World Applied Sciences Journal 21 (Mathematical Applications in Engineering): 68-72, 2013
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2013.21.mae.99923

Fmilp Formulation for Aggregate Production Planning

[1]*Mohamed K. Omar*, [2,3]*Muzalna Mohd Jusoh and* [3]*Mohd Omar*

[1]Nottingham University Business School,
University of Nottingham (Malaysia Campus), Jalan Broga, 43500 Semenyih, Selangor, Malaysia
[2]Faculty of Manufacturing Engineering, Universiti Teknikal Malaysia Melaka,
Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia
[3]Institute of Mathematical Sciences, University of Malaya, 50603 Kuala Lumpur, Malaysia

Abstract: This paper considers a specialty chemical plant that produces multiple batch product families that involve setup time and cost each time the production run is switched from one product family into another. The aim of the paper is to introduce the manufacturing environment and explain the challenges facing the planners when dealing with the aggregate production planning (APP) preparations. Moreover, this paper introduces a fuzzy mixed-integer linear programming (FMILP) modeling approach that the authors developed to deal with the multi-product APP problems confronting the specialty chemical plant. The objective of the proposed model is to minimize the sum of production, set-up, inventory, backorder and workforce costs. The model formulation incorporates the fuzzy set theory and possibilistic theory to define the uncertainties that appear in the model's objective and constraints.

Key words: Aggregate production planning · Fuzzy mathematical programming · Possibilistic linear programming · Uncertainty

INTRODUCTION

Aggregate production planning (APP) is a process of developing firm plans to meet the forecast demand for the forthcoming period, up to approximately 12 months into the future, all in aggregate terms. APP combines all the similar product costs and demand seasonality into a group and determines the best production levels for each product group to meet the uncertain demand by adjusting the controllable variables, such as inventory levels, backorder, workforce levels and others.

A study of the literature on APP modeling reveals that there are two models available to formulate the APP problems. These models are deterministic modeling and fuzzy modeling. Basically, the deterministic model assumes that all required data inputs can be uniquely determined while the fuzzy modeling considers some of data input could be uncertain, in which the values of variable cannot be precisely identified. In practical, APP problem is a medium range planning with lots of uncertain elements involved, such as customer demand, production variation and operating cost. Therefore, the fuzzy modeling approach is more sufficient to model the uncertainties faced in real world rather than the deterministic modeling.

In 1970, Bellman and Zadeh [1] first introduced the concept of fuzzy set theory to deal with fuzziness that appears in the decision making problem. They define fuzzy set as a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. In this regard, all the imprecisely defined parameters can be defined by preference-based membership function. The concept of fuzzy sets has been successfully applied in many areas, including APP problem. Tang *et al*. [2] formulated a multi-product APP problem as a fuzzy quadratic programming with both fuzzy demand and fuzzy constraint appears in the same model. The solution procedure based on fuzzy optimization approach has been proposed to construct a variant of choices of APP plan towards the decision-maker. Wang and Fang [3] presented a fuzzy linear programming model to solve multi-objective APP problem. The proposed model assumed the product price, unit cost to subcontract, work force level, production capacity and

Corresponding Author: Muzalna Mohd Jusoh, Faculty of Manufacturing Engineering, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia.

market demands are fuzzy parameters that represent by trapezoidal form. Fung *et al*. [4] and Wang and Liang [5] adopted a fuzzy approach to formulate a multi-product APP problem under the environment of fuzzy demands. In 2005, Wang and Liang [6] developed an interactive possibilistic linear programming model to solve the real-world multi-product APP problem. Liang *et al*. [7] developed a fuzzy mathematical programming model to solve multi-product and multi-periods APP problems with fuzzy parameters.

