for the investors and it has gotten high attention in financial time series and financial researchers. Forecasting stocks market forecasting is difficult because unlike demand series, price series present such characteristics as inconstant mean and variance and significant outliers.

The developing economies are facing many impediments in their financial markets and with many other factors, such that the change point [1], high volatility in prices which also considered as high risk or uncertainty is a major factor of erosion of capital from markets. As due to this the investors becomes fearful and run away from the market. Though it is not the sign of inefficiency of market but it poses a threat to 'crash' the market due to high volatility. High volatility creates a high uncertainty in a stock market and individual security prices and these may curtail down the prices and associated return. The stock market volatility caused by number of factors such as; credit policy, inflation rate, interest, financial leverage, corporate earnings, dividends yield policies, bonds prices and many other macroeconomic, social and political variables are involved [2]. Madhavan defines volatility in terms of price variance. Low volatility is preferred as it reduces unnecessary risk borne by investors thus enables market traders to liquidate their assets without large price movements [3]. Glen defines volatility as the frequency and magnitude of price movements and comparing the various microstructure attributes argues that liquid and efficient markets have less volatility than illiquid and inefficient markets.

The three main purposes of forecasting volatility are for risk management, for asset allocation and for taking bets on future volatility. A large part of risk management is measuring the potential future losses of a portfolio of assets and in order to measure these potential losses, estimates must be made of future volatilities and correlations.

Recently, wavelet transforms are used for filtering time series representing. Wavelet analysis has grows very quickly in the recent years and more recently Wall Street analysts have started using mathematical models to analyze their financial data. Moreover, wavelet analysis has been used in signal processing (time scale analysis), frequency analysis, regression function, pattern recognition, decomposition, approximation techniques, quantum field and Image Compression. For more details and examples refer to [4, 5, 6]. The wavelet transforms convert the financial series in a set (typically three to six) of constitutive series. These series show a better behavior

Corrsponding Author: S. Al Wadi, School of Mathematical Science, Universiti Sains Malaysia, 11800 Minden, Penang, Malaysia.
E-mail: sadam_alwadi@yahoo.co.uk & mtahir@cs.usm.my.

than the original price series. In other hands, more stable in variance and no outliers. Wavelet transforms are more efficient than Fourier transform [7-12]. This is because wavelet transforms can be used to analyze nonlinear and non-stationary time series signals, useful in identifying transient events, used to filtering the de-noise data to get more accurately data, provided decomposition of a time series into several components from different scale and appears their correlation as a function on scale and time (localized in both). Although, wavelet transform utilizes to decompose cloudy images into several frequency level components a new technique was developed to enhance cloud-associated shadow areas in satellite images while preserving details underneath these areas [13].

The fundamental and novel contribution of the paper is to use the wavelet transforms, to decompose the volatility data into a set of better-behaved approximation series. The forecasting results based on wavelet transforms methods and ARIMA models will compare by using some statistical criteria. MATLAB 2008a and SAS 9.1 programs have been used to get significant results and fair comparison.

This paper is organized as follows. The next section describes the principle of the mathematical framework. Section 3 provides a description of data set. In Section 4 the comparison of the experimental results is presented. In Section 5 we summarize our contributions and mention the conclusion. And finally we mention the acknowledgement.

MATERIALS AND METHODS

Wavelet Analysis: Wavelet analysis is a mathematical model that transforms the original signal (especially with time domain) into a different domain for analysis and processing [11, 14, 15] This model is very suitable with the non-stationary data, i.e. mean and autocorrelation of the signal are not constant over time, that is well know, most of the financial time series data is non- stationary, that is why we applied wavelet transform.

In mathematical literature, Fourier transforms decomposed the original signal into a linear combination as a sine and cosine function whereas by wavelet transform the signal is decomposed as a sum of a more flexible function called wavelet that is localized in both time and frequency. The wavelet transforms were used to adopt a wavelet prototype function (mother wavelet). Temporal analysis is constructed with a contracted, high-frequency version of prototype wavelet, whereas frequency analysis is performed with a dilated, low frequency version of the prototype wavelet. Because the function can be represented in terms of a wavelet expansion (using coefficients in a linear combination of the wavelet functions), data decompositions can be constructed by just using the corresponding wavelet coefficients. There are several types of wavelet transforms. Depending on the applications, regarding the continuous input signal, the time and scale parameters can be continuous, leading to the continuous wavelet transform (CWT). On the other hand, the discrete wavelet transform (DWT) can also be used for discrete time signals [11].

