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Fuzzy Method for Identification of Aggregate Weights in Ordered Weighted Averaging Operators

Sadegh Derakhshandeh and Nasser Mikaeilvand

Department of Mathematics, Islamic Azad University - Ardabil Branch, Ardabil, Iran

Abstract: In 1988 Ronald R. Yager [2] introduced a new aggregation technique based on the ordered weighted averaging (OWA) operators. The goal of this paper is to present a short survey of OWA operators and illustrate their applicability for risk processing. In this article, we present new method for obtain of weights w_i in OWA and we use OWA operators for decision making on selection of counter-measures for reduction of risks.

Key words: Decision making · OWA operator · Probabilities · Risk modification

INTRODUCTION

Provision of information security in modern information systems is based on information security risk management. Risk management process contains risk analysis, risk assessment, risk evaluation, risk processing and informing the users about risks [1]. OWA operators fill the gap between the operators Min and Max and can be verified that are commutative, increasing monotonous and idempotent, but in general not associative [2].

A process of selection and realization of actions by modification of risk is named as risk processing. Risk processing actions can include acceptance, rejection, reduction, transfer or insurance of risk.

One of the processing mechanisms of information security risks is reduction of risks by using correct selection of counter-measures against threats. While choosing counter-measures it's necessary to consider several criterions. In this article ordered weighted averaging operators are used for risks processing of information security [3]. OWA operators consider decision making person's behavior (risk avoidance or risk acceptance) and interaction among criterions and from this perspective OWA method has a supremacy in comparison with other multi-criteria decision making models (Multi Criteria Decision Making), also TOPSIS (Technique for Order Preferences by Similarity to Ideal Solution) and AHP (Analytic Hierarchy Process).

A very efficient for information combination method OWA was suggested by R. Yager [3]. Since then OWA operators are studied from different aspects and applied in engineering and different fields of artificial intellect [4-9].

OWA Operators

Definition: An OWA operator of dimension n with an associated vector $W = (w_1,...w_n,)$ is a mapping $F : R^n \to R$ defined as

$$F(a_1,...,a_n) = \sum_{j=1}^{n} w_j b_j \tag{1}$$

Where b_j is the j-th largest element of the of the bag

$$< a_1,...,a_n > w_j \in [0,1] \sum_{j=1}^{n} w_j = 1$$

For example, the value of OWA operator which is given with the vector $W = (0.4{:}\ 0.3{:}\ 0.2{:}\ 0.1)^T$ for the bag <0.7, 1.0, 0.2, 0.6> will be calculated as following:

$$F(0.7, 1.0, 0.2, 0.6) = 0.4 \times 1.0 \times + 0.3 \times 0.7 + 0.2 \times 0.6 + 0.1 \times 0.2 = 0.75$$

The fundamental aspect of this operator is the re-ordering step, in particular an aggregate a_i is not associated with a particular weight w_i but rather a weight is associated with a particular ordered position of aggregate.

It is noted that different OWA operators are distinguished by their weighting function. R. Yager pointed out three important types of OWA operators:

Corresponding Author: Sadegh Derakhshandeh, Department of Mathematics, Islamic Azad University - Ardabil Branch, Ardabil, Iran, E-mail: smdk364@yahoo.com.

$F^*: W = W^* = (1; 0; ...; 0)^T$ And $F^*(a_1,...,a_n) = \max\{a_1,...,a_n\}$

$F_*: W = W_* = (0; 0; ...; 1)^T$ And $F_*(a_1,...,a_n) = \min\{a_1,...,a_n\}$

$F_{mean}: W = W_A = (1/n; 1/n; ...; 1/n)^T$

And $F_{mean}(a_1,...,a_n) = \frac{a_1 + ... + a_n\}}{n}$

There are several important properties (commutative, monotonicity, idempotency and limitation) of OWA operators. Let's have a short look on limitation characteristics. Each OWA operator meets an inequality

$$F_*(a_1,...,a_n) \le F(a_1,...,a_n) < F^*(a_1,...,a_n),$$

In other words, value of operator is between $\{a_1,...,a_n\}$ and $\{a_1,...,a_n\}$,.

OWA operators have an important parameter identified by *orness* function; it can be also defined as a degree of risk acceptance. R.Yager defined *orness* function for W weight vector as following [3]:

$$orness(W) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i. \quad (2)$$

It can be shown that, 0≤. A little value of ≤1 illustrates risk avoidance, big value illustrates order acceptance of risk.

