World Applied Sciences Journal 10 (1): 40-42, 2010
ISSN 1818-4952

Some New Second-order Balanced Designs on Square Lattice for Observing Variety Competition

[1]*Muhammad Nawaz,* [2]*Muhammad Zafar-Yab and* [3]*Munir Akhtar*

[1]Department of Statistics, The Islamia University of Bahawalpur, Pakistan
[2]Department of Statistics, University of Gujrat, Pakistan
[3]COMSATS institute of Information Technology, Wah, Pakistan

Abstract: The idea of constructing second-order balanced designs on square lattice is present in literature since 1980 but it remained ignored for almost three decades. A family of second-order balance designs on square lattice is introduced. The designs are balance with respect to both first and second order opposite neighbours separately. They are computer aided designs constructed for 21-hills, 28-hills and 36-hills.

Key words: Variety competition · Balanced designs · Elementary arrays · Test hill · Complementary half

INTRODUCTION

The type of interaction between plants when grown in mixtures may be termed as competition. Competition among plants is thought to be one of the primary factors determining plant performance in the field and has, therefore, been the most studied ecological interactions in plant communities.

If the lack of resources limits the growth of an individual then that individual has suffered from competition [1]. According to [2] components of a mixture use limiting resources more efficiently than pure stands. It has been observed in small grains when one component of a mixture is less susceptible to lodging it provides support for the second component [3]. The ability of plants to avoid competition through suitable place differentiation is quite limited, although plants can use resources in different proportions [4] at different depths in the soil [5, 6] or at different times of the year [7, 8] assessed productivity of mixtures of oats and wheat and compared two different approaches used in plant competition studies.

[9] presented a class of balanced designs, with respect to first order neighbours, having 50-50 mixture for two varieties sown on a square lattice. The designs consist of monoculture rows and rows on which two varieties are strictly alternating. There are equal numbers of hills having each of 0, 1, 2, 3 and 4 hills of the opposite variety amongst their nearest neighbours. On a square lattice a plant has four nearest (first-order) neighbours at a distance d and next nearest four at (second order neighbours). The plants at different distances from the central plant are likely to produce different effects. [10] assumed that the effect of second and higher order neighbours is negligible but it is possible only if plantation is not at close proximity e.g. mango or banana trees. However, for plantation at close proximity (cereals or vegetables) it seems unnatural to ignore second order neighbours, as they are not too far away to be neglected. Thus, it is obligatory to develop designs that are balanced with respect to both the first as well as the second order opposite neighbours separately. The term second-order balanced array/design here is used for balanced with respect to both first and second-order nearest opposite neighbours separately. There are 10 such levels of competition on a square lattice and 14 levels on a triangular lattice. Let 0 and 1 represent the two varieties. In the arrangement, the plantation of single plant is called a *hill plot* or a *hill*. All hills except the border hills are *testable hills*. An array in which the plants of a variety at testable sites are immediately surrounded by exactly 0 . .. 4 (1st and 2nd order separately) plants of the opposite variety is said to be a *second-order basic-balanced-array*. The array will be *self buildable* if it can be built into a larger design by rearrangements in its isomorphism class by some overlapping.

Construction of Balanced Arrays of 21 Hills: In searching for self buildable arrays attention is confined to balanced arrays in complementary halves.

Corresponding Author: Muhammad Nawaz, Department of Statistics, The Islamia University of Bahawalpur, Pakistan.

Consider a 3-row array where each row consists of 7 hills as shown in Fig. 2.1. The possible configurations of 0's and 1's to be considered are 2^{21}. Without loss of generality assigning arbitrarily variety 0 (say) to the leftmost hill of the second row, thus configurations of 0's and 1's to be investigated reduced to 2^{20}. For the present study balanced arrays are constructed through a computer aided search incorporating the knowledge of combinatorics. The first order neighbours are the neighbouring hills in the same column and in the same row of the considered test-hill whereas the second order neighbours are the neighbouring hills in the diagonals of the considered test-hill.

For the construction of a second order symmetric balanced array, containing three rows, the possible numbers of second order nearest neighbours that can be obtained are either zero, two or four. These can't result into balanced array. Therefore, the smallest possible array that can conceivably be balanced is of 21-hills having three different rows (*i.e.* non-symmetric about major axis). There are 16 isomorphism classes of such arrays.

Considering the Array Number N=14 in Table 2.1.: The first and the second element in the subscript of testable hills are numbers of first and second order opposite neighbours respectively. Their respective configurations are (0, 1, 4, 2 and 3) and (2, 0, 4, 1 and 3). It can be seen that the array in Fig. 2.2 is balanced both for first and second-order opposite neighbours separately. Three rows of binary digits converted to octal base that determines the array are presented in Table 2.1. Let N identifies an array number in this study. Each of these arrays and its complement when placed side by side give rise to basic balanced designs. Such designs have a test ratio of 23.81% and are categorized as Type Z in Table 5.1. All other arrays can generate better test-ratio designs when their respective complement is allowed to overlap some of its columns.

Rules for Building Basic Design and Their Extension to Larger Size Design: Let Δ_N, Θ_N, Γ_N and Ω_N denote the elementary balanced array, its complement and its reflection in minor axis and reflection in major axis or mirror image respectively. Also let Δ_N, $\grave{E}_N$, Γ_N and Ω_N denote the basic balanced design, its complement, its reflection in minor axis and reflection in major axis respectively.

Rule L: for the Construction of a Basic Balanced Design Δ_n Take One of the Following Actions: Overlap the last column of Δ_N with the

L_1 : First column of $\grave{E}_N$.
L_2 : First column of complement of Ω_N.