In the wavelet transforms case, consider that the time domain is the original domain. Although, wavelet transforms is the transformation process from time domain to time scale domain, these processes are known as signal decomposition because a given signal is decomposed into several other signals with deferent levels of resolution. These processes allow recovering the original time domain signal without losing any information. Wavelet transforms has reverse process which is called the inverse wavelet transform or signal reconstruction [16].

The wavelet transform is implemented using a multiresolution pyramidal decomposition technique. In fact, a recorded digitized time signal x_t can be analyzed into its detailed cD1(n) and smoothed (approximations) cA1 (n) signals using high-pass filter (HiF-D) and low-pass filter (LoF-D), respectively. High-pass filter has a band-pass response. Consequently, the filter signal cD1(n) is a detailed coefficient of x_t and contains higher frequency components. While the approximation signal cA1 (n) has a low-pass frequencies filter response. The decomposition of x_t into cA1 (n) and cD 1(n) is the first scale decomposition. Inversely, that is possible to perform the original signal from the approximations and details coefficients [17].

In this paper we will focus in the most famous types of discrete wavelet transforms which are Haar wavelet transform and Daubechies wavelet transform. The wavelets having compact support or narrow window function are suitable for local analysis of the signal. Daubechies wavelets and Haar wavelet are compactly supported orthonormal wavelets and are the most appropriate for treating a non-stationary series.

Definition: [18] suppose that $\psi(t)$ is the mother wavelet, then the sequence of Wavelets can be defined, by using the translations and dilations of $\psi(t)$ as the following:

$$\psi_{a,b}(t) = |a|^{\frac{-1}{2}} \psi(\frac{t-b}{a}).$$

Where a: real number ($a \neq 0$) and represents the dilation parameter and b: real number also and represent the translation parameter. t: Represents the time.

In some special cases of a, b and the mother wavelet ($\psi(t)$), $\psi_{a,b}$ constitute an orthonormal basis for $L_2(R)$ more specifically, if we suppose that $a = 2^j$, $b = k2^j$ then there exits ψ, such that the following functions constitute an orthonormal basis for $L_2(R)$:

$$\psi_{j,k}(t) = \psi_{a,b}(t) = 2^{\frac{-j}{2}} \psi(2^{-j} t - k),$$
$$j,k \in Z;\ z = \{0,1,2,....\}.$$

Generally, the wavelet transforms were evaluated by using dilation equations, given as:

$$\phi(t) = \sqrt{2} \sum_k l_k \phi(2t-k),$$
$$\psi(t) = \sqrt{2} \sum_k h_k \phi(2t-k).$$

Father and mother wavelets were defined by the last two equations where $\phi(2t-k)$ represents the father wavelet and $\psi(t)$ represents the mother wavelet. Father wavelet gives the high scale approximation components of the signal, while the mother wavelet shows the deviations from the approximation components. This is because the father wavelet generates the scaling coefficients and mother wavelet evaluates the differencing coefficients. Father wavelet defines the lower pass filter coefficients (h_k). High pass filters coefficients (l_k) are defined as [19].

$$l_k = \sqrt{2} \int_{-\infty}^{\infty} \phi(t)\phi(2t-k)dt,$$

$$h_k = \sqrt{2} \int_{-\infty}^{\infty} \psi(t)\psi(2t-k)dt.$$

Note: the mother wavelet $\psi(t)$ satisfies the following conditions [20]:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0,\ \int_{-\infty}^{\infty} |\psi(t)| < \infty,\ \int_{-\infty}^{\infty} \frac{|\psi_1(\omega)|^2}{|\omega|} d\omega < \infty.$$

Where $\psi_1(\omega)$ presents the Fourier transform, that is,

$$\psi_1(\omega) = \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt.$$

Haar wavelet transform is the oldest and simplest example in the wavelet transforms and is defined as:

$$\psi^{(H)}(t) = \begin{cases} 1, & 0 \le t < \frac{1}{2} \\ -1, & \frac{1}{2} \le t < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Daubechies wavelet transforms: Since Haar wavelet is the simplest and oldest wavelet transform; it was improved by Daubechies in 1992 [21]. He developed the frequency – domain characteristics of the Haar wavelet. However, we do not have a specific formula for this method of wavelet transform. So, we tend to use the square gain function of their scaling filter, the square gain function was defined as [11].

$$g(f) = 2\cos^l(\pi f) \sum_{l=0}^{\frac{l}{2}-1} \binom{\frac{l}{2} - 1 + l}{l} \sin^{2l}(\pi f).$$

l: Positive number and represents the length of the filter, for more details [5].

