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On a Subclass of Starlike Univalent Functions

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Abstract: The aim of this paper is to introduce and study new classes of analytic functions by using a well-known convolution operator $L(a, c)$ which was introduced by Carlson and Shaffer [1]. Sharp coefficient bound and some inclusion results are discussed. Invariance of these classes under convolution with convex functions has also been examined.

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INTRODUCTION

Let A be the class of functions f analytic in the open unit disk $E = \{z: |z| < 1\}$ and

$$f(z) = z + \sum_{n=2}^{\infty} b_n z^n. \tag{1.1}$$

Let the incomplete beta function $\phi(a, c)$ be defined as

$$\phi(a, c)(z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} z^{n+1}, \tag{1.2}$$

Where $z \in E,\ a \in \mathbb{C},\ c \in \mathbb{C} \setminus \mathbb{Z}_0^-,\ \mathbb{Z}_0^- = \{0, -1, -2, \ldots\}$ and $(\upsilon)_n$ is the Pochhammer symbol (or the shifted factorial) defined (in terms of Gamma function) by

$$(\upsilon)_n = \frac{\Gamma(\upsilon+n)}{\Gamma(\upsilon)} = \begin{cases} 1, & \text{if } n = 0,\ \upsilon \in \mathbb{C} \setminus \{0\}, \\ \upsilon(\upsilon+1)(\upsilon+2)\ldots(\upsilon+n-1), & \text{if } n \in \mathbb{N},\ \upsilon \in \mathbb{C}. \end{cases} \tag{1.3}$$

The convolution (or Hadamard product) of two analytic functions $h_1(z) = \sum_{n=1}^{\infty} b_n z^n$ and $h_2(z) = \sum_{n=1}^{\infty} c_n z^n$ is defined as

$$(h_1 * h_2)(z) = \sum_{n=1}^{\infty} b_n c_n z^n. \tag{1.4}$$

By using the function $\phi(a, c)$ and convolution, Carlson and Shaffer [1] introduced a linear operator $L(a, c): A \to A$. It is defined as

$$L(a, c)f(z) = \phi(a, c)(z) * f(z) \tag{1.5}$$

$$= z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} b_n z^n. \tag{1.6}$$

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Where f is defined by (1.1). It can easily be seen from (1.3) and (1.6) that $L(2, 1)f(z) = zf'(z)$ and

$$z(L(a, c)f(z))' = aL(a+1, c)f(z) - (a-1)L(a, c)f(z). \tag{1.7}$$

Furthermore, we note that

$$L(\delta+1,1)f(z) = D^{\delta}f(z), \; (\delta > -1), \tag{1.8}$$

Where the symbol D^{δ} denotes the well-known Ruscheweyh derivative for $\delta \in \mathbb{N}_0 = \mathbb{N}\cup\{0\}$, which was introduced by Ruscheweyh [19]. Noor [10] and Noor and Noor [14] defined and studied an integral operator $I_n : A \to A$ analogous to $D^{\delta} f$ as follows:

Let $f_n(z) = \frac{z}{(1-z)^{n+1}}, n \in \mathbb{N}_0$ and let $f_n^{(-1)}$ be defined such that

$$f_n(z) * f_n^{(-1)}(z) = \frac{z}{1-z}. \tag{1.9}$$

Then

$$I_n f = f_n^{(-1)} * f. \tag{1.10}$$

If $a \notin \mathbb{Z}_0^-$, then $L(a, c)$ has a continuous inverse $L(c, a)$ Clearly, $L(a, a)$ is the unit (identity) operator and

$$L(a, c) = L(a, b)L(b,c) = L(b, c)L(a, b). \tag{1.11}$$

This convolution operator $L(a, c)$ provides a convenient representation of differentiation and integration, that is, if $g(z) = zf'(z)$, then $g(z) = L(2, 1)f(z)$ and $f(z) = L(1, 2)\, g(z)$. For more details of this operator, see [7, 9, 16]. Using these facts, Noor integral operator can be expressed in term of $L(a, c)$ as