In this paper, a fuzzy mixed-integer linear programming (FMILP) model is developed for the APP problem where setups occur when switching from one product family into another. The proposed model intends to minimize the total production, set-up, inventory and backorder costs, in a fixed workforce size. Silva *et al*. [8] indicated that model with a constant workforce size might reduce the total significant cost in a realistic environment. This paper is organized as follows. Firstly, section 2 describes the research environment. Section 3 then develops the FMILP model with the detailed assumptions. Section 4 discusses the solution procedure of the model and finally, conclusions are presented in Section 5.

Production Environment: In this section, the production environment of a resin manufacturing company in South-East Asia originally introduced by Omar and Teo [9] will be addressed. The company produces over 100 finished products per annum and only 20 products out of the available range are considered as fast moving products. These fast-moving products are categorised by five families. Family 1 has 10 products; Family 2 has 6 products; Family 3 has 2 products and both Families 4 and 5 have a single product each. The plant has two production lines having identical machines. Families 1 to 3 can be produced at both production lines while Family 4 and Family 5 are only produced on line two. The plant operates on three shifts and each shift requires seven people to run the process and the workers involved cannot be fired or lay-off. Due to storage limitations, the company allows the safety stock within a restricted fixed period of time only. When the demand estimates for the next year are ready, marketing division passes these estimates to the production division to prepare the operational budget for the next year. The order batching process starts when the production planner receives customer's orders with due dates. The ultimate objective of this process is to meet the customers' due dates and minimize set-up activities. Resource sharing, while preparing the production plans and schedules, is practised. Likewise, stiff competition limits the firms' ability to adjust delivery dates for confirmed orders. This situation leads to a very high level of plant utilization giving the production manager no alternative but to allow backordering of unfulfilled demands.

Model Formulation: In this section, we developed a FMILP formulation based on the ordinary linear programming model proposed by Omar and Teo [9]. The FMILP model aims to minimize the total costs with reference to production, set-up, inventory, backorder and workforce costs. Due to incompleteness or unavailability of required inputs data over the planning horizon, some of the parameters and constraints are considered as uncertainties. The proposed model has two varieties of uncertainties that appear in the same model. These uncertainties are an imprecise parameters denoted by a tilde, ~ and a fuzzy constraints denoted by a 'hat', ^, in order to differentiate the entities. Possibility distribution is used to define the imprecise parameters and a preference-based membership function represents the fuzzy constraints. There are various types of fuzzy numbers and triangular fuzzy number is one of the most adopted in literature due to its simplicity in data acquisition and computational efficiency [10].

The FMILP model formulated here is based on the following assumptions: (1) A setup time and setup cost are considered whenever production changes from one product family to another. These setups are negligible among products within the same family. (2) The process is assumed to be a single stage having identical machines in parallel configuration. The model's parameters, decision variables and FMILP formulation are presented below.

Parameters:

$\hat{Z}_{it}$ Unit production cost for product family i (excluding labour) in period t
$\hat{V}_{it}$ Production changeover cost for product family i in period t
$\hat{H}_{it}$ Unit inventory holding cost for product family i in period t
$\hat{CB}_{it}$ Unit backorder cost for product family i in period t
$\hat{CR}_{t}$ Manpower cost in period t
$\hat{D}_{it}$ Demand for product family i in period t
SC_t Maximum available storage capacity in period t
$\hat{Q}_{tl}$ Capacity available for production line l in period t
$P_i^{\min}$ Minimum batch size for product family i
$\hat{A}_i$ Unit process time for product family i (man-hour/units)
G_i Production changeover time required for product family i
$\hat{TR}_t$ Total regular time available in period t
M_t Upper bound on production of family i in period t

Decision Variables

x_{itl} Production level of product family i in line t in period t

h_{it} Inventory level of product family i in period t

b_{it} Backorder level of product family i in period t

ϕ_{itl} Binary changeover variable for product family i in line l in period t

s_t Time consumed in set-up activities in period, t, $s_t = \sum_{i=1}^{N}\sum_{l=1}^{L} G_i\phi_{itl}$

w_t Time consumed in production activities in period, t, $w_t = \sum_{i=1}^{N}\sum_{l=1}^{L} \hat{A}_i x_{itl}$