ARIMA Model: Application of nonlinear regression to price forecasting has not been reported so far. Other approaches of econometric modeling are univariate time series methods like auto regressive moving average (ARMA) [22, 23]. ARMA is a suitable model for the stationary time series data, although most of the software uses least square estimation which requires stationary. To overcome this problem and to allow ARMA model to handle non-stationary data, the researchers investigate a special class for the non-stationary data. This model is called Auto-regressive Integrated Moving Average (ARIMA). This idea is to separate a non-stationary series one or more times until the time series becomes stationary and then find the fit model. ARIMA model has got very high attention in the scientific world. This model is popularized by George Box and Gwilym Jenkins in 1970s, for more details and examples refer to [24].

There are a huge variety of ARIMA models. The general non-seasonal model is known as ARIMA (p, d, q) [24].

AR:
P = Order of the autoregressive part.
I: d = degree of first degree involved.
MA: q = order of the moving average part.

Note that, if there is no differencing been done (d = 0), Then ARMA model can be got from ARIMA model.

If non- stationary is added to a mixed ARMA model and then the general ARIMA (p, d, q) is obtained. The equation for the simplest case ARIMA (1, 1, 1) is as following:

$$(1-\Phi_1 B)(1-B)Y_t = c + (1-\theta_1 B)e_t.$$

The model building process involves the following steps [25]:

Model Identification: The first step is to determine whether the time series data is stationary or non-stationary. The stationarity can be assessed either using Dickey Fuller test or run sequence plots. If the original series has no trend (stationary) then the series is an ideal candidate for ARIMA. If the original series has trend (non-stationary), the series can be converted to stationary by differencing the series. The order of differencing is zero for a stationary series and greater than zero for non- stationary series.

Model Parameter Estimation: The estimation of parameters is very importance in the model building. The parameters thus obtained are estimated statistically by the method of least squares. A t-statistic shall be employed to test the parameters significance.

Model Diagnostics: Once the parameters are statistically estimated, before forecasting the series, it is necessary to check the adequacy of the tentatively identified model. The model is declared adequate if the residuals cannot improve forecast anymore. In other words, residuals are random. To check the overall model adequacy, the Ljung-Box Statistic is employed which follows a Chi-Square distribution. The null hypothesis is either rejected or not rejected based on the low or high p-value associated with the Statistic.

Forecasting: Once the model adequacy is established the series in question shall be forecasted for specified period. It is always advisable to keep track on the forecast errors and depending on the magnitude of errors, the model shall be re-evaluated.

The model building process involves the following steps; Model identification, Model parameter estimation, Model Diagnostics and Forecasting. For more details refer to [24].

In order to apply ARIMA model, the data should be stationary. Therefore, the return data can be considered for this comparison. Because in the financial literature, that is well known the return series is a stationary. Moreover, this result can be checked empirically provided that is a sufficient number of historical returns are available [26]. Moreover, the returns are serially uncorrelated which means that the data are stationary and suitable to apply ARIMA model with the return data directly without any treatments [27].

The daily logarithmic return, r_t for all market prices can be calculated using the definition of historical volatility as [25]:

$$r_t = \text{In}(p_t) - \text{In}(p_{t-1}$$

Where p_t indicates to the price information at time t.

Volatility: Researchers have already improved a lot of the definitions about volatility. In this paper we use the most popular and modern technique definition. In other words, it is defined as the absolute value of the daily return. It is mathematically expressed as [11]:

$$v_t = |r_t| = |\log(x_t) - \log(x_{t-1})|.$$

Where x_t represents the time series and r_t represents the daily return. stochastic volatility or time varying stochastic volatility model is very important in the financial mathematical fields to evaluate the derivative securities, such as options. the name derives from the models' treatment of the underlying security's volatility as a random process, governed by state variables such as the price level of the underlying security, the tendency of volatility to revert to some long-run mean value and the variance of the volatility process itself, among others. the volatility in a stochastic volatility model is changing randomly according to some stochastic differential equation or some discrete random processes [28].

In general, both the conditional volatility models and Time-Varying long memory in volatility are estimated under the assumption that the returns follow a t-distributional because this distribution performs far better than normal distribution. Moreover, the autocorrelation function plot (ACF) is used to identify the orders of an ARIMA process and then we obtain an appropriate model fitted to the data [29].