$$I_n f(z) = L(1, n+1)f(z), \quad n \in \mathbb{N}_0. \tag{1.12}$$

For applications of Noor integral operator, see [2, 5, 8, 11, 12, 15].
Let $P[A, B]$ be the class of functions h, analytic in E with $h(0) = 1$ and

$$h(z) \prec \frac{1+Az}{1+Bz}, \; -1 \le B < A \le 1, \tag{1.13}$$

Where the symbol $\prec$ stands for subordination. This class was introduced by Janowski [4]. Polatoğlu [17] defined the class $P[A, B, \alpha]$ as:
Let $P[A, B, \alpha]$ be the class of functions p analytic in E with $p(0) = 1$ and

$$p(z) \prec \frac{1+\left[(1-\alpha)A+\alpha B\right]z}{1+Bz}, -1 \le B < A \le 1, 0 \le \alpha < 1, \; z \in E. \tag{1.14}$$

From this, one can easily write $p \in P[A, B, \alpha]$ as

$$p(z) = (1-\alpha)h + \alpha, \; h \in P[A, B], \quad 0 \le \alpha < 1. \tag{1.15}$$

It is noted that $P[1,-1,0] \equiv P$, the well-known class of analytic functions in E with positive real part.

Using the concept of convolution operator $L(a, c)$ and the class $P[A, B, \alpha]$ we define the followings, which is the main motivation of this paper.

Definition 1.1: A function $f \in A$ is in the class $S^*_{a,c}[A,B,\alpha]$, if and only if,

$$\frac{z\left(L(a,c)f(z)\right)'}{L(a,c)f(z)} \in P[A,B,\alpha], \quad z \in E. \tag{1.16}$$

Definition 1.2: A function $f \in A$ is in the class $C_{a,c}[A, B, \alpha]$ if and only if,

$$\frac{\left(z\left(L(a,c)f(z)\right)'\right)'}{\left(L(a,c)f(z)\right)'} \in P[A,B,\alpha], \quad z \in E. \tag{1.17}$$

Now we discuss some special cases of the class $S^*_{a,c}[A,B,\alpha]$.

- $S^*_{a,c}[1,-1,0] = S^*_{a,c} \equiv S^*(a,c)$, The well-known class, introduced by Noor [13].
- $S^*_{a,a}[1,-1,0] = S^*$, The well-known class of starlike univalent functions in E.
- $S^*_{a,a}[A,B,\alpha] = S^*[A,B,\alpha]$, The class of janowski starlike functions of order á, introduced by Polatoğlu [17].

In a similar way, one can obtain several special cases of $C_{a,c}[A, B, \alpha]$ One can also show that

$$f \in C_{a,c}[A,B,\alpha] \iff z f' \in S^*_{a,c}[A,B,\alpha]. \tag{1.18}$$

Preliminary Lemmas: We need the following lemmas which will be used in our main results.

Lemma 2.1: Let $p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n \in P[A,B,\alpha]$. Then, for all $n \geq 1$,

$$|p_n| \leq (1-\alpha)(A-B). \tag{2.1}$$

This inequality is sharp.

The proof is immediate when we use the coefficient result for the class $P[A, B]$, (see [13]) and (1.15).

Lemma 2.2 [18]: Let $a_2 \geq a_1 > 0$. If $a_2 \geq > 2$ or $a_1 + a_2 \geq 3$, then the function $\phi(a_1, a_2)$ defined by (1.2) is convex univalent.

Lemma 2.4 [6]: Let $f \in C$ and $g \in S^*$. Then for any analytic function F with $F(0) = 1$ in E,

$$\frac{(f * Fg)}{(f * g)}(E) \subset \overline{co}\, F(E), \tag{2.2}$$

Where $\overline{co}\, F(E)$ denotes the convex hull of $F(E)$ (the smallest convex set which contains $F(E)$).