Fuzzy Mixed-integer Linear Programming Model

$$Min \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{l=1}^{L}\left(\hat{Z}_{it}x_{itl} + \hat{V}_{it}\phi_{itl}\right) + \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\hat{H}_{it}h_{it} + \hat{C}B_{it}b_{it}\right) + \sum_{t=1}^{T}\hat{C}R_t\left(w_t + s_t\right) \quad (1)$$

subject to

$$\sum_{I=1}^{L} x_{itl} + h_{i,t-1} - b_{i,t-1} - h_{it} + b_{it} \cong \hat{D}_{il} \quad \forall_{i,t} \quad (2)$$

$$\sum_{1}^{L} h_{it} \le SC_t \quad \forall_t \quad (3)$$

$$\sum_{1}^{N} x_{itl} \lesssim \hat{Q}_{tl} \quad \forall_{t,l} \quad (4)$$

$$x_{itl} \ge P_i^{\min}\phi_{itl} \quad \forall_{i,t,l} \quad (5)$$

$$x_{itl} \le M_i\phi_{itl} \quad \forall_{i,t,l} \quad (6)$$

$$\sum_{1}^{N}\sum_{1}^{L}\hat{A}_i x_{itl} + \sum_{1}^{N}\sum_{1}^{L} Gi\phi itl \lesssim \hat{T}R_t \quad \forall_t \quad (7)$$

$$x_{itl} = 0 \quad \text{for} \quad l = 1 \quad \text{and} \quad i > 3 \quad \forall_t \quad (8)$$

$$\phi_{itl} + \phi_{i,t-1,l} = 1 \quad \text{for} \quad l = 2 \quad i>3, t \ge 2 \quad (9)$$

$$x_{itl}, h_{it}, b_{it}, s_t, w_t \ge 0 \quad (10)$$

$$\phi_{itl} \in \{0, 1\} \quad (11)$$

The objective function (1) is to minimize the sum of production, setup, inventory, backorder and workforce costs for N products family over the planning horizon of T periods. All the objective function coefficients $\left(\hat{Z}_{it}, \hat{V}_{it}, \hat{H}_{it}, \hat{C}B_{it}, \hat{C}R_t\right)$ are assumed to be imprecise parameters characterized by triangular possibility distribution. For example, in deterministic formulation, production costs for each family in each period $\left(\hat{Z}_{it}\right)$ is very hard to determine and as a result and to be more realistic this parameter was made to be imprecise to reflect real-life situation. Equation (2) expresses the relationship between production, inventory and backorder with family product demand. The families' product demand is always unknown and need to be predicted. In deterministic formulation, the demand is assumed to be known and will not change during the planning period, unrealistic assumption that bound to be very costly. For this reason, in our formulation, we propose that the demand is imprecise. Equation (3) is storage limitation that ensures an enough space to store the safety stock and once more, finding the exact inventory space per planning period.. Equation (4) state the capacity limitation involving the imprecise parameter in the right-hand side with soft inequalities. In deterministic formulation, the capacity is assumed to be available in each planning period, however, in real life industrial situation, usually it is not the case and often; the true available capacity is imprecise. Equations (5) and (6) enforce a minimum batch size requirement for each product family in each production line in each time the planning period. Equation (7) states that the total labour capacity for each product family in each planning time period is sufficient for both production and setup activities. This fuzzy constraint also involves the imprecise parameter in both left and right-hand sides. Equations (8) and (9) are designed to ensure that product families 4 and 5 can only be produced in production line two and once in each two months. Equation (10) is **a** non-negativity constraint and equation (11) defines the binary variable.

Fmilp Model Solution Approaches: In order to solve the proposed FMILP model, two main steps are considered as the solution procedure as presented next.