Mathematical Criteria: Two mathematical criteria have used in this comparison paper to justify the best model; root mean square error and mean absolute error [25]:

RMSE can be defined by:

$$RMSE = \left(\frac{\sum_{i=1}^{N} (\text{actual value-predicted value})^2}{N} \right)^{\frac{1}{2}}.$$

MAE can be defined by:

$$MAE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{\text{actual value-predicted value}}{\text{actual value}} \right|.$$

Where N represents the number of observations used for analysis.

DATA DESCRIPTION

In order to illustrate the effectiveness of Haar wavelet transforms and Daubechies wavelet transform, the Amman Stock Market data sets are selected for discussion. We consider volatility data for the time period from April 1993 (the days when stock exchanges were open) until December 2009 with a total of 4096 observations. The total number of observations for mathematical convenience is suggested to be divisible by 2^j [21]. It means that the data should satisfy the condition of observations 2^j. For more details refer to [25, 30].

The volatility data can be considered for this comparison without any treatment. Because in the mathematical literature, it is well known that ARIMA models can be used for the stationary and non-stationary

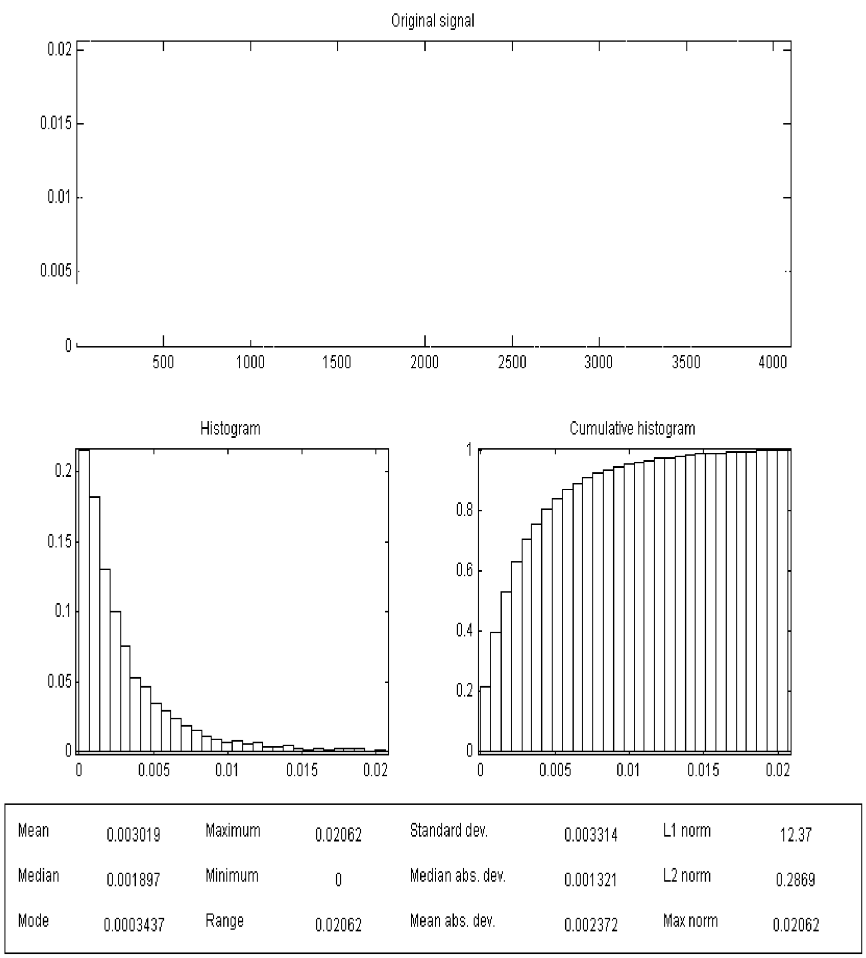

Fig. 1: Statistical analysis for the original data

data. Moreover, in financial mathematics literature many researcher showed that the forecasting based on wavelet-ARIMA model is better than the forecasting based on ARIMA directly and gives more accuracy results, in 2002 the forecasting financial data in Saudi stock market has discussed and they showed that forecasting based on wavelet-ARIMA model is better than the forecasting based on ARIMA directly [17], as well as 2005 this results have emphasized [31]. So that in this paper we will discuss the comparison of the forecasting in the orthogonal wavelet transforms methods deeply by using the approximation series data, since approximation series contains the main component of the transformed data and it shows all the important and sufficient information about the original series.

The following figure gives the basic statistical data description with its distributions for the original data set. Wavelet tool was used in this decomposition.