Main Results

Theorem 3.1: Let $f(z) = z + \sum_{n=2}^{\infty} b_n z^n \in S^*_{a,c}[A,B,\alpha]$. Then, for all $n \geq 2$,

$$|b_n| \leq \frac{|(c)_{n-1}|}{|(a)_{n-1}|} \prod_{k=0}^{n-2} \frac{|(1-\alpha)(A-B) - kB|}{k+1}. \tag{3.1}$$

This result is sharp and equality holds for the extremal function f_0 for which

$$L(a,c)f_0(z)=\begin{cases} z(1+Bz)^{\frac{(1-\alpha)(A-B)}{B}}, & B\neq 0,\\ ze^{(1-\alpha)Az}, & B=0.\end{cases} \tag{3.2}$$

Proof: Since $f(z)=z+\sum_{n=2}^{\infty}b_nz^n\in S^*_{a,c}[A,B,\alpha]$, there exists a function $p \in P[A, B, \alpha]$ such that

$$\frac{z\left(L(a,c)f(z)\right)'}{L(a,c)f(z)}=p(z). \tag{3.3}$$

If

$$p(z)=1+\sum_{n=1}^{\infty}p_nz^n,$$

then

$$z+\sum_{n=2}^{\infty}n\frac{(a)_{n-1}}{(c)_{n-1}}b_nz^n=\left(z+\sum_{n=2}^{\infty}\frac{(a)_{n-1}}{(c)_{n-1}}b_nz^n\right)\left(1+\sum_{n=1}^{\infty}p_nz^n\right).$$

Equating coefficients of z^n on both sides, we have

$$n\frac{(a)_{n-1}}{(c)_{n-1}}b_n=\frac{(a)_{n-1}}{(c)_{n-1}}b_n+\frac{(a)_{n-2}}{(c)_{n-2}}b_{n-1}p_1+\ldots+\frac{(a)_1}{(c)_1}b_2p_{n-2}+p_{n-1}$$

That is,

$$(n-1)\frac{\left|(a)_{n-1}\right|}{\left|(c)_{n-1}\right|}\left|b_n\right|\le\sum_{k=1}^{n-1}\frac{\left|(a)_{n-k-1}\right|}{\left|(c)_{n-k-1}\right|}\left|b_{n-k}\right|\left|p_k\right|,\qquad b_1=1.. \tag{3.4}$$

Using Lemma 2.1, we have

$$\left|b_n\right|\le\frac{1}{n-1}\frac{\left|(c)_{n-1}\right|}{\left|(a)_{n-1}\right|}(1-\alpha)(A-B)\sum_{k=1}^{n-1}\frac{\left|(a)_{k-1}\right|}{\left|(c)_{k-1}\right|}\left|b_k\right|. \tag{3.5}$$

Now we prove that

$$\frac{\left|(c)_{n-1}\right|}{\left|(a)_{n-1}\right|}\frac{(1-\alpha)(A-B)}{n-1}\sum_{k=1}^{n-1}\frac{\left|(a)_{k-1}\right|}{\left|(c)_{k-1}\right|}\left|b_k\right|\le\frac{\left|(c)_{n-1}\right|}{\left|(a)_{n-1}\right|}\prod_{k=0}^{n-2}\frac{\left|(1-\alpha)(A-B)-kB\right|}{k+1}. \tag{3.6}$$

For this, we use induction method,

For $n = 2$;

From (3.5), we have

$$\left|b_2\right|\le\frac{|c|}{|a|}(1-\alpha)(A-B). \tag{3.7}$$

From (3.1), we have

$$\left|b_2\right|\le\frac{|c|}{|a|}(1-\alpha)(A-B). \tag{3.8}$$

For $n = 3$;

From (3.5), we have

$$|b_3| \le \frac{1}{2}\frac{|(c)_2|}{|(a)_2|}(1-\alpha)(A-B)\left\{|b_1| + \frac{|a|}{|c|}|b_2|\right\}$$

$$\le \frac{1}{2}\frac{|(c)_2|}{|(a)_2|}(1-\alpha)(A-B)\{1+(1-\alpha)(A-B)\}. \tag{3.9}$$