Overlap the last two columns of Δ_N with the

L_3 : First two columns of complement of Γ_N.
L_4 : First two columns of complement of both reflections; Γ_N and Ω_N applied successively on the same Δ_N.

Rule M: for the Construction of Larger Designs Based on Δ_n Select One of the Following Actions: Join a copy of Δ_N by superimposing the last column of Δ_N and the first column of

M_1 : Δ_N for the designs of Types L_1 and L_2. Continue the process to the desired length.
M_2 : $\grave{E}_N$ for the designs of Types L_3. Continue the process by taking complement of the preceding copy attached.
M_3 : Complement of Ω_N for the designs of Type L_4. Continue the process by attaching complement of reflection in major axis of the preceding copy.

The designs thus obtained are given the same names as the building rules they follow for basic design as well as larger designs. Single overlap column consists of non testable hills where as the pair of overlap columns consist of test-hills.

Building Large Size Design: Let h and v be the numbers of horizontal and vertical copies of Δ_N. The basic balanced design is realized by placing side by side an elementary array and its complement. Larger designs can be obtained by repeating the basic design either vertically or horizontally or in both directions. To study the effect of second-order opposite neighbours, basic balanced design is rotated downwards about the last row to obtain *nearly self-buildable designs* vertically. In these designs all the hills except the internal hills of even row numbers are non-testable hills. Thus there are (r-v-2) rows of test-hills, where r is the number of rows in an array. Nearly self buildable arrays are obtained for all the arrays in the sections that follow. The 16 elementary arrays in Table 2.1, containing five testable hills, are divided into five types as given in Table 3.1.

Type Z: Array number N=3 is horizontally non-buildable, however, it is nearly self-buildable vertically to any depth as shown in Fig. 3.1.

Only elementary array portion is shown in Fig. 3.1. Basic balanced design is obtained by placing elementary array and it complement side by side.

Type L_2M_1 : In Fig. 3.2 the construction of basic design is realized by rule L_2 and its extension into larger size design is realized by rule M_1 and thus named as Type L_2M_1. Construction of nearly buildable designs is illustrated with the help of figures in three stages. In Fig. 3.2(a) elementary balanced array is shown then Fig. 3.2(b) shows basic balanced design and at the last stage Fig. 3.2 shows its extension into lager designs.

Type L_4M_3: Consider array number N=13.
In Fig. 3.3, the construction of basic design (Δ) is realized by rule L_4 and its extension into larger size design is realized by rule M_3.

The Construction of 28-hill Second Order Balanced Arrays: In the hunt for second-order balanced arrays in complementary halves the next larger array that can conceivably be balanced is of 28 hills. The hills are arranged in four rows containing 7 hills each. There are only two isomorphic classes of second order balanced arrays and they are presented in Table 4.1.

Both of the above arrays have different buildability structure in both horizontal and vertical directions.

Type Z: The buildability structure of this array is same as that of Section 3.1.1.

Type L_2M_1: The design shown in Fig. 4.1 has maximum test ratio among all second order designs constructed in the present study.

Construction of Balanced Arrays of 36 Hills: Consider 36 hill plots arranged in three rows, each row consisting of 12 hills. There are 364 isomorphic classes of second order balanced arrays which are non-symmetric. They can be divided into two types with respect to their horizontal buildability vis-à-vis the ones which do not and the other which do so. They are presented in Table 1 (Appendix A) in their octal representation.

Type Z: The buildability structure of these 196 arrays is same as that of Section 3.1.1.

Type L_1M_1: Amongst 364 isomorphism classes there are 168 isomorphism classes of Type L_1M_1. These arrays are self-buildable in horizontal direction but nearly self-buildable in vertical direction. Those are presented in Table 1 of Appendix at the end.

Consider Array Number N=50: The construction of basic design is realized by rule L_1 and its extension into larger size designs is realized by rule M_1. The test ratio formulae for different number of hills for all design types in previous sections are given in Table 5.1.

REFERENCES

1. Stoll. P. and J. Weiner, 2000. A Neighborhood view of interactions among individual plants. In: The Geometry of Ecological Interactions: Simplifying Spatial complexity, eds. Dieckmann U, Law R and Metz JAJ, Cambridge University Press. pp: 11-27.
2. Bulson, H.A.J., R.W. Snaydon and C.E. Stopes, 1997. Effects of plant density on intercropped wheat and field beans in an organic farming system. J. Agric. Sci. Camb., 128: 59-71.
3. Sobkowicz, P., 2003. Interspecific competition in mixtures of spring cereals. Zesz. Nauk. AR Wroc. 458, Rozprawy, Wydawnictwo Akademii Rolniczej we Wroclawiu. 1-105 (in Polish, abstr. in English).
4. Tilman, D., 1982. On the meaning of competition and mechanism of competitive superiority. Funct. Ecol., 1: 304-315.
5. Parrish, J.A.D. and F.A. Bazzaz, 1976. Underground niche separation In successional plants. Ecol., 57: 1281-1288.
6. Parrish, J.A.D. and F.A. Bazzaz, 1985. Ontogenetic niche shifts in old-field annuals. Ecol., 66:1296-1302.
7. Eissenstat, D.M. and M.M. Caldwell, 1988. Seasonal timing of root growth in favorable microsites. Ecol., 69: 870-873.
8. Sobkowicz, P. and E. Tendziagolska, 2005. Competition and Productivity in Mixture of Oats and Wheat. J. Agronomy and Crop Sci., 191: 377-385.
9. Veevers, A. and Zafar-Yab, 1980. Balanced designs for two-component competition experiments on a square lattice. Euphytica. 29: 459-464.
10. Zafar-Yab, M., 1980. Construction and Statistical analysis of experimental designs for observing variety competition. Unpublished Ph.D. Thesis, University of Liverpool.,