PROCEDURE AND RESULTS

Wavelet Levels and Coefficients: Figures 2 and 3 shows the distributions of the wavelet coefficients until level 2 based on Haar wavelet and Daubechies wavelet. The data set can be decomposed until level 12 since the total number is 4096 (4096=212 observations). However, similar results are returned from level 2 onwards, so only the data up to level 2 are decomposed, as suggested in [21].

Forecasting Procedures: The Prediction technique for the volatility time series data taken from Amman stocks market works as follows:

First Step: Transform the original data through the wavelet transform based on Haar wavelet transform and Daubechies wavelet transform.

Second Step: Evaluate the volatility data for the transformed data (based on Haar and Daubechies wavelets).

Third Step: After that evaluate the return data for the volatility data.

Fourth Step: Select the fitted ARIMA models for the approximation Haar wavelet series and Daubechies wavelet series, after that make the forecasting for the future data for each approximation series.

Fifth Step: Comparison all of these results and decide the best model depending on statistical criteria.

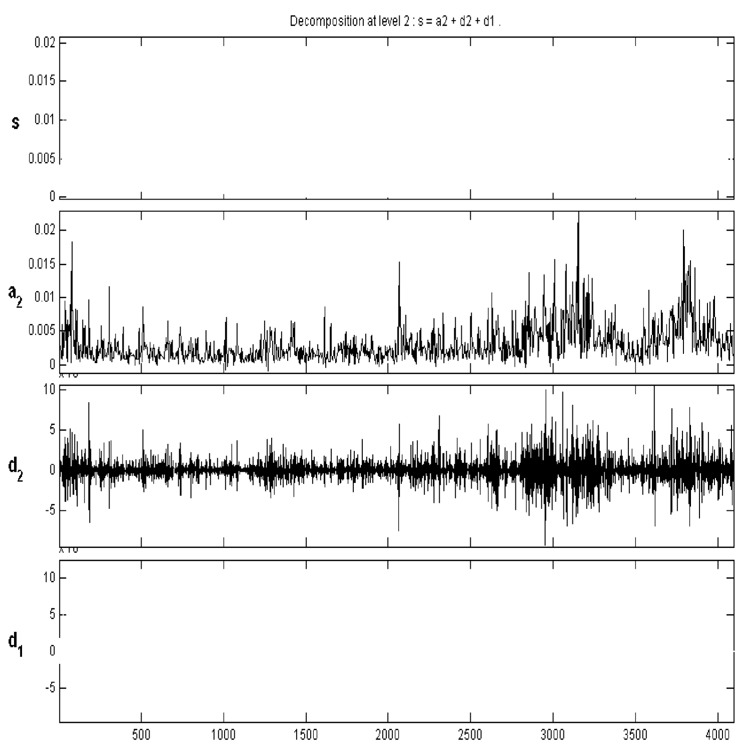

Fig. 2: Wavelet levels for Daubechies wavelet

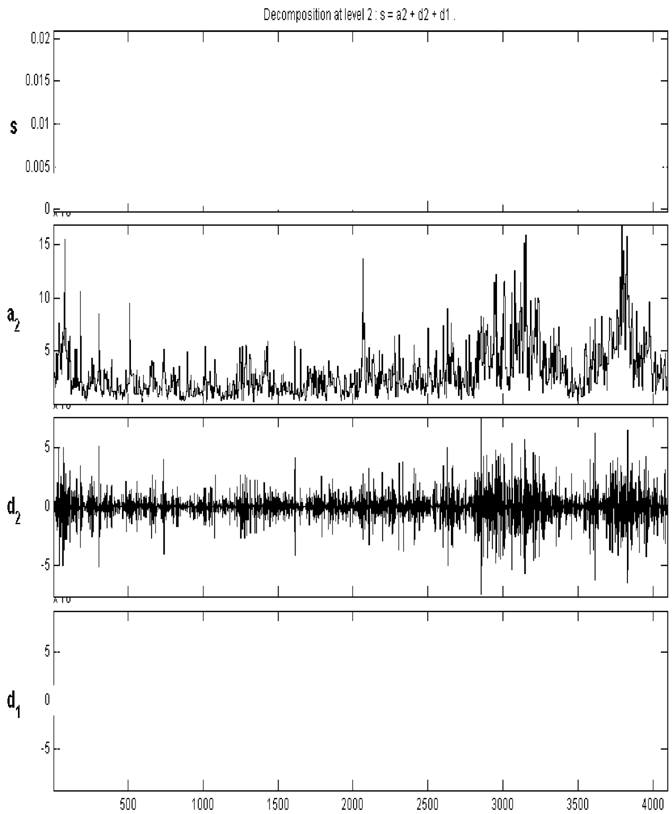

Fig. 3: Wavelet levels for Haar wavelet

ARIMA Forecasting Results: In this paper, the minimum value of RMSE and MAE is considered to select the best ARIMA model of the daily volatility data. All choices of ARIMA models for the volatility data are included in this test between (0,0,0) and (2,2,2). If we choose more than two, then there are more complicated conditions that should be satisfied. Also, if p and q are more than two, then Autocorrelation function (ACF) and partial Autocorrelation function (PACF) will be presented as an exponential decay. This means that ARIMA model becomes worthless and there is no importance.