From (3.1), we have

$$|b_3| \le \frac{|(c)_2|}{|(a)_2|}(1-\alpha)(A-B)\frac{|(1-\alpha)(A-B)-B|}{2}$$

$$\le \frac{1}{2}\frac{|(c)_2|}{|(a)_2|}(1-\alpha)(A-B)\{1+(1-\alpha)(A-B)\}. \tag{3.10}$$

Let the hypothesis be true for $n = m$. From (3.5), we have

$$|b_m| \le \frac{|(c)_{m-1}|}{|(a)_{m-1}|}\frac{(1-\alpha)(A-B)}{m-1}\sum_{k=1}^{m-1}\frac{|(a)_{k-1}|}{|(c)_{k-1}|}|b_k|. \tag{3.11}$$

From (3.1), we have

$$|b_m| \le \frac{|(c)_{m-1}|}{|(a)_{m-1}|}\prod_{k=0}^{m-2}\frac{|(1-\alpha)(A-B)-kB|}{k+1}$$

$$\le \frac{|(c)_{m-1}|}{|(a)_{m-1}|}\prod_{k=0}^{m-2}\frac{(1-\alpha)(A-B)+k}{k+1}. \tag{3.12}$$

By induction hypothesis, we have

$$\frac{|(c)_{m-1}|}{|(a)_{m-1}|}\frac{(1-\alpha)(A-B)}{m-1}\sum_{k=1}^{m-1}\frac{|(a)_{k-1}|}{|(c)_{k-1}|}|b_k| \le \frac{|(c)_{m-1}|}{|(a)_{m-1}|}\prod_{k=0}^{m-2}\frac{(1-\alpha)(A-B)+k}{k+1}. \tag{3.13}$$

Multiplying both sides by $\frac{|c-(m-1)|}{|a-(m-1)|}\frac{(1-\alpha)(A-B)+(m-1)}{m}$, we have

$$\frac{|(c)_m|}{|(a)_m|}\prod_{k=0}^{m-1}\frac{(1-\alpha)(A-B)+k}{k+1}$$

$$\ge \frac{|(c)_m|}{|(a)_m|}\frac{(1-\alpha)(A-B)}{m-1}\left\{\frac{(1-\alpha)(A-B)+(m-1)}{m}\right\}\sum_{k=1}^{m-1}\frac{|(a)_{k-1}|}{|(c)_{k-1}|}|b_k|$$

$$= \frac{|c-(m-1)|}{|a-(m-1)|}\frac{(1-\alpha)(A-B)}{m}\left\{\frac{|(c)_{m-1}|}{|(a)_{m-1}|}\frac{(1-\alpha)(A-B)}{m-1}\sum_{k=1}^{m-1}\frac{|(a)_{k-1}|}{|(c)_{k-1}|}|b_k|\right\}$$

$$+ \frac{|(c)_m|}{|(a)_m|}\frac{(1-\alpha)(A-B)}{m}\sum_{k=1}^{m-1}\frac{|(a)_{k-1}|}{|(c)_{k-1}|}|b_k|$$

$$\ge \frac{|c-(m-1)|}{|a-(m-1)|}\frac{(1-\alpha)(A-B)}{m}|b_m| + \frac{|(c)_m|}{|(a)_m|}\frac{(1-\alpha)(A-B)}{m}\sum_{k=1}^{m-1}\frac{|(a)_{k-1}|}{|(c)_{k-1}|}|b_k|$$

That is,

$$= \frac{\left|(c)_m\right|}{\left|(a)_m\right|}\frac{(1-\alpha)(A-B)}{m}\left\{\frac{\left|(a)_{m-1}\right|}{\left|(c)_{m-1}\right|}\left|b_m\right| + \sum_{k=1}^{m-1}\frac{\left|(a)_{k-1}\right|}{\left|(c)_{k-1}\right|}\left|b_k\right|\right\}. \tag{3.15}$$

Which shows that the inequality (3.6) is true for $n = m + 1$. Hence the required result. ■

Special Case: As a special case, we note that, when $A = 1$ $B = -1$, $B = -1$, $\alpha = 0$ and $a = c$, we obtain the well known coefficient bound for starlike functions.