- Converting the FMILP model into its equivalent crisp model.
- Converting the crisp multi-objective model into its equivalent single-objective model to obtain a preferred solution.

In this paper, to achieve step (i), we used the approach by Lai and Hwang [11], together with fuzzy ranking method proposed by Ramik and Rimanek [12]. Triangular possibility distribution and triangular membership function is introduced for modeling the imprecise data and fuzzy inequality/equality, respectively. On the other hand, steps (ii) were accomplished using Zimmermann's approach [13] and Torabi and Hassini [14] approach. The problem was solved using LINGO 11.0, while the input data and solutions was exported and imported by Microsoft Excel.

RESULTS AND DISCUSSION

To evaluate the model, we compare the best results of TH method with the results obtained by deterministic model reported by Omar and Teo [9]. The objective of this evaluation is to demonstrate the benefits of applying fuzzy approach to solve real-life APP problem. This paper considered four out of the five parameters originally proposed by Mula *et al*. (2008) to evaluate the model's performance. The four parameters are: (i) the service level; (ii) the levels of inventory; (iii) total costs and (iv) computational efficiency. Moreover, we consider a new parameter define as average capacity utilization introduced by Mula *et al.* (2010) in another research work, but with different formula is presented by equation (12). Average capacity utilization.

$$(\%) = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} x_{itl}}{\sum_{t=1}^{T} Q_{tl}} \times 100 \qquad \forall l \qquad (12)$$

The results obtained from the deterministic model and the fuzzy model is shown in Tables 1 and 2 respectively. Considering these tables results, it can be seen that fuzzy model is the winner in terms of obtaining results with minimum planning total costs. The optimal solution when applying FMILP model was 28,287,092 (MU). The result was improved about 10.60% or 3.2 Million (MU) compared with the deterministic model's result.

The computational efficiency of the best FMILP solution and the deterministic model solution are presented in Tables 3 and 4. The results indicate that the fuzzy model consumed more time to reach the optimal solution since this model has more constraints than the deterministic. However, the increase of the computational time is within an acceptable computational time in real industrial applications.

Table 1: MILP and FMILP Results

Model	Production Cost	Setup Cost	Holding Cost
Deterministic	31332254	116100	78215
FMILP	28105810	107500	65428

Table 2: MILP and FMILP Results

Model	Backorder Cost	Manpower Cost	Total Cost
Deterministic	0	9345	31535914
FMILP	0	8354	28287092

Table 3: Models Efficiency Comparison

Model	Variables	Integer	Constraint
Deterministic	485	120	400
FMILP (α = 0.55)	408	120	528

Table 4: Models Efficiency Comparison

Model	Non-zero	Time (seconds)
Deterministic	1592	0.01
FMILP (α = 0.55)	2967	0.05

Table 5: Average Capacity Utilization

	Average capacity utilization (%)	
α	Production line 1	Production line 2
0	68.74	50.96
0.1	46.04	73.28
0.2	61.30	60.68
0.3	54.29	73.01
0.4	64.13	69.26
0.5	75.22	59.14
0.6	62.47	77.47
0.7	77.51	67.06
0.8	96.63	48.18
0.9	80.43	70.99
1.0	95.37	58.70
Deterministic model	83.22	70.30

Unlike the MILP deterministic model, employing the FMILP approach to provide solutions to the aggregate production planning, the decision maker could select appropriate values for (α) that express the minimal acceptance level of satisfaction can determine the desired level of capacity utilization level. Therefore, the results shown in Table 5 provides a general good idea on how to manage available production lines utilization based on the demand and the minimal acceptance level of satisfaction.