Before forecasting with the final equation, it is necessary to perform various diagnostic tests in order to validate the goodness of fit of the model. A good way to check the adequacy of a Box-Jenkins model is to analyze the residuals $(Y_t - \hat{Y}_t)$. If the residuals are truly random, the autocorrelations and partial autocorrelations calculated using the residuals should be statistically equal or approximately to zero. If they are not, this is an indication that we have not fitted the correct model to the data. When this is the case, the residual of the ACF and PACF are contained information about which alternative models to consider.

Figures 4 and 5 showed the residuals of ACF and PACF for the return volatility data. All these values have significant t-test (less than 2). Thus, the residuals are random and the model is a good fit to the data. Also, the spikes are within the confidence limits.

The volatility data for Amman stocks market has been used as case study. Price forecasting is performed using daily data. Basically, the forecasting by using ARIMA (p, d, q) models under the wavelet transforms is better than the forecasting directly as well as it gives more accuracy results. So that, as a new contribution in this paper, the approximation series data have selection to make comparison fairly, also the same sample data are selected (From 1993-2009) for transforms and forecasting. The fit ARIMA model for the transform data by using Haar wavelet transform is selected as ARIMA (1,0,1) with root mean square error equal to 0.00238as presented in Table 1 also. Although the fit ARIMA model for the transform data by using Daubechies wavelet transform is selected as ARIMA (2,2,0) with root mean square error equal to 0.00349 Table 1 shows some other criteria about these results. All of these criteria explain that the Haar wavelet transform gives more sufficient result and better

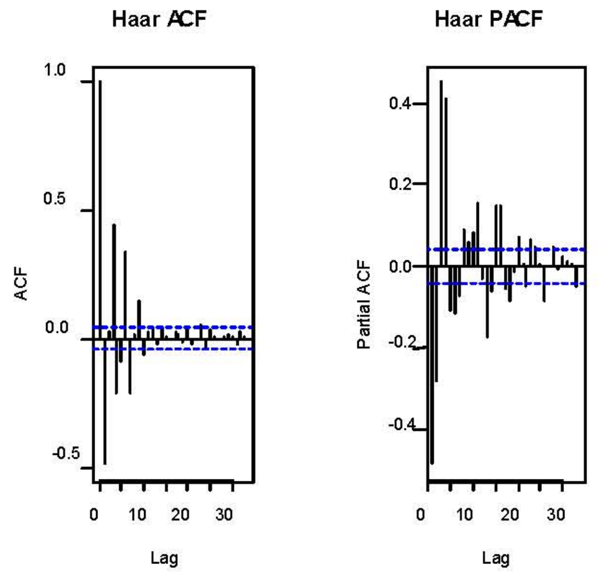

Fig. 4: Shows the Autocorrelation and partial Autocorrelation functions for the return volatility data based on Haar Wavelet

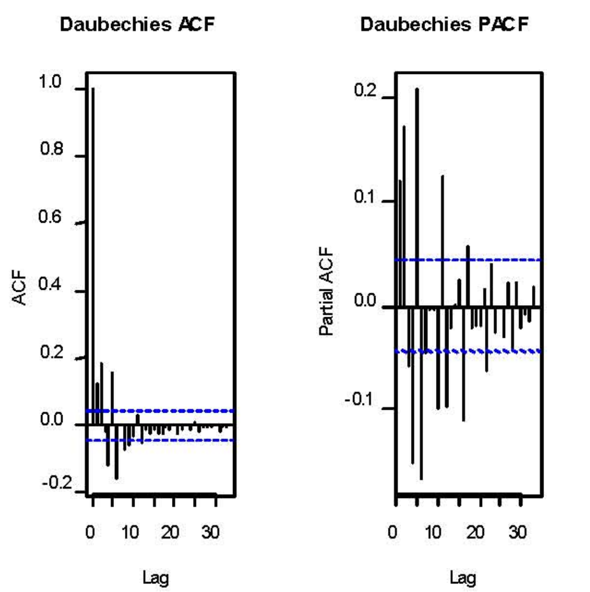

Fig. 5: Shows the Autocorrelation and partial Autocorrelation functions for the return volatility data based on Daubecies Wavelet