Theorem 3.2: If $f \in S^*_{a,c}[A,B,\alpha]$, then for $|z| = r < 1$,

$$\left.\begin{matrix} r(1-Br)^{\frac{(1-\alpha)(A-B)}{B}}, & B \neq 0, \\ re^{-(1-\alpha)Ar}, & B = 0 \end{matrix}\right\} \leq \left|L(a,c)f(z)\right| \leq \left\{\begin{matrix} r(1+Br)^{\frac{(1-\alpha)(A-B)}{B}}, & B \neq 0, \\ re^{(1-\alpha)Ar}, & B = 0. \end{matrix}\right. \tag{3.16}$$

This result is sharp.
The proof is immediate and we omit the details.

Theorem 3.3: Let $a_2 \geq a_1 > 0,\ c \in \mathbb{R} \setminus \mathbb{Z}_0^-$. If $a_2 \geq 2$ or $a_1 + a_2 \geq 3$, then

- $S^*_{a_2,c}[A,B,\alpha] \subset S^*_{a_1,c}[A,B,\alpha]$.
- $C_{a_2,c}[A,B,\alpha] \subset C_{a_1,c}[A,B,\alpha]$.

Proof:

- Let $f \in S^*_{a_2,c}[A,B,\alpha]$. Then it follows that

$$\frac{z\left(L(a_2,c)f(z)\right)'}{L(a_2,c)f(z)} = F(z) \in P[A,B,\alpha]. \tag{3.17}$$

Now, from (1.5) and (1.11), we have

$$\begin{aligned}\frac{z\left(L(a_1,c)f(z)\right)'}{L(a_1,c)f(z)} &= \frac{z\left(\phi(a_1,c)(z)*f(z)\right)'}{\phi(a_1,c)(z)*f(z)} \\ &= \frac{z\left[\phi(a_1,a_2)(z)*\phi(a_2,c)(z)*f(z)\right]'}{\phi(a_1,a_2)(z)*\phi(a_2,c)(z)*f(z)} \\ &= \frac{\phi(a_1,a_2)(z)*z\left(L(a_2,c)f(z)\right)'}{\phi(a_1,a_2)(z)*L(a_2,c)f(z)} \\ &= \frac{\phi(a_1,a_2)(z)*F(z)L(a_2,c)f(z)}{\phi(a_1,a_2)(z)*L(a_2,c)f(z)}.\end{aligned}$$

Since $\phi(a_1,a_2) \in C$, by Lemma 2.2 and $L(a_2,c)f \in S^*[A,B,\alpha] \subset S^*$, so by using Lemma 2.3, we have

$$\frac{z\left(L(a_1,c)f(z)\right)'}{L(a_1,c)f(z)} \in P[A,B,\alpha]. \tag{3.18}$$

This implies that $f \in S^*_{a_1, c}[A,B,\alpha]$ and the proof if (i) is complete.

- Using the relation given in (1.18),

$$\begin{aligned} f \in C_{a_2, c}[A,B,\alpha] &\Leftrightarrow z f'(z) \in S^*_{a_2, c}[A,B,\alpha] \\ &\Rightarrow z f'(z) \in S^*_{a_1, c}[A,B,\alpha] \\ &\Leftrightarrow f(z) \in C_{a_1, c}[A,B,\alpha], \end{aligned}$$

and this proves (ii). ∎

Theorem 3.4: Let $a \in \mathbb{R}$, $c^2 \geq c_1 > 0$. If $c_2 \geq 2$ or $c_1 + c_2 \geq 3$, then

- $S^*_{a, c_1}[A,B,\alpha] \subset S^*_{a, c_2}[A,B,\alpha]$.
- $C_{a, c_1}[A,B,\alpha] \subset C_{a, c_2}[A,B,\alpha]$.