As we can see from definition of OWA operator, identification of aggregate weights w_i is an essential issue [10]. There are several methods for calculation of aggregate weights; the most used is a method suggested by R. Yager based on linguistic quantifier. Decision makers identify Q linguistic quantifier (for example, "many"). Linguistic quantifier Q can be illustrated as a fuzzy subset of I single interval, for every $r \in I$ value of $Q(r)$ shows in what degree r meets a concept marked as Q. If Q is a regularly growing monotone qualifier, then aggregate weights can be calculated with following formula:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad (3)$$

Following formula is widely used as Q linguistic quantifier:

$$Q(r) = r^{\alpha}, \alpha \ge 0 \quad (4)$$

The *orness* function of calculated aggregate weights is as following:

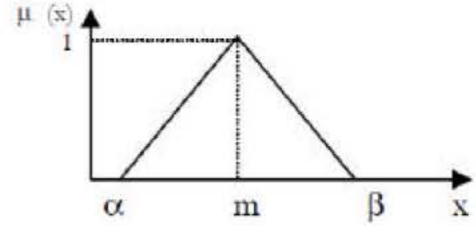

Fig. 1 Triangular numbers

$$orness(w) = \int_0^1 Q(r)dr = \int_0^1 r^{\alpha}dr = \frac{1}{\alpha+1} \quad (5)$$

If $a > 1$, it will be $orness(w) < 0.5$ and it illustrates the avoidance of decision makers from risk. If $a = 1$, it will be $orness(w)$ 0.5 and illustrates neutrality of decision maker against risk. If $a > 1$, it will be $orness(w)$ 0.5 and it illustrates secure risk acceptance of decision maker.

Fuzzy Owa Approach for Risk Treatment: In this section we produce a new approach for identification of aggregate weights w_i in OWA. And we use it in risk treatment process.

- Asumme that we have n experts in security team (D_1, D_2,...D_n) according to k (C_1, C_2,....C_k) criteria security team have to evaluate m (A_1, A_2,...A_M) alternative .
- Security team use linguistic and triangle numbers for evaluation of criteria. triangular numbers (a, m, β) and member function[14]:

$$\mu(x) = \begin{cases} \frac{x-\alpha}{m-\alpha} \to\to a \le x \le m \\ \frac{x-\beta}{m-\beta} \to\to m \le x \le \beta \\ 0 \to\to o.w \end{cases} \quad (6)$$

Security Team Use Triangular Average Formula:

$$\hat{w}_{c_t} = \frac{w_{c_{t1}} + w_{c_{t2}} + ... + w_{c_{tn}}}{n}$$
$$= \left(\frac{1}{n}\sum_{i=1}^{n}\alpha^{(t)}, \frac{1}{n}\sum_{i=1}^{n}m^{(t)}, \frac{1}{n}\sum_{i=1}^{n}\beta^{(t)}\right) \quad (7)$$

Here w_{ci} is value of expert i for criteria t and $\hat{w}_{c_t}$ is average value of criteria t.

Attention that value of $\hat{w}_{c_t}$ is triangular and we have to change to normal number with using formula (9):

$$
\begin{aligned}
&if\,|m-\alpha| = |m-\beta| \rightarrow w_{C_t} = \frac{1}{n}\sum_{i=1}^{n} m^{(t)} \\
&if\,|m-\alpha| > |m-\beta| \rightarrow w_{C_t} = \frac{\alpha^{(t)} + m^{(t)} + \beta^{(t)}}{3} \\
&if\,|m-\alpha| < |m-\beta| \rightarrow w_{C_t} = \frac{\alpha^{(t)} + 4m^{(t)} + \beta^{(t)}}{6}
\end{aligned} \tag{8}
$$

According to definition of OWA operators $w_j \in [0,1], \sum_{j=1}^{n} w_j = 1$ now we have to find normal value of w_{C_t}. And we use (9) for this object:

$$Nw_{C_t} = \frac{w_{C_t}}{\sum_{t=1}^{k} w_{C_t}} \tag{9}$$

RESULTS

Decision making person's behavior (risk avoidance or risk acceptance) is considered by OWA operators. Identification of aggregate weights w_i is an essential issue. In this paper we present new method for obtain of weights w_i in OWA. This method considers experts opinion and it can solve main problems in OWA method.

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