than Daubechies wavelet transform in the forecasting. However, in some statistical literature, Daubechies wavelet transform is better than Haar wavelet in the decomposition, however, experimentally in this paper we found a negative result, the reason is related to the data set since just the approximation coefficients series have used in the comparison.

Table 1: Shows all of the ARIMA options based on Haar wavelet transforms

Model (ARIMA)	RMSE	Model (ARIMA)	RMSE
(1,0,0)	0.0023824	(1,2,0)	0.002695
(1,0,1)	0.0023823	(1,2,1)	0.0027675
(1,0,2)	0.0023825	(1,2,2)	Not fitted
(1,1,0)	0.0024111	(2,0,0)	0.0023824
(1,1,1)	Not fitted	(2,0,1)	0.0023824
(1,1,2)	0.0033855	(2,0,2)	Not fitted
(2,1,0)	0.002468	(2,2,0)	0.0027351
(2,1,1)	0.003080	(2,2,1)	0.0028409
(2,1,2)	0.0030198	(2,2,2)	Not fitted
(0,0,1)	0.004990	(0,1,2)	0.0024200
(0,1,1)	0.0042154	(0,1,0)	0.0024096
(0,0,2)	0.0036271	(0,2,0)	0.0028595
(0,2,1)	Not fitted	(0,2,2)	Not fitted

Table 2: Shows all of the ARIMA options based on Daubechies wavelet transforms

Model (ARIMA)	RMSE	Model (ARIMA)	RMSE
(1,0,0)	0.00359	(1,2,0)	0.0035313
(1,0,1)	0.00357	(1,2,1)	0.003531
(1,0,2)	0.00359	(1,2,2)	0.003539
(1,1,0)	0.00372	(2,0,0)	0.003600
(1,1,1)	0.00436	(2,0,1)	0.00429
(1,1,2)	Not fitted	(2,0,2)	0.00356
(2,1,0)	Not fitted	(2,2,0)	0.00349
(2,1,1)	0.00371	(2,2,1)	0.00357
(2,1,2)	Not fitted	(2,2,2)	Not fitted
(0,0,1)	0.00525	(0,1,2)	0.00700
(0,1,1)	0.003711	(0,1,0)	0.003725
(0,0,2)	0.00430	(0,2,0)	0.00416
(0,2,1)	0.00364	(0,2,2)	0.00352

Table 3: Statistical of fit ARIMA model based on wavelet transforms

Statistical fit	Daubechies wavelet transform	Haar wavelet transform
RMSE	0.0027239	0.00238
MAE	0.0002457	0.0002200

CONCLUSION

As conclusion for this article, if the Wavelet transform is used for the volatility data, then there is no outlier, seasonal effects and other irregular effects. Also In this work, ARIMA based method for volatility forecasting involving application of wavelet transform has been presented. Wavelet transform has been applied to volatility to convert it into its constitutive series and their statistical properties are more like a normal distribution curve than the original series and can be utilized for better prediction. Therefore, experimentally the proposed model based on Haar wavelet transform gives better accuracy than the proposed model based on Daubecies wavelet transform in forecasting.