Proof:

- Let $f \in S^*_{a, c_1}[A,B,\alpha]$. Then it follows that

$$\frac{z\left(L(a, c_1) f(z)\right)'}{L(a, c_1) f(z)} = F(z) \in P[A,B,\alpha].$$

Now, from (1.5) and (1.11), we have

$$\begin{aligned} \frac{z\left(L(a, c_2) f(z)\right)'}{L(a, c_2) f(z)} &= \frac{z\left(\phi(a, c_2)(z) * f(z)\right)'}{\phi(a, c_2)(z) * f(z)} \\ &= \frac{z\left[\phi(a, c_1)(z) * \phi(c_1, c_2)(z) * f(z)\right]'}{\phi(a, c_1)(z) * \phi(c_1, c_2)(z) * f(z)} \\ &= \frac{\phi(c_1, c_2)(z) * z\left(L(a, c_1) f(z)\right)'}{\phi(c_1, c_2)(z) * L(a, c_1) f(z)} \\ &= \frac{\phi(c_1, c_2)(z) * F(z) L(a, c_1) f(z)}{\phi(c_1, c_2)(z) * L(a, c_1) f(z)}. \end{aligned}$$

Since $\phi(c_1, c_2) \in C$, by Lemma 2.2 and $L(a, c_1) f \in S^*[A, B, \alpha] \subset S^*$, so by using Lemma 2.3, we have

$$\frac{z\left(L(a, c_2) f(z)\right)'}{L(a, c_2) f(z)} \in P[A,B,\alpha]. \tag{3.20}$$

This implies that $f \in S^*_{a,c_2}[A,B,\alpha]$ and this completes the proof.

- This can be proved in similar way as in Theorem 3.3(ii). ■

Corollary 3.5: Let $a_2 \geq a_1 > 0$, $c_2 \geq c_1 > 0$. If $a_2 \geq \min[2, 3-a_1]$and $c_2 \geq \min[2, 3-c_1]$, then

- $S^*_{a_2,c_1}[A,B,\alpha] \subset S^*_{a_2,c_2}[A,B,\alpha] \subset S^*_{a_1,c_2}[A,B,\alpha]$.
- $C_{a_2,c_1}[A,B,\alpha] \subset C_{a_2,c_2}[A,B,\alpha] \subset C_{a_1,c_2}[A,B,\alpha]$.

We shall now prove that the classes $S^*_{a,c}[A,B,\alpha]$ and $C_{a,c}[A, B, \alpha]$ are invariant under convolution with convex functions.

Theorem 3.6: Let $a > 0,\ c \in \mathbb{C} \setminus \mathbb{Z}_0^-$ and let g be convex in E. Then

- $f \in S^*_{a,c}[A,B,\alpha] \Rightarrow (f*g) \in S^*_{a,c}[A,B,\alpha]$.
- $f \in C_{a,c}[A,B,\alpha] \Rightarrow (f*g) \in C_{a,c}[A,B,\alpha]$.

Proof:

- Let $f \in S^*_{a,c}[A,B,\alpha]$. Then, we have

$$\frac{z\left(L(a,c)f(z)\right)'}{L(a,c)f(z)} = F(z) \in P[A,B,\alpha].$$

Now, for $g \in C$, consider

$$\begin{aligned}\frac{z\left(L(a,c)(f*g)(z)\right)'}{L(a,c)(f*g)(z)} &= \frac{z\left(\phi(a,c)*f(z)*g(z)\right)'}{\phi(a,c)*f(z)*g(z)} \\ &= \frac{g(z)*z\left(L(a,c)f(z)\right)'}{g(z)*L(a,c)f(z)} \\ &= \frac{g(z)*F(z).\left(L(a,c)f(z)\right)}{g(z)*L(a,c)f(z)}.\end{aligned}$$

We now use Lemma 2.3 to have $(f*g) \in S^*_{a,c}[A,B,\alpha]$.

- Proof of (ii) follows in similar way.

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