CONCLUSION

In this paper, the authors presented a fuzzy mixed integer linear programing model (FMILP) for aggregate production problem with multi-product and multiple-time periods. In addition, the presented model considers setups activities every time a production is switched from one product family into another, making our formulation approach much closer to a real-life industrial problem. The proposed FMILP formulation incorporates the concept of fuzzy sets and possibilistic theory to deal with the uncertainties that appears in the parameters and constraints. Validation of the proposed FMILP was carried out using the real data taken from a resin manufacturing plant and the obtained results were compared with an ordinary deterministic MILP. Comparing the results of the two planning approaches indicated that the proposed FMILP model could produce a less planning costs of the two approaches. Besides, the authors compared both models computational efficiency and the comparison shows that both models almost are similar in the computational efforts. Furthermore, our paper shows the advantage that FMILP in which the decision maker could select appropriate values for (α) that express the minimal acceptance level of satisfaction can determine the desired level of capacity utilization level.

An opportunity for further research is the development of a fuzzy mathematical model for the development of master production schedule that aims to disaggregate the aggregate planning decisions developed in this paper.

ACKNOWLEDGMENTS

This research is supported by the Ministry of Higher Education (MOHE), Malaysia and the Universiti Teknikal Malaysia Melaka, Malaysia.

REFERENCES

1. Bellman, R.E. and L.E. Zadeh, 1970. Decision-making in a fuzzy Environment. Management Science, 17: 141-164.
2. Tang, J., D. Wang and R.Y.K. Fung, 2000. Fuzzy formulation for multi-product aggregate production planning. Production Planning & Control: The Management of Operations, 11(7): 670-676.
3. Wang, R.C. and H.H. Fang, 2001. Aggregate production planning with multiple objectives in a fuzzy environment. European Journal of Operational Research, 133: 521-536.
4. Fung, R.Y.K., J. Tang and D. Wang, 2003. Multiproduct aggregate production planning with fuzzy demands and fuzzy capacities. IEEE Trans. Syst., Man, Cybern. A, Syst., Humans, 33: 302-313.
5. Wang, R.C. and T.F. Liang, 2004. Application of fuzzy multi-objective linear programming to aggregate production planning. Computer and Industrial Engineering, 46: 17-41.
6. Wang, R.C. and T.F. Liang, 2005. Applying possibilistic linear programming to aggregate production planning. International Journal of Production Economics, 98: 328-341.
7. Liang, T.F., H.W. Cheng, P.Y. Chen and K.H. Shen, 2011. Application of fuzzy sets to aggregate production planning with multiproducts and multitime periods. IEEE Transactions on Fuzzy Systems, 19: 465-477.
8. Paulo Silva, J., J. Lisboa and P. Huang, 2000. A labour-contrained model for aggregate production planning. International Journal of Production Research, 38(9): 2143-2152.
9. Omar, M.K. and S.C. Teo, 2007. Hierarchical production planning and scheduling in a multi-product, batch process environment. International Journal of Production Research, 45: 1029-1047.
10. Liang, T.F., 2011. Application of fuzzy sets to manufacturing/distribution planning decisions in supply chains. Information Sciences, 181: 842-854.
11. Lai, Y.J. and C.L. Hwang, 1992. A new approach to some possibilistic linear programming problems. Fuzzy Sets and Systems, 49: 121-133.
12. Ramik, J. and J. Rimanek, 1985. Inequality relation between fuzzy numbers and its use in fuzzy optimization. Fuzzy Sets and Systems, 16: 123-138.
13. Zimmermann, H.J., 1978. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems, 1: 45-55.
14. Torabi, S.A. and E. Hassini, 2008. An interactive possibilistic programming approach for multiple objective supply chain master planning. Fuzzy Sets and Systems, 159: 193-214.
15. Mula, J., R. Poler and J.P. Garcia-Sabater, 2008. Capacity and material requirement planning modeling by comparing deterministic and fuzzy models. International Journal of Production Research, 46(20): 5589-5606.
16. Mula, J., D. Peidro and R. Poler, 2010. The effectiveness of fuzzy mathematical programming approach for supply chain production planning with fuzzy demand. International Journal of Production Economics, 128: 137-143.