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REFERENCES

1. Fazlollah Lak, 2008. Bayesian Estimation of Change-Point in Unobserved-ARCH Models, World Appl. Sci. J., 3(3): 487-491.
2. Madhavan, A., 1992. Trading mechanisms in securities markets, J. Finance, 2: 607-641.
3. Glen, J., 1994. An introduction to the microstructure of emerging markets. International Finance Corporation Discussion. Working Paper No. 24 Washington.
4. Anuj Bhardwaj and Rashid Ali, 2009. Image Compression Using Modified Fast Haar Wavelet Transform, World Appl. Sci. J., 7(5): 647-653.
5. Afshari, M., 2008. Thresholding Kernel-regression Estimation and Empirical Distribution of Wavelet Coefficients for Regularity Regression Function, World Appl. Sci. J., 4(4): 605-609.
6. Mohammad Ali Lotfollahi-Yaghin and Mohammad Amin Hesari, 2008. Using Wavelet Analysis in Crack Detection at the Arch Concrete Dam under Frequency Analysis with FEM, World Appl. Sci. J., 3(4): 691-704.
7. Razdan, A., 2004. Wavelet correlation coefficient of strongly correlated time series, Physics A., 333: 335-342.
8. Brandon Whitchera, F. Peter and Peter Brownc, 2005. Time-varying spectral analysis in neurophysiologic time series using Hilbert wavelet pairs, Signal Processing, 85: 2065-2081.
9. Newland, D.E., 1993. An Introduction to Random Vibrations Spectral and Wavelet Analysis, Third edition, Prentice-Hall. Englewood Cliffs.
10. Manchanda, P., J. Kumar and A.H. Siddiqi, 2007. Mathematical methods for modeling price fluctuations of financial time series, J. Franklin Institute, 344: 613-363.
11. Gencay, R., F. Seluk and B. Whitcher, 2002. An Introduction to Wavelets and Other Filtering Methods in Finance and Economics, Academic Press, New York.
12. Mallat, S., 2001. A Wavelet Tour of Signal Processing, Academic Press, San Diego.
13. Abd-Elrahman, A., I.F. Shaker, A.K. Abdel-Gawad and A. Abdel-Wahab, 2008. Enhancement of Cloud-Associated Shadow Areas in Satellite Images Using Wavelet Image Fusion, World Appl. Sci. J., 4(3): 363-370.
14. Yao, S.J., Y.H. Song, L.Z. Zhang and X.Y. Cheng, 2000. Wavelet Transform and Networks for Short-term Electrical Load Forecasting, Energy Conversion and Management, 41(18): 1975-1988.
15. Dong, Z., X. Guo and J. Zheng, 2001. Calculation of Noise Resistance by Use of the Discrete Wavelets Transform, Electrochemistry Communications, 3: 561-565.
16. In-Keun Yu, Chang-Il Kim, Y. H. Song, 2000.A Novel Short-Term Load Forecasting Technique Using Wavelet Transform Analysis, Electric Machines and Power Systems, 24: 537–549.
17. Rumaih M. Alrumaih and Mohammad A. Al- Fawzan, 2002. Time series forecasting using wavelet denoising an application to Saudi stocks index, J. King Saudi Univ., 14(2): 221-234.
18. Chang Chiann and A. Pedro, 1998. A Wavelet Analysis for Time Series, Nonparametric Statistics, 10: 1-46.
19. Zbigniew, R., 2001. Wavelet Methods in (Financial) Time Series Processing. Physica A., 296: 307-319.
20. Chang, Chiann and A. Pedro, 1998. A wavelet analysis for time series, Nonparametric Statistics, 10: 1-46.
21. Daubechies, I., 1992. Ten Lectures on Wavelets, SIAM and Philadelphia.
22. Swider, D.J. and C. Weber, 2007. Extended ARMA models for estimating price developments on day-ahead electricity markets, Electric Power Systems Res., 77: 583-93.
23. Contreras, J., R. Espinola, F.J. Nogales and A.J. Conejo, 2005. ARIMA models to predict next-day electricity prices, IEEE Transactions on Power Systems, 18: 1014-20.
24. Spyros Makridakis, C. Steven and J. Rob, 1998. Forecasting methods and applications, Third edition, John Wiley& Sons, Inc, New York.
25. Todd Wittman, 2002. Time-Series Clustering and Association Analysis of Financial Data, CS 8980 Project.
26. Tim Bollerslev and Steve Lange, 1999. Forecasting financial market volatility: Sample frequency vis-à-vis forecast horizon, J. Empirical Finance, 6: 457-477.

27. Ruey S., 2005. Analysis of financial time series, Second edition, John Wiley & Sons, Inc., Publication, pp: 25.
28. Fouque Jean-Pierre, Papanicolaou George and Sircar K. Ronnie, 2000. Derivatives in financial markets with stochastic volatility, Cambridge University Press.
29. McAleer, M. and B.D. Veiga, 2008b. Single index and portfolio models for forecasting value-atrisk thresholds, J. Forecasting, 27: 217-235.
30. Mallat, S., 2001. A Wavelet Tour of Signal Processing, Academic Press, San Diego.
31. Antonio J. Conejo, Miguel A. Plazas, Rosa Espínola and Ana B. Molina, 2005. Day-Ahead Electricity Price Forecasting Using the Wavelet Transform and ARIMA Models. IEEE Transactions on Power Systems, 20(2): 1035